The Joint Location Inventory Replenishment Problem at a Supermarket Chain under Stochastic Demand

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Abstract

We consider a joint location inventory replenishment problem involving a chain of supermarkets at designated locations. Associated with each supermarket is stochastic stationary demand where inventory replenishment periods are uniformly fixed over the supermarkets. Considering inventory positions of the supermarket chain, we formulate a finite state Markov decision process model where states of a Markov chain represent possible states of demand for milk powder product. The unit replenishment cost, shortage cost, demand and inventory positions are used to generate the total inventory cost matrix; representing the long run measure of performance for the Markov decision process problem. The problem is to determine for each supermarket at a specific location an optimal replenishment policy so that the long run inventory costs are minimized for the given state of demand. The decisions of replenishing versus not replenishing at a given location are made using dynamic programming over e finite period planning horizon. We test the model using data from two supermarket locations. The model demonstrates the existence of an optimal state-dependent replenishment policy and inventory costs for two selected supermarket locations.

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1 Introduction

In joint location inventory replenishment problems, timely and concerted efforts are needed to deliver products to designated locations in order to satisfy customers. In the given system, systematic planning and coordination of procurement, production and distribution functions are critical to support the inventory management framework at various locations of the supermarket chain. In this study, we consider an inventory system consisting of a milk powder production plant producing and supplying milk powder product to supermarkets in a chain at designated locations. As a joint location inventory replenishment problem, a chain of supermarkets receive milk powder from the production plant. Customers demand the product at various locations of the supermarket. Inventory costs are incurred at each supermarket and such costs consist of ordering costs, carrying (holding) costs and shortage costs. Demand is a critical issue that arises when coordinating or managing inventory of a supermarket chain. A drastic increase in demand can impose pressure on supermarkets and create delays that usually result in higher costs and lower customer satisfaction. On the other hand, if demand is less than expected, supermarkets face unnecessary inventory ordering costs. In a typical competitive market place, cost reduction is paramount for the growth and survival of supermarkets. Taking a close look at daily business transactions, managing demand uncertainty requires persistent inventory monitoring in order to achieve efficient customer service levels. Supermarket managers no wonder rank demand uncertainty as one of the most top challenges that hamper achievement of their goals and objectives. This demands a better understanding of the sources of uncertainty so as to plan appropriately for customers.

As competition among supermarkets continue to rise, enterprises are constantly looking for ways to maximize their efficiency and gain better control over inventory items. Supermarket managers must ensure that the store runs smoothly, that items are priced competitively and that customers are satisfied. Having a thorough understanding of the key concepts for effective supermarket inventory management is imperative for any manager dedicated to the success of supermarket operations. Concerted efforts are required to link the supply chain professionals and the supermarket inventory managers in order to gain competitive advantage. It is imperative therefore those supermarkets identify suitable sources of committed suppliers. The systematic investigation and comparison of sources, the evaluation and monitoring of performance of supply sources and the development of appropriate procedures with suppliers are vital to support business performance. Where there is some uncertainty in demand and/or the time between placing an order and the receipt of the order, it becomes necessary to use some safety stock or buffer stocks in order to prevent a stock out situation. This may however become risky in case of perishable items.

2 Literature Review

Zheng [1] discussed the difference between optimal solution under deterministic EOQ policy and a stochastic model. His research indicated that at large quantities, the difference between deterministic and stochastic models is small and does not exceed one eighth. In the model presented by Miller [2], a hypothesis was tested that inventory behavior for American non fat dry milk was consistent with dynamic cost minimization by dairy manufacturers. Results indicated that firms chose their dairy product inventories so as to minimize quadratic output and inventory carrying costs subject to autoregressive cost shocks. In related work by Cheung and Powell [3], a two stage model was formulated that minimized the cost of stochastic demand. The first stage dealt with moving inventory from the plant to the warehouses based on forecasted demand. The second stage was moving the inventory from the warehouses to the customers when they send an order. Using an experimental case, the model indicated that having two warehouses per customer was more efficient than having one warehouse per customer. A related model for analyzing demand for dairy products was developed by Hein and Wessels [4]. The structure of dairy products demand was estimated under the assumption of two-stage budgeting procedure and a complete demand system for food incorporating demand graphic effects was explained. Eynan and Kropp [5] examined a periodic review system under stochastic demand using a single product. A simple solution procedure gave an almost optimal solution where results were extended to the joint replenishment problem for multiple items and the simple heuristic developed provided promising results. A study conducted by Yu and Li [6] developed a model that would help decision makers about uncertainty in the supply chain. The method reduced the number of variables and made the model more robust. The approach by Kamath and Pakkala [7] was based on a Bayesian model that covered long term planning. It was noted that previous information on demand trends can help significantly to reduce the total cost of the supply chain because the cost drops as the variation of demand declines. Miranda and Garrido [8] examined the network design of a supply chain with stochastic demand and risk pooling. The model developed maximized the reduction of total cost as the variability and holding costs increased because the number of warehouses and inventory costs decreased. Related research of a periodic inventory model at a supermarket chain by Gaur and Fischer [9] examined the problem as a vehicle routing and delivery scheduling problem where hourly demand forecasts for each store, travel times and distances, cost parameters and various transportation constraints were considered. The system resulted in savings of distribution costs and also had tactical and strategic advantages for the firm. Shu, Teo and Shen [10] analyzed the joint location inventory model by incorporating uncertainty in demand and solving the problem using variable fixing. Stochastic programming as a modeling tool was presented by Santoso, Goetshalckx and Shapiro [11]. The model is based on Sample Average Approximation (SAA) with an accelerated bender decomposition. The SAA method was based on solving a model for a number of random samples followed by deciding the confidence intervals for different solutions. In a similar context, a replenishment inventory model by Broekmeullen and Van Donselarr [12] analyzed a perishable inventory system based on item aging and retrieval behavior; and the paper had profound insights in terms of random demand. A periodic review inventory model by Zhao and Katehakis [13] examined the case in which the ordering quantity was either zero or at least a minimum order size. The objective in this study was to characterize the inventory policies that minimize total discounted ordering, holding and backorder penalty costs over a finite time horizon. In a related study, the concept of inventory planning under stochastic demand and the problems encountered was presented by Bollapragada [14]. His argument was that it is quite common to experience out-of-stock situations or overstocking which is costly on either side. Snyder, Daskin and Teo [15] presented a stochastic extension of the inventory-location model by including the probability of different scenarios based on demand and cost. This extension was referred to as the stochastic location model with risk pooling (SLMRP). The model proved to be cost effective compared to deterministic models by having low regret values. Tanonkou, Benyoucef and Xie [16] explained the stochastic model of a joint location inventory system. In the given model, a number of distribution centers and customers with random demand and supply lead times were considered. The model was solved using Langrangian relaxation – based approach. In a related study, Roychowdhury [17] determined an optimal policy for a stochastic inventory model of deteriorating items with time-dependent selling price. The rate of deterioration of the items was constant over time and the selling price decreased monotonically at a constant rate with deterioration of items. Diabat and Richard [18] formulated the economic order quantity policy and investigated the strategic and tactical decisions previewed as the number of distribution centers in inventory management. A stochastic model by Kizito, Kariko and Lating [19] was proposed to determine the optimal decision of ordering versus not ordering milk powder in supermarkets. Results from the study yielded an optimal state-dependent optimal ordering policy. The inventory model by Aref, Klib and Babi [20] presented important results in relation to the multi-period location inventory optimization problem characterized by uncertain demand and supply lead time. In this paper, a stochastic two-stage mathematical model maximizing the total expected supply network profits was presented. The generic modeling approach integrated key features of the inventory planning decisions with uncertainty.

On a comparative note, the stochastic joint location-inventory replenishment model offers interesting results for discussion when compared to existing models. Most models have very important aspects in characterizing random demand and the degree of uncertainty varies from situation to situation. In particular, the model presented by Aref, Kliba and Babi [20] vividly brings out the problem at hand, especially in the context of the stochastic multi-location inventory problem.

While prior research has attempted to study the problem, this research is the first to show that Markov decision processes offer a plausible approach to modeling of joint location inventory replenishment policies within the context of a supermarket chain. The benefits of using the model include the model's implicit ability to handle the stochastic dynamics of demand despite the similarities in supermarket facility set ups that form the chain.

3 Model Description

We consider a joint location inventory system consisting of a milk powder production plant producing and storing milk powder for a given number of supermarkets in a chain at designated locations. The demand during each time period over a fixed planning horizon at a given location L is described as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state top another may be described by means of a Markov chain.

Suppose one is interested in determining an optimal course of action, namely to replenish additional units of milk powder (a decision denoted by R = 1) or not to replenish additional units of milk powder (a decision denoted by R = 0) during each time period over the planning horizon, where R is a binary decision variable. Optimality is defined such that the minimum inventory costs are accumulated at the end of N consecutive time periods spanning the planning horizon under consideration. In this paper, a two-location (L = 2) and two-period (N = 2) planning period is considered.

3.1 Notation

Sets

i, j Set of states of demand

L Set of locations

R Set of replenishment policies

Parameters

Demand

D Demand matrix

M Demand transition matrix

Inventory

O On-hand inventory matrix

Costs

V Inventory cost matrix

e Expected inventory costs

a Accumulated inventory costs

c_r Unit replenishment cost

c_h Unit holding cost

c_s Unit shortage cost

Probabilities

$M^R{}_{ij}$ Probability that demand changes from state i to state j given replenishment policy R

Others

n, N Stages

C Customer matrix

F Favorable demand

U Unfavorable demand

3.2 Finite-Period Dynamic Programming Model

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal replenishment policy can be expressed as a finite period dynamic programming model. Assuming Z_n(i, L) denotes the optimal expected inventory costs accumulated at the end of periods *n, n*+1,............***N*** given that the state of the system at the beginning of period *n* is i ε{F, U}. The recursive equation relating Z_n and Z_{n+1} is

$$Z_n(i,L) = \min_R[M_{iF}^R(L)V_{iF}^R(R) + Z_{n+1}(F,L), M_{iU}^R(L)V_{iU}^R(R) + Z_{n+1}(U,L)]$$

$$\mathrm{i}\,\varepsilon\,\{\mathrm{F,U}\},\quad \mathrm{L}=\{1,2\},\quad \mathrm{n}=1,2,\ldots\ldots\ldots\ldots\ldots\mathrm{N} \tag{1}$$

together with the conditions

$$Z_{N+1}(F,L) = Z_{n+1}(U,L) = 0$$

This recursive relationship may be justified by noting that the cumulative inventory costs $\mathrm{V_{ij}^R(L)} + \mathrm{Z_{N+1}(j)}$ resulting from reaching state j ε{F,U} at the start of period *n*+1 from state i ε {F,U} at the start of period *n* occurs with probability $\mathrm{M_{ij}^R(L)}$

$$\text{Clearly } \mathrm{e^R(L) = [M^R(L)][V^R(L)]^T, R\,\varepsilon\{0,1\}, L = \{1,2\}} \tag{2}$$

where "T" denotes matrix transposition. Hence, the dynamic programming recursive equations

$$Z_N(i,L) = \min_R[e_i^R(L) + M_{iF}^R(L)Z_{N+1}(F) + M_{iU}^R(L)Z_{N+1}(U)] \tag{3}$$

$$Z_N(i,L) = \min_R[e_i^R(L)] \tag{4}$$

result where (4) represents the Markov chain stable state.

3.3 Computing $M^R(L)$ and $V^R(L)$

The demand transition probability from state i ε {F,U} to state j ε {F,U}, given replenishment policy R ε {0,1} may be taken as the number of customers observed at location L with demand initially in state I and later with demand changing to state j, divided by the sum of customers over all states. That is,

$$M_{ij}^R(L) = C_{ij}^R(L)/[C_{iF}^R(L) + C_{iU}^R(L)] \tag{5}$$

$$\mathrm{i}\,\varepsilon\,\{\mathrm{F,U}\}, \qquad \mathrm{R}\,\varepsilon\,\{0,1\}, \qquad \mathrm{L} = \{1,2\}$$

When demand outweighs on-hand inventory, the inventory cost matrix $\mathrm{V^R(L)}$ may be computed by means of the relation

$$V^R(L) = (c_r + c_h + c_s)[D^R(L) - O^R(L)]$$

Therefore,

$$V_{ij}^R(L) = \begin{cases} (c_r + c_h + c_s)[D_{ij}^R(L) - O_{ij}^R(L) & \text{if } D_{ij}^R(L) > O_{ij}^R(L) \\ c_h[O_{ij}^R(L) - D_{ij}^R(L) & \text{if } D_{ij}^R(L) \le O_{ij}^R(L) \end{cases}$$

$$\text{for all} \quad \mathrm{i,j}\,\varepsilon\,\{\mathrm{F,U}\},\ \mathrm{L} = \{1,2\} \quad \text{and} \quad \mathrm{R}\,\varepsilon\,\{0,1\} \tag{6}$$

The justification for expression (6) is that $D^R_{ij}(L) - O^R_{ij}(L)$ units must be replenished to meet excess demand. Otherwise replenishment is cancelled when demand is less than or equal to on-hand inventory.

The following conditions must, however hold:

1. R = 1 when $c_r > 0$ and R = 0 when $c_r = 0$.
2. $c_s > 0$ when shortages are allowed and $c_s = 0$ when shortages are not allowed.

4 Optimization

The optimal replenishment policy and inventory costs are found in this section for each period over supermarket location L separately.

4.1 Optimization During Period 1

When demand is favorable (i.e. in state F), the optimal replenishment policy during period 1 is

$$R = \begin{cases} 1 & \text{if } e_f^1(L) < e_F^0(L) \\ 0 & \text{if } e_f^1(L) \ge e_F^0(L) \end{cases}$$

The associated inventory costs are then

$$Z_1(F,L) = \begin{cases} e_F^1(L) & \text{if } \; R=1 \\ e_F^0(L) & \text{if } \; R=0 \end{cases}$$

Similarly, when demand is unfavorable (i.e. in state U), the optimal replenishment policy during period 1 is

$$R = \begin{cases} 1 \;\text{ if } \; e_U^1(L) < e_U^0(L) \\ 0 \text{ if } \; e_U^1(L) \geq e_U^0(L) \end{cases}$$

In this case, the associated inventory costs are

$$Z_1(F,L) = \begin{cases} e_U^1(L) & \text{if } \; R=1 \\ e_U^0(L) & \text{if } \; R=0 \end{cases}$$

4.2 Optimization During Period 2

Using (3) and (4) and recalling that a$^Z{}_i$(L) denotes the already accumulated inventory costs at the end of period 1 as a result of decisions made during that period, it follows that

$$\begin{aligned} a_i^R(L) &= e_i^R(L) + M_{iF}^R(L)\min[e_f^1(L), e_f^0(L)] \\ &\quad + M_{iU}^R(L)\min\left[e_{\overline{U}}^{1}(L), e_U^0(L)\right] \\ a_i^R(L) &= e_i^R(L) + M_{iF}^R(L)Z_2(F,L) + M_{iU}^R(L)Z_2(U,L) \end{aligned}$$

Therefore when demand is favorable (i.e. in state F), the optimal replenishment policy during period 2 is

$$R = \begin{cases} 1 \;\text{ if } \; a_F^1(1) < a_F^0(L) \\ 0 \;\text{ if } \; a_F^1(1) < a_F^0(L) \end{cases}$$

while the associated inventory costs are

$$Z_2(F,L) = \begin{cases} a_F^1(L) & \text{if } \; R=1 \\ a_F^0(L) & \text{if } \; R=0 \end{cases}$$

Similarly, when demand is unfavorable (i.e. in state U), the optimal replenishment policy during period 2 is

$$R = \begin{cases} 1 & \text{if } \; a_U^1(1) < a_U^0(L) \\ 0 & \text{if } \; a_U^0(1) \geq a_U^0(L) \end{cases}$$

In this case, the associated inventory costs are

$$Z_2(U, L) = \begin{cases} a_U^1(L) & \text{if } \; R = 1 \\ a_U^0(L) & \text{if } \; R = 0 \end{cases}$$

5 A Case Study about the Shoprite Supermarket Chain

In order to demonstrate use of the model in §3-4, a real case application form *Sameer Agriculture and Livestock Ltd,* a production plant of milk powder product and two *Shoprite supermarket* locations in Uganda are presented in this section. The production plant stores and supplies milk powder to Shoprite locations; and customers come to supermarkets for milk powder product. The demand for milk powder fluctuates every week at all locations. The production plant and supermarkets want to avoid excess inventory when demand is unfavorable (state U) or running out of stock when demand is favorable (state F) and hence, seek decision support in terms of an optimal replenishment policy and the associated inventory cost of milk powder in a two-week planning period. The network topology of the joint location inventory system for milk powder at the plant and supermarket locations are illustrated in Figure 1.

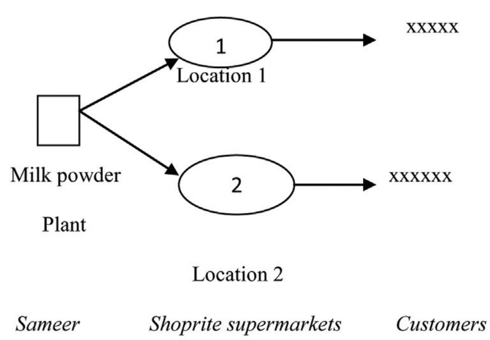

Figure 1 A joint location inventory replenishment network at the plant and supermarkets.

5.1 Data Collection

Samples of customers demand and inventory levels were taken for 400gms milk powder product (in thousand packets) at Shoprite supermarket locations. The state-transitions and the respective replenishment policies were examined over twelve weeks. The data is presented in the Tables 1–3.

5.2 Computation of Model Parameters

Using (5) and (6), the state-transition matrices and inventory costs (in million UGX) at each respective location are

$$M^1(1) = \begin{bmatrix} 0.5617 & 0.4383 \\ 0.8312 & 0.1688 \end{bmatrix} \qquad V^1(1) = \begin{bmatrix} 333.5 & 78 \\ 77 & 53 \end{bmatrix}$$

Table 1 Customers versus state transitions at Shoprite supermarket locations

		Ordering Policy 1		Ordering Policy 0	
Supermarket Location	States	F	U	F	U
1	F	182	142	164	60
	U	126	26	110	50
2	F	90	118	108	80
	U	118	26	90	22

Table 2 Demand (in packets) versus state transitions at Shoprite supermarket locations

		Ordering Policy 1		Ordering Policy 0	
Supermarket Location	States	F	U	F	U
1	F	156	15	123	78
	U	107	11	78	15
2	F	93	60	72	77
	U	59	11	75	11

Table 3 On-hand inventory (in packets) versus state transitions at Shoprite supermarket locations

		Ordering Policy 1		Ordering Policy 0	
Supermarket Location	States	F	U	F	U
1	F	190	186	87	90
	U	186	188	93	91
2	F	290	80	162	157
	U	71	159	158	157

$$M^1(2) = \begin{bmatrix} 0.4327 & 0.5673 \\ 0.8194 & 0.1806 \end{bmatrix} \qquad V^1(2) = \begin{bmatrix} 52 & 110 \\ 125.26 & 68.5 \end{bmatrix}$$

for the case when additional units were replenished (R = 1) during week 1;while these matrices are given by

$$M^1(2) = \begin{bmatrix} 0.7322 & 0.2678 \\ 0.6875 & 0.3125 \end{bmatrix} \qquad V^0(1) = \begin{bmatrix} 437.26 & 181.5 \\ 173.26 & 39.5 \end{bmatrix}$$

$$M^0(2) = \begin{bmatrix} 0.5745 & 0.4255 \\ 0.8036 & 0.4964 \end{bmatrix} \qquad V^0(2) = \begin{bmatrix} 9.0 & 1.5 \\ 4.5 & 67.5 \end{bmatrix}$$

for the case when additional units were not replenished (R = 0) during week 1.

When additional units were replenished (R = 1), the matrices $M^1(1)$, $V^1(1)$, $M^1(2)$ and $V^1(2)$ yield the expected inventory costs (in million UGX) in week 1 for the two locations:

Location 1:

$$e_F^1(1) = (0.5617)(335.6) + (0.4383)(78) = 222.64$$
$$e_U^1(1) = (0.8312)(77) + (0.1688)(83) = 75.02$$

Location 2:

$$e_F^1(2) = (0.4327)(52) + (0.5673)(110) = 84.90$$
$$e_U^1(2) = (0.8194)(129.26) + (0.1906)(68.50) = 118.28$$

When additional units were not replenished (R = 0), the matrices $M^0(1)$, $V^0(1)$, $M^0(2)$ and $V^0(2)$ yield the expected inventory costs (in million UGX) in week 1 for the two locations:

Location 1:

$$e_F^0(1) = (0.7322)(437.26) + (0.2678)(81.50) = 368.50$$
$$e_U^0(1) = (0.6875)(163.26) + (0.3125)(30.50) = 121.77$$

Location 2:

$$e_F^0(2) = (0.5745)(9.0) + (0.4255)(1.50) = 5.80$$
$$e_U^0(2) = (0.8036)(4.50) + (0.1964)(67.5) = 16.88$$

When additional units were replenished (R = 1), the accumulated inventory costs (in million UGX) at the end of week 2 are calculated as follows for the two locations:

Location 1:

$$a_F^1(1) = 222.64 + (0.5617)(222.64) + (0.4383)(78.02) = 381.89$$
$$a_U^1(1) = 78.02 + (0.8312)(222.64) + (0.1688)(78.02) = 276.24$$

Location 2:

$$a_F^1(2) = 84.90 + (0.4327)(222.64) + (0.5673)(78.02) = 225.88$$
$$a_U^1(2) = 118.28 + (0.8194)(222.64) + (0.1806)(78.02) = 314.80$$

When additional units were not replenished (R = 0), the accumulated inventory costs (in million UGX) at the end of week 2 are calculated as follows for the two locations:

Location 1:

$$a_F^0(1) = 368.5 + (0.7322)(368.5) + (0.2678)(78.02) = 659.2$$
$$a_U^0(1) = 128.64 + (0.6875)(368.5) + (0.3125)(78.02) = 406.36$$

Location 2:

$$a_F^0(2) = 5.80 + (0.6745)(368.5) + (0.4255)(78.02) = 250.70$$
$$a_U^0(2) = 516.88 + (0.8036)(368.5) + (0.1964)(78.02) = 328.32$$

5.3 The Optimal Joint Location Inventory Replenishment Policy at supermarkets

Week 1:
At location 1, since $222.64 < 368.50$, it follows that R = 1 is an optimal replenishment policy for week 1 with associated inventory costs of 222.64 M.UGX for the case of favorable demand. Since $75.02 < 121.77$, it follows that R = 1 is an optimal replenishment policy for week 1 with associated inventory costs of 75.02 M.UGX for the case when demand is unfavorable.

At location 2, since $5.80 < 84.90$ follows that R = 0 is an optimal replenishment policy for week 1 with associated inventory costs of 5.80 M.UGX for the case of favorable demand. Since $16.88 < 118.28$, it follows that R = 0 is an optimal replenishment policy for week 1 with associated inventory costs of 16.88 M.UGX for the case when demand is unfavorable.

Week 2:
At location 1, since $381.8 < 659.2$, it follows that R = 1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 381.8 M.UGX for the case of favorable demand. Since $276.24 < 406.46$, it follows that R = 1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 278.24 M.UGX for the case when demand is unfavorable.

At location 2, since $225.88 < 250.70$ follows that R = 1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 225.88 M.UGX for the case of favorable demand. Since $314.80 < 328.32$, it follows that R = 1 is an optimal replenishment policy for week 2 with associated inventory costs of 314.89 M.UGX for the case when demand is unfavorable.

6 Conclusions and Discussion

The joint location inventory replenishment model with stochastic demand was presented in this paper. The model determines an optimal replenishment policy and inventory costs of milk powder product under demand uncertainty. The decision of whether or not to replenish additional units at a specific supermarket location is made using dynamic programming over a finite period planning horizon. Results from the model indicate optimal replenishment policies and inventory costs for the given problem at each location. As a cost minimization strategy in joint location inventory systems, computational efforts of using Markov decision process approach provide promising results. However, further extensions of the research are vital in order to analyze the impact of non stationary demand on replenishment policies. In the same spirit, the model developed raises a number of salient issues to consider: lead time of milk powder during the replenishment cycle and customer response to abrupt changes in price of the product at the chosen locations that form the supermarket chain. Special interest is also sought in further extending the model by considering replenishment policies for minimum inventory costs in the context of continuous time Markov Chains (CTMC). The model developed is therefore expected to be amenable to some formal, logical or systematic analysis. The stochastic joint location inventory replenishment model developed is to allow its application in the specific form in order to assist users achieve specific purposes. As noted in the study, inventory cost comparisons were vital in determining the optimal replenishment policy for the two supermarket locations. By the same token, classification of demand as a two-state Markov chain facilitated modeling and optimization process of replenishment policies for the chosen locations.

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