A Multi-Depot Logistics Distribution Routing Model for Unexpected Events

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Abstract

This paper discusses a multi-depot vehicle routing problem for dispatching vehicles in multi-depots to send relief material to the affected areas in unexpected events. Due to the occurrence of the unexpected events, people's demands in the affected areas and the vehicles' travel times on roads lack historical data and are given by experts' estimations. Uncertain variables are used to describe these parameters. By using uncertainty theory, a multi-depot logistics distribution routing model is developed. To solve the problem, the equivalent of the proposed model is also presented. As an illustration, an example is also presented and discussed.

Keywords: Multi-depot vehicle routing problem, emergency logistics, uncertain programming, uncertainty theory, genetic algorithm.

1 Introduction

Unexpected destructive events always cause great losses to human beings. Logistics plays an important role in mitigating pains and losses in these unexpected events and has attracted many scholars' attention. The first study on logistics models for unexpected events was in the late 1970s aiming at

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handling the maritime disasters in the late 1960s and 1970s. Most early models were developed for dealing with oil spills or maritime disasters, e.g., Charnes et al. (1976), Psaraftis and Zogas (1985), etc. Later, research has extended to explore more logistics distribution problems concerning a variety of disaster events such as earthquakes (Viswanath and Peeta, 2003; Linet et al., 2004), hurricanes (Lodree and Taskin, 2009; Horner and Down, 2010), floods (Garrido, et al., 2015), and man-made catastrophes (Wang, et al., 2012).

In the past, studies concerning logistics distribution for unexpected events mainly focused on addressing the specialness of the problem from normal logistics distribution and the way to handle it. Some examples include Yuan and Wang (2009) and Zhang, et al. (2013) who studied distribution routing problems for logistics with travel speed varying with time due to occurrence of the disasters, Zhang et al. (2012) who proposed a mixed integer programming model to handle primary and secondary disasters, and Liberatore et al. (2014) who proposed a method for repairing a broken transportation network for distribution of relief, etc. However, the authors argue that the greatest speciality of the problem should lie in the feature that no past data about the parameters are available because the events are unexpected and unusual. Then in the situation, all available are experts' estimations about the parameters. Those estimations cannot be exact numbers. Since men's estimations are not random in nature, men's estimations can be quite different from probabilities either (Tversky and Kahneman, 1974). In order to model men's estimations, Liu (2007) proposed an uncertainty theory and further refined it (Liu, 2010). Along with the introduction and development of uncertainty theory, the application of the theory has also been researched. Some important works include Liu (2009b) who proposed the uncertain programming, and Huang (2010) who proposed an uncertain portfolio selection theory by introducing uncertainty theory into portfolio selection. So far, uncertainty theory has been applied to solve optimization problems concerning belief degrees in a variety of fields, e.g., in shortest path (Gao, 2011), inventory (Qin and Kar, 2013), production planning (Ning, et al., 2013), finance (Huang and Zhao, 2014a), capital budgeting (Zhang, et al., 2011; Zhang, et al., 2015), project selection and scheduling (Huang and Zhao, 2014b), and facility location problems (Gao, 2012; Huang and Di, 2015), etc. In this paper, we will use uncertainty theory to explore a multi-depot logistics distribution problem in unexpected events in which travel times on road and the demands of the affected areas are uncertain and given by experts' estimations.

The paper proceeds as follows. For easy understanding of the paper, we will briefly review the fundamentals of uncertainty theory in Section 2. In Section 3 we will propose a multi-depot logistics distribution routing model

for unexpected events. In order to solve the problem, we will also present the model equivalent in the section. Finally, we will conclude the paper.

2 Fundamentals of Uncertainty Theory

Uncertainty theory is a branch of mathematics based on the following four axioms.

Definition 1 Let L be a σ -algebra over a nonempty set Γ . Each element $\Lambda \in L$ is called an event. A set function $M\{\Lambda\}$ is called an uncertain measure if it satisfies the following four axioms (Liu, 2007; 2009a):

- (i) $M\{\Gamma\} = 1$.
- (ii) $M\{\Lambda\} + M\{\Lambda^c\} = 1$.
- (iii) For every countable sequence of events $\{\Lambda_i\}$, we have

$$M\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} M\{\Lambda_i\}.$$

The triplet (Γ, L, M) is called an uncertainty space.

- (iv) Let (Γ_k, L_k, M_k) be uncertainty spaces for $k = 1, 2, \dots$. The product uncertain measure is

$$M\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} M_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from L_k for $k = 1, 2, \dots$, respectively.

Definition 2 (Liu, 2007) An uncertain variable is a measurable function ξ from an uncertainty space (Γ, L, M) to the set of real numbers.

Definition 3 (Liu, 2007) The uncertainty distribution $\Phi : \Re \rightarrow [0, 1]$ of an uncertain variable ξ is defined by

$$\Phi(t) = M\{\xi \leq t\}.$$

In application, an uncertain variable is characterized by an uncertainty distribution function. Regarding the way to obtain the uncertainty distribution function via experts' estimations, please refer to Liu (2010), Huang (2012) and Huang and Zhao (2016).

Definition 4 (Liu, 2010) An uncertainty distribution $\Phi(t)$ is said to be regular if it is a continuous and strictly increasing function with respect to t at which $0 < \Phi(t) < 1$, and

$$\lim_{t \rightarrow -\infty} \Phi(t) = 0, \quad \lim_{t \rightarrow +\infty} \Phi(t) = 1.$$

To measure the size of an uncertain variable, the expected value of the uncertain variable is defined as follows.

Definition 5 (Liu, 2007) The expected value of an uncertain variable ξ is defined by

$$E[\xi] = \int_0^\infty M\{\xi \geq r\}dr - \int_{-\infty}^0 M\{\xi \leq r\}dr \quad (1)$$

provided that at least one of the two integrals is finite.

The operational law of the uncertain variables is given by Liu (2010) as follows:

Theorem 1 (Liu, 2010) Let $\xi_1, \xi_2, \dots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \dots, \Phi_n$, respectively. If $f(t_1, t_2, \dots, t_n)$ is strictly increasing with respect to $t_1, t_2, \dots, t_n$, then

$$\xi = f(\xi_1, \xi_2, \dots, \xi_n)$$

is an uncertain variable that has the following inverse uncertainty distribution function

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \dots, \Phi_n^{-1}(\alpha)), \quad 0 < \alpha < 1. \quad (2)$$

Theorem 2 (Liu, 2010) Let ξ and η be independent uncertain variables with finite expected values. Then for any real numbers a and b , we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]. \quad (3)$$

3 A Multi-Depot Logistics Distribution Routing Model for Unexpected Events

In the problem, suppose we have p numbers of depots, each having m vehicles. The decision maker (DM) needs to dispatch vehicles to distribute relief material from the p depots to the n affected areas. The affected areas' demands

and the travel times on roads are uncertain and lack historical data because of the impact of the accidents. Therefore, these parameter values are estimated by experts and treated as uncertain variables in the paper. Assume that (1) total vehicles in p depots can fully serve all the n numbers of affected areas but none of one depot can individually serve them well; (2) each vehicle begins and ends both at its own depot; (3) the vehicles will leave for the affected areas immediately once they receive the order so that they can send the material to the affected areas as early as possible; (4) the relief material will be unloaded immediately when it arrives at the affected areas; (5) each affected area will be served by one and only one vehicle; (6) a vehicle will serve only one route on which there may be several affected areas.

For expression convenience, the model parameters are introduced as follows.

$D_1, D_2, \dots, D_p$: the depots;

$i = 1, 2, \dots, n$: the affected areas;

$j = 1, 2, \dots, m, m+1, \dots, pm$: vehicles, among which the vehicles $1, 2, \dots, m$ belong to the 1st depot, the vehicles $m+1, m+2, \dots, 2m$ belong to the 2nd depot, $\dots$, and the vehicles $(p-1)m+1, (p-1)m+2, \dots, pm$ belong to the p -th depot;

Q_j : the capacities of vehicles $j, j = 1, 2, \dots, m, \dots, pm$;

S_{D_k} : the inventory levels of the relief material of the depots $D_k, k = 1, 2, \dots, p$;

$\tilde{T}_{ik}$: the travel times from the depots i to the affected areas $k, i = D_1, D_2, \dots, D_p, k = 1, 2, \dots, n$, which are uncertain variables;

t_{ik} : the travel times from the areas i to $k, i, k = 1, 2, \dots, n$, which are uncertain variables;

$\tilde{d}_i$: the demands at affected areas $i, i = 1, 2, \dots, n$, which are uncertain variables;

U_i : the unloading times at affected areas $i, i = 1, 2, \dots, n$;

We use two decision vectors $\mathbf{x}$ and $\mathbf{y}$ to describe a routing plan. The vector $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is an integer vector representing n affected areas with $1 \leq x_i \leq n$ and $x_i \neq x_k$ for all $i \neq k, i, k = 1, 2, \dots, n$. That is, $\mathbf{x}$ is the sequence of a rearrangement of $\{1, 2, \dots, n\}$. The vector $\mathbf{y} = (y_1, y_2, \dots, y_m, y_{m+1}, \dots, y_{pm-1})$ is another integer vector with $y_0 \equiv 0 \leq y_1 \leq y_2 \leq \dots \leq y_m \leq y_{m+1} \leq \dots \leq y_{pm-1} \leq n \equiv y_{pm}$. A routing plan is thus determined as follows: For each $j, 1 \leq j \leq pm$, if $y_j = y_{j-1}$, the vehicle j is not used; if $y_j > y_{j-1}$ and $(k-1)m < j \leq km$ for $1 \leq k \leq p$ the vehicle j is used and starts at the k -th depot, and the routing of the vehicle j is

$$D_k \rightarrow x_{y_{j-1}+1} \rightarrow x_{y_{j-1}+2} \rightarrow \cdots \rightarrow x_{y_j} \rightarrow D_k.$$

It is seen that the above two decision vectors $\mathbf{x}$ and $\mathbf{y}$ together ensure that (1) each vehicle will be used at most one time; (2) each vehicle routing begins and ends at this vehicle's own depot; (3) each area will be served by one and only one vehicle; and (4) no sub-routing exists.

Let $f_i(\mathbf{x}, \mathbf{y})$ denote the arrival times at areas i under the routing plan of $(\mathbf{x}, \mathbf{y})$. Then if vehicle j at depot k is used, i.e., if $y_j > y_{j-1}$ and $(k-1)m < j \leq km$ for $1 \leq k \leq p$, the arrival time of this vehicle at the area $x_{y_{j-1}+1}$ is

$$f_{x_{y_{j-1}+1}}(\mathbf{x}, \mathbf{y}) = \tilde{T}_{D_k x_{y_{j-1}+1}}, \quad (4)$$

and the arrival times at the areas $x_{y_{j-1}+l}$, $2 \leq l \leq y_j - y_{j-1}$ are

$$\begin{aligned} f_{x_{y_{j-1}+l}}(\mathbf{x}, \mathbf{y}) &= f_{x_{y_{j-1}+l-1}}(\mathbf{x}, \mathbf{y}) + U_{x_{y_{j-1}+l-1}} \\ &\quad + \tilde{t}_{x_{y_{j-1}+l-1} x_{y_{j-1}+l}}. \end{aligned} \quad (5)$$

Let u_i be the unloading time of unit material at areas i , $i = 1, 2, \dots, n$. Then the Equation (5) becomes

$$\begin{aligned} f_{x_{y_{j-1}+l}}(\mathbf{x}, \mathbf{y}) &= f_{x_{y_{j-1}+l-1}}(\mathbf{x}, \mathbf{y}) + \tilde{d}_{x_{y_{j-1}+l-1}} \cdot u_{x_{y_{j-1}+l-1}} \\ &\quad + \tilde{t}_{x_{y_{j-1}+l-1} x_{y_{j-1}+l}}. \end{aligned} \quad (6)$$

Thus the total arrival time at the n areas is

$$T = \sum_{i=1}^n f_i(\mathbf{x}, \mathbf{y}). \quad (7)$$

Since the $\tilde{T}_{ik}$'s, $\tilde{t}_{ik}$'s and u_i 's in (4) and (6) are uncertain variables, the total arrival time T is also uncertain and cannot be minimized directly. Then the DM can set the objective as minimizing the expected value of the total arrival time, i.e., the DM can set the objective as

$$\min E \left[\sum_{i=1}^n f_i(\mathbf{x}, \mathbf{y}) \right]$$

where $f_i(\mathbf{x}, \mathbf{y})$ are determined by Equations (4) and (6).

When making routing plan, the DM must consider the vehicle capacity constraints and the depot inventory level constraints. Since the demands $\tilde{d}_i$ at

areas $i, i = 1, 2, \dots, n$, are uncertain, the DM can ask that the total demands in a route should not exceed the capacity of the vehicle serving the route at a preset high enough confidence level β . That is, we can express the vehicle capacity constraints mathematically as follows,

$$M\left\{\sum_{i=y_{j-1}+1}^{y_j} \tilde{d}_{x_i} \leq Q_j\right\} \geq \beta, \quad j = 1, 2, \dots, pm. \quad (8)$$

In the meantime, it should be ensured that the depot inventory level of the relief material can satisfy the amount of the material sent out by the vehicles from the depot at a preset high enough confidence level β' . That is,

$$M\left\{\sum_{j=(k-1)m+1}^{km} \sum_{i=y_{j-1}+1}^{y_j} \tilde{d}_{x_i} \leq S_{D_k}\right\} \geq \beta', \quad k = 1, 2, \dots, p. \quad (9)$$

Thus, if the DM wants to minimize the expected value of the total arrival time at all affected areas subject to the vehicle capacity constraints and the depot inventory level constraints, he or she can dispatch the vehicles in p depots according the following model:

$$\left\{ \begin{array}{l} \min E\left[\sum_{i=1}^n f_i(\mathbf{x}, \mathbf{y})\right] \\ \text{subject to:} \\ M\left\{\sum_{i=y_{j-1}+1}^{y_j} \tilde{d}_{x_i} \leq Q_j\right\} \geq \beta, \quad j = 1, 2, \dots, pm \\ M\left\{\sum_{j=(k-1)m+1}^{km} \sum_{i=y_{j-1}+1}^{y_j} \tilde{d}_{x_i} \leq S_{D_k}\right\} \geq \beta', \\ k = 1, 2, \dots, p \\ 1 \leq x_i \leq n, \quad i = 1, 2, \dots, n, \quad x_i \neq x_k, \\ i \neq k, i, k = 1, 2, \dots, n \\ 0 \leq y_1 \leq y_2 \leq \dots \leq y_{pm-1} \leq n \\ x_i, y_j, i = 1, 2, \dots, n, j = 1, 2, \dots, pm - 1, \text{ integers} \end{array} \right. \quad (10)$$

where $f_i(\mathbf{x}, \mathbf{y})$ are determined by Equations (4) and (6) for $i = 1, 2, \dots, n$.

Theorem 3 Suppose the travel times $\tilde{T}_{ik}, i = 1, 2, \dots, n, k = 1, 2, \dots, p$, and $\tilde{t}_{ik}, i, k = 1, 2, \dots, n$, and the uncertain demands $\tilde{d}_i$ at areas $i, i = 1, 2, \dots, n$, are independent and have regular uncertainty distributions. Let $e_{D_k, i}, k = 1, 2, \dots, p, i = 1, 2, \dots, n$ denote the expected values of the travel times between the depots D_k and the affected areas i and $\mu_{ij}, i \neq j, i, j = 1, 2, \dots, n$ the expected values of the travel times between the affected areas i and j . Let $e_{d_i}, i = 1, 2, \dots, n$ the expected values of the demands at areas i , and Φ_i denote the regular uncertainty distributions of the uncertain demands $\tilde{d}_i$ for $i = 1, 2, \dots, n$. Then the model (10) is equivalent to the following model:

$$\left\{ \begin{array}{l} \min \sum_{i=1}^n E[f_i(\mathbf{x}, \mathbf{y})] \\ \text{subject to:} \\ \sum_{i=y_{j-1}+1}^{y_j} \Phi_{x_i}^{-1}(\beta) \leq Q_j, \quad j = 1, 2, \dots, pm \\ \sum_{j=(k-1)m+1}^{km} \sum_{i=y_{j-1}+1}^{y_j} \Phi_{x_i}^{-1}(\beta') \leq S_{D_k}, \quad k = 1, 2, \dots, p \\ 1 \leq x_i \leq n, \quad i = 1, 2, \dots, n, \quad x_i \neq x_k, \\ i \neq k, \quad i, k = 1, 2, \dots, n \\ 0 \leq y_1 \leq y_2 \leq \dots \leq y_{pm-1} \leq n \\ x_i, y_j, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, pm-1, \text{ integers} \end{array} \right. \quad (11)$$

where $E[f_i(\mathbf{x}, \mathbf{y})]$ are obtained from the following equations

$$\begin{aligned} E[f_{x_{y_{j-1}+1}}(\mathbf{x}, \mathbf{y})] &= e_{D_k, x_{y_{j-1}+1}}, \quad k = 1, 2, \dots, p, \\ (k-1)m &< j \leq km \end{aligned} \quad (12)$$

and for $2 \leq l \leq y_j - y_{j-1}$,

$$\begin{aligned} E[f_{x_{y_{j-1}+l}}(\mathbf{x}, \mathbf{y})] &= E[f_{x_{y_{j-1}+l-1}}(\mathbf{x}, \mathbf{y})] + \\ &u_{x_{y_{j-1}+l-1}} \cdot e_{x_{y_{j-1}+l-1}} + \mu_{x_{y_{j-1}+l-1} x_{y_{j-1}+l}}. \end{aligned} \quad (13)$$

Proof: From Equations (4) and (6) we know that the formula $\sum_{i=1}^n f_i(\mathbf{x}, \mathbf{y})$ is the sum function of independent uncertain variables $\tilde{T}_{ik}$ for $i = D_1, D_2, \dots, D_p, k = 1, 2, \dots, n$, $\tilde{t}_{ik}$ for $i, k = 1, 2, \dots, n$, and $\tilde{d}_i$ for $i = 1, 2, \dots, n$. Therefore, from Theorem 2 we get that $E[\sum_{i=1}^n f_i(\mathbf{x}, \mathbf{y})] = \sum_{i=1}^n E[f_i(\mathbf{x}, \mathbf{y})]$.

Since $\sum_{i=y_{j-1}+1}^{y_j} \tilde{d}_{x_i}$ is strictly increasing with respect to $\tilde{d}_{x_i}$, according to Theorem 1 we can get that the inverse uncertainty distribution of the total demand amount served by vehicle j , i.e., the inverse uncertainty distribution of $\sum_{i=y_{j-1}+1}^{y_j} \tilde{d}_{x_i}$ at β is

$$\sum_{i=y_{j-1}+1}^{y_j} \Phi_{x_i}^{-1}(\beta), \quad j = 1, 2, \dots, pm.$$

Then from the increasing property of the uncertainty measure we can get that the vehicle capacity constraint (8) can be converted into the following form:

$$\sum_{i=y_{j-1}+1}^{y_j} \Phi_{x_i}^{-1}(\beta) \leq Q_j, \quad j = 1, 2, \dots, pm.$$

Similarly, the depot inventory level constraint (9) can be converted into the following form:

$$\sum_{j=(k-1)m+1}^{km} \sum_{i=y_{j-1}+1}^{y_j} \Phi_{x_i}^{-1}(\beta') \leq S_{D_k}, \quad k = 1, 2, \dots, p.$$

Thus, the theorem is proved.

Huang and Song (2016) provided a cellular integrated genetic algorithm (CGA) to solve a single depot emergency logistics distribution routing problem. We revise the algorithm to solve the problem of model (11).

4 An Example

A rural emergency command center has 2 distribution depots with each depot having 6 vehicles. The capacities of the 12 vehicles are given in Table 1 where vehicles 1 to 6 belong to depot 1 and vehicles 7 to 12 belong to depot 2.

Table 1 Vehicle capacities (kilogram)

Vehicle	1	2	3	4	5	6	7	8	9	10	11	12
Capacity	1000	1100	1200	1300	1400	1500	1000	1100	1200	1300	1400	1500

Depots 1 and 2 both have 7500 kg inventory of relief material. The center is responsible for 30 areas material distribution in the case of unexpected events. When an unexpected event happens, the center must decide how to dispatch vehicles in the two depots to send needed materials to the affected areas. Though lacking historical data, experts can take likely chaos, traffic jams and other bad situations into account to estimate the road travel times of vehicles from each depot to the affected areas and the times from one area to another area. Suppose the experts believe these parameters are normal uncertain variables and present their estimations in Tables 2 and 4. In Table 2, D_1 represents depot 1 and D_2 depot 2. In addition, the experts estimate the people's demands in the affected areas according to the judgement on the severity of the accident and present them in Table 3.

Table 2 Normal uncertain travel times between depots and the affected areas (μ, σ) (minute)

LCTs	1	2	3	4	5	6	7	8	9	10
D_1	(40,10)	(50,5)	(50,10)	(55,10)	(55,10)	(50,10)	(30,10)	(55,10)	(45,10)	(46,10)
D_2	(60,5)	(38,10)	(65,15)	(70,15)	(60,10)	(45,10)	(60,10)	(50,10)	(45,10)	(50,10)
LCTs	11	12	13	14	15	16	17	18	19	20
D_1	(40,10)	(35,10)	(50,10)	(50,15)	(55,10)	(50,10)	(65,10)	(55,10)	(45,10)	(42,10)
D_2	(70,5)	(50,10)	(65,15)	(70,10)	(60,10)	(45,10)	(60,10)	(50,10)	(45,10)	(50,10)
LCTs	21	22	23	24	25	26	27	28	29	30
D_1	(45,10)	(45,10)	(50,10)	(55,15)	(55,10)	(50,10)	(50,5)	(55,10)	(45,10)	(40,10)
D_2	(35,5)	(50,10)	(65,15)	(70,10)	(60,10)	(45,10)	(60,10)	(50,10)	(45,10)	(45,5)

Table 3 The normal uncertain demands (μ', σ') (kilogram)

Affected Area	Demand	Affected Area	Demand	Affected Area	Demand
1	(230,20)	11	(240,25)	21	(240,25)
2	(300,21)	12	(220,25)	22	(310,30)
3	(215,25)	13	(300,35)	23	(240,25)
4	(360,30)	14	(195,25)	24	(290,25)
5	(190,25)	15	(200,20)	25	(320,30)
6	(190,35)	16	(260,30)	26	(240,25)
7	(410,35)	17	(210,20)	27	(370,35)
8	(220,30)	18	(375,30)	28	(330,30)
9	(410,35)	19	(300,20)	29	(260,20)
10	(220,30)	20	(375,30)	30	(280,35)

Table 4 Normal uncertain travel times between two affected areas (μ, σ) (minute)

LCTs	1	2	3	4	5	6	7	8	9	10
2	(15,7)									
3	(40,15)	(25,10)								
4	(45,10)	(20,10)	(27,12)							
5	(40,10)	(27,10)	(25,10)	(50,15)						
6	(20,10)	(50,10)	(25,5)	(20,10)	(26,10)					
7	(35,10)	(25,10)	(27,10)	(25,10)	(25,5)	(20,7)				
8	(20,15)	(30,5)	(25,10)	(35,10)	(25,5)	(27,10)	(23,10)			
9	(55,15)	(20,10)	(30,10)	(30,10)	(26,10)	(25,7)	(27,5)	(30,12)		
10	(15,5)	(27,10)	(35,10)	(30,10)	(30,10)	(30,10)	(26,5)	(27,10)	(25,10)	
11	(65,15)	(30,15)	(35,15)	(25,10)	(40,12)	(20,10)	(27,12)	(25,10)	(20,10)	(25,10)
12	(15,5)	(25,10)	(25,5)	(45,15)	(30,10)	(27,10)	(25,10)	(30,10)	(30,15)	(20,10)
13	(65,20)	(25,10)	(40,15)	(35,10)	(25,10)	(35,15)	(23,10)	(35,14)	(15,5)	(15,10)
14	(20,15)	(30,12)	(50,20)	(40,15)	(27,10)	(45,14)	(30,14)	(25,10)	(20,10)	(10,5)
15	(65,15)	(35,10)	(26,10)	(50,20)	(65,10)	(40,10)	(27,12)	(30,12)	(20,10)	(25,10)
16	(55,10)	(45,10)	(30,10)	(35,14)	(35,10)	(30,10)	(27,12)	(35,10)	(25,10)	(20,10)
17	(15,5)	(25,10)	(35,10)	(55,15)	(30,10)	(25,10)	(30,10)	(27,10)	(25,10)	(15,10)
18	(55,10)	(20,10)	(30,12)	(30,10)	(25,10)	(25,7)	(27,5)	(30,12)	(20,10)	(10,5)
19	(15,5)	(27,10)	(35,12)	(30,15)	(30,10)	(30,10)	(26,5)	(27,10)	(30,15)	(25,10)
20	(65,10)	(30,15)	(35,15)	(25,10)	(40,12)	(20,10)	(27,12)	(25,10)	(15,5)	(20,10)
21	(15,10)	(20,10)	(25,10)	(45,15)	(30,14)	(27,10)	(25,10)	(30,10)	(20,10)	(15,10)
22	(65,15)	(25,10)	(40,15)	(35,15)	(25,10)	(35,15)	(23,10)	(35,14)	(20,10)	(10,5)
23	(20,15)	(30,10)	(50,20)	(40,10)	(27,10)	(45,14)	(30,14)	(25,10)	(25,10)	(25,10)
24	(65,15)	(95,30)	(85,30)	(120,50)	(65,25)	(100,30)	(80,35)	(90,35)	(75,30)	(70,25)
25	(55,10)	(65,10)	(30,10)	(35,15)	(35,10)	(30,10)	(27,12)	(35,10)	(20,10)	(15,10)
26	(15,5)	(25,10)	(35,14)	(55,20)	(30,10)	(25,10)	(30,10)	(27,10)	(30,15)	(10,5)
27	(20,15)	(25,5)	(50,20)	(40,15)	(27,10)	(45,14)	(30,14)	(25,10)	(15,5)	(25,10)
28	(65,15)	(35,10)	(26,10)	(50,20)	(65,10)	(40,10)	(27,12)	(30,12)	(20,10)	(20,10)
29	(55,10)	(55,10)	(30,10)	(35,15)	(35,10)	(30,10)	(27,12)	(35,10)	(20,10)	(15,10)
30	(15,5)	(23,10)	(35,14)	(55,10)	(30,10)	(25,10)	(30,10)	(27,10)	(25,10)	(10,5)
LCTs	11	12	13	14	15	16	17	18	19	20
12	(55,25)									
13	(30,10)	(45,7)								
14	(15,7)	(30,10)	(20,10)							
15	(65,10)	(20,10)	(25,10)	(35,15)						
16	(25,10)	(15,5)	(30,5)	(40,20)	(30,10)					
17	(55,7)	(25,5)	(45,7)	(30,5)	(15,5)	(26,12)				
18	(20,10)	(30,15)	(35,10)	(55,10)	(25,5)	(30,14)	(25,10)			
19	(20,7)	(27,10)	(65,10)	(55,7)	(23,10)	(35,14)	(23,10)	(27,10)		
20	(65,20)	(30,10)	(35,15)	(25,10)	(40,10)	(20,10)	(27,12)	(25,10)	(25,10)	
21	(15,7)	(26,10)	(26,14)	(45,15)	(30,10)	(27,10)	(25,10)	(30,10)	(15,10)	(15,5)
22	(65,15)	(20,5)	(40,14)	(35,10)	(25,10)	(45,20)	(23,10)	(35,14)	(15,10)	(20,5)
23	(20,15)	(30,12)	(50,20)	(40,10)	(20,10)	(45,15)	(30,14)	(25,10)	(25,10)	(25,10)
24	(65,15)	(35,10)	(26,10)	(50,20)	(65,20)	(40,10)	(27,10)	(30,12)	(10,5)	(30,10)
25	(55,10)	(55,15)	(30,10)	(35,15)	(35,10)	(30,10)	(27,10)	(35,10)	(25,10)	(35,10)
26	(15,7)	(23,10)	(35,14)	(55,10)	(30,10)	(25,10)	(30,10)	(27,10)	(15,10)	(15,5)
27	(20,15)	(30,12)	(50,20)	(40,14)	(25,10)	(45,15)	(30,15)	(25,10)	(15,10)	(20,5)
28	(65,15)	(95,30)	(85,30)	(120,50)	(65,25)	(100,30)	(80,35)	(90,35)	(75,30)	(70,25)
29	(55,10)	(65,10)	(30,10)	(35,15)	(35,10)	(30,10)	(27,10)	(30,12)	(10,5)	(30,10)
30	(15,7)	(23,10)	(35,14)	(55,10)	(30,10)	(25,10)	(30,10)	(27,10)	(25,10)	(35,10)

(Continued)

Table 4 Continued

LCTs	21	22	23	24	25	26	27	28	29
22	(65,7)								
23	(20,15)	(30,12)							
24	(65,15)	(35,10)	(25,10)						
25	(55,10)	(60,10)	(30,10)	(35,14)					
26	(15,7)	(23,10)	(35,14)	(55,10)	(30,10)				
27	(20,15)	(30,12)	(50,20)	(40,15)	(25,10)	(45,15)			
28	(65,15)	(35,10)	(26,10)	(50,15)	(60,10)	(40,10)	(27,10)		
29	(55,10)	(45,10)	(30,10)	(35,10)	(35,10)	(30,10)	(25,12)	(35,10)	
30	(15,7)	(23,10)	(35,14)	(55,10)	(30,10)	(25,10)	(30,10)	(27,10)	(25,5)

The manager of the center sets the confidence levels at $\beta = 0.90$ and $\beta' = 0.95$. Running the algorithm ($pop_size = 30, P_c = 0.6, P_m = 0.5$, and the parameter in the rank based evaluation function $\nu = 0.05$) 50,000 generations we get the optimal routing plan and present it in Table 5. The objective value is 1090 minutes.

To test the effectiveness of the algorithm, we change the P_c and P_m values in the CGA and do the experiments. The results are provided in Table 6. To compare the results, we give an index called relative error, i.e. (optimal objective value – actual objective value)/optimal objective value $\times 100\%$, where the optimal objective value is the minimum one of all the objective values obtained. It can be seen from Table 6 that the relative errors do not exceed 4%, which shows that the designed algorithm is robust to the parameter settings and effective for solving the problem.

Table 5 Optimal routing plan for twelve vehicles in two depots to thirty areas

Vehicles	Vehicle Routing
Vehicle 1	$D_1 \rightarrow 4 \rightarrow D_1$
Vehicle 2	$D_1 \rightarrow 14 \rightarrow 10 \rightarrow 25 \rightarrow D_1$
Vehicle 3	$D_1 \rightarrow 7 \rightarrow D_1$
Vehicle 4	$D_1 \rightarrow 3 \rightarrow 28 \rightarrow D_1$
Vehicle 5	$D_1 \rightarrow 12 \rightarrow 16 \rightarrow D_1$
Vehicle 6	$D_1 \rightarrow 22 \rightarrow D_1$
Vehicle 7	$D_2 \rightarrow 29 \rightarrow 19 \rightarrow 24 \rightarrow D_2$
Vehicle 8	$D_2 \rightarrow 26 \rightarrow 11 \rightarrow 18 \rightarrow D_2$
Vehicle 9	$D_2 \rightarrow 2 \rightarrow 13 \rightarrow 9 \rightarrow D_2$
Vehicle 10	$D_2 \rightarrow 21 \rightarrow 20 \rightarrow D_2$
Vehicle 11	$D_2 \rightarrow 30 \rightarrow 1 \rightarrow 17 \rightarrow 15 \rightarrow 23 \rightarrow D_2$
Vehicle 12	$D_2 \rightarrow 6 \rightarrow 5 \rightarrow 8 \rightarrow 27 \rightarrow D_2$

Table 6 Optimal objective values with different P_c and P_m values in CGA (minutes)

Pop_size	P_c	P_m	Objective Value	Relative Error (%)
30	0.3	0.2	1108	1.65
30	0.4	0.4	1112	2.02
30	0.5	0.2	1119	2.67
30	0.6	0.5	1090	0.00
30	0.6	0.6	1098	0.73
30	0.7	0.4	1126	3.30
30	0.7	0.5	1114	2.20
30	0.7	0.7	1121	2.84
30	0.8	0.3	1109	1.74
30	0.8	0.4	1125	3.21
30	0.8	0.8	1111	1.93
30	0.9	0.3	1126	3.21
30	0.9	0.5	1120	2.75

To show the significance of our multi-depot distribution routing model, we study the case where unexpected events have attacked only a smaller number of areas, sending relief material to which can be well done by either one depot. Suppose the former 12 areas in Table 4 are attacked by the unexpected events. We search the optimal vehicle routing plans in the following three cases. In case one, only vehicles in depot 1 serve the 12 affected areas, in case two, only vehicles in depot 2 serve the 12 affected areas, and in case three, vehicles in both depot 1 and depot 2 can serve the 12 affected areas. Running CGA 50,000 generations, we get that the objective values in case one, two and three are 490.5, 428.75 and 421.25 minutes, respectively, which implies that even in the case where either one depot has capacity to serve the affected areas in unexpected events, two-depot routing plan still has advantage: It can send relief material to the affected areas earlier than either one-depot routing plan.

5 Conclusion

This paper has studied a multi-depot vehicle routing problem for dispatching vehicles in multi-depots to distribute relief material to the affected areas in unexpected events. Due to the occurrence of the unexpected events, people's demands in the affected areas and the vehicles' travel times on roads lack historical data and are given by experts' estimations rather than historical data. Uncertain variables have been used to describe these parameters and

an emergency multi-depot logistics distribution routing model has been developed. To solve the problem, the equivalent of the proposed model has also been given. The results of the example show that the model is effective for solving the proposed multi-depot vehicle routing problem.

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