Location Routing Inventory Problem with Transshipment Points Using p -center

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Abstract

Location Routing Inventory Problem with Transshipment (LRIP-T) is a collaboration of the three components in the supply chain which are location-allocation; vehicle routing and inventory management problems that allow transshipment process, in a way that the total system cost and the total operational time are minimized. This study is to determine a set of customer points to act as the transshipment point as and when it is necessary, based on the surplus quantities it has, the quantities to ship to the needing customers and the sequence in which customers are replenished by homogeneous fleet of vehicles. The transshipment point is selected from the existing customers using p -center. The performance of the selection is evaluated using a set of benchmark data and a real life data. For the real life data, sensitivity analysis based on the number of distribution centres and size of lorries are presented. Results show important savings achieved when compared to the existing model in solving the supply chain problem.

Keywords: Location Inventory Routing Problem with Transshipment, p -center.

1 Introduction

Location routing inventory problem (LRIP) is an integration of three keys logistics decision problems which are location-allocation, vehicle routing and inventory management. LRIP arises when decisions on the three problems must be taken simultaneously. According to [1, 3], LRIP in the distribution systems is to allocate depots from several potential locations to schedule the routes for vehicle in meeting the customer's demand and to determine the inventory policy based on the information of customer's demands in order to minimize the system's total costs. Location inventory routing problem (LRIP) is a branch of logistic study that is not widely explored by the researchers due to the interrelated decision areas. However, excellent integration consequently presents significant saving costs as simultaneous decisions imposed in solving the problems.

Not having enough stock to fulfill the customer's demand or stock out is one of issues in the supply chain distribution. This unexpected situation occurred when there is an excess demand that lead to falling of inventories. When this happened, customers tend to purchase goods at another store or does not purchase at all. In addition, when a substitution is made, the retailer and supplier lose their potential sale as customers start to switch to another substitute and they may experience customer's dissatisfaction, worst outcome is losing a customer. It is vice versa of surplus where surplus is described as a situation to retain excess inventory. Surplus can cause profit loss to a company. When there is a surplus, it takes up space and increased holding costs. We have seen the impact of both issues in supply chain, thus we consider incorporating the transshipment process in order to prevent it from happening.

Transshipment has important role in the supply chain as it allows the goods to be shipped from depot to a customer or from a customer with excess stock to be shipped to another customer. It is profitable to the supplier as they can save the transportation cost in terms of delivering goods to the respective customer and at the same time, customer can also save cost in terms of storage space and obsolete the access stock. Several studies implemented transshipment and proved that it is effective compared with the LRIP without transshipment. Researchers tend to integrate transshipment with inventory routing problem (IRP) such as researchers [2, 4–6]. IRP is the combination of two problems in operation research; vehicle routing and inventory and IRPT is the extension of IRP with insertion of transshipment. In both studies they used the customers as the transshipment points and we also implemented the similar technique. The transshipment is only allowed when the location

of transshipment point with retailer are nearer compared to the location of supplier. It is profitable to the supplier as they can save the transportation cost because of the short distance of travel to the customers. In this study, we consider LRIP-T or Location Routing Inventory Problem with Transshipment by applying p -center to select the transshipment center and apply the model to solve a real world problem.

2 Research Methods

Model

The model used is adapted from the earlier study [9] in which the transshipment cost is incorporated into total system cost of existing model adapted from [8]. p -center model is used to select the best transshipment point. In p -center problem the maximum distance between the distribution point and customer's node is minimized.

where

r_{ij} : the cost (unit) of transshipping product from i to j

Q_{vij} : the quantity transshipped from customer i in period t

$MaxD$: the total cost at the maximum distance,

$$\text{Maximize } \sum_{t=1}^l \sum_{k=1}^K \sum_{h=1}^{N+M} \sum_{g=1}^{N+M} D_{hg} \cdot W_{hgkt}$$

The formulation of the objective function is as below:

Minimize

$$MaxD + \sum_{j=N+1}^{N+M} (E_j + \left(\sum_{t=1}^l (B_j + Y_j) H_j(t) + q_j \cdot \frac{P}{l} \cdot \sum_{i=1}^l (V_j(t) + \alpha_j \gamma_j(t)) + B_j \right) \cdot A_j$$

Subject to:

$$\sum_{k=1}^K \sum_{h=1}^{N+M} W_{gikt} = 1, \text{ for all } i, t \quad (1)$$

$$\sum_{g=1}^{N+M} W_{ghki} - \sum_{g=1}^{N+M} W_{hgki} = 0, \text{ for all } h, k, \text{ and } I \quad (2)$$

$$\sum_{h=1}^{N+M} \sum_{g=1}^{N+M} W_{ghki} \leq 1, \text{ for all } k, \text{ and } t \quad (3)$$

$$\beta_k^t(i) \leq CK, \text{ where } i \in \{0, 1, 2, \dots, N\} \quad (4)$$

$$\tau_i(t) \leq U(f_i(t)), \text{ for all } I \quad (5)$$

$$\sum_{h=1}^{N+M} W_{ihkt} - \sum_{h=1}^{N+M} W_{jhkt} - S_{ij} \leq 1, \text{ for all } i, j, k \text{ and } t \quad (6)$$

$$W_{ghkt} \in \{0, 1\}, \text{ for all } g, h, k \text{ and } t \quad (7)$$

$$A_j \in \{0, 1\}, \text{ for all } j \quad (8)$$

$$S_{ij} \in \{0, 1\}, \text{ for all } i \text{ and } j \quad (9)$$

$$O_{jkt} \in \{0, 1\}, \text{ for all } j, k, \text{ and } t \quad (10)$$

$$O_j \in \{0, 1\}, \text{ for all } j \quad (11)$$

The objective function is to minimize the total system cost expressed by the summation of logistics center's set up cost, transportation cost and inventory cost. Constraint (1) ensures that each customer appears in only one route during period t . Constraint (2) states that every point entered should be the same point the vehicle leaves. Constraint (3) insures that each route only served by one logistics center. Constraint (4) and (5) are the statement for any period, in any point the total load is less than the vehicle capacity and the actual volume is equal or less than expected volume. Constraint (6) states that a customer can be allocated to a logistics center if there is a route passed by the customer. Constraint (7–11) insures the decision variable's integrality.

The following notations are used to describe the formulation.

N : number of customer points

M : number of logistics centers

h : index of customer point or logistic centers ($1 \leq h \leq N + M$)

g : index of customer point or logistic centers ($1 \leq g \leq N + M$)

i : index of customer point ($1 \leq i \leq N$)

j : index of logistics center ($N + 1 \leq j \leq N + M$)
 k : index of vehicles or routes ($1 \leq k \leq K$)
 t : index of time periods of planning horizon (p) ($1 \leq t \leq p$)
 E_j : cost to establish the logistics center j
 D_{hg} : distance between point h and point g
 P : length of the planning horizon
 CK : capacity of vehicles
 c : unit cost of vehicles
 B_j : cost of dispatching the product from factory to logistics center j
 α_j : probability of being reused after a circulation
 q_j : holding cost of products (in unit) at logistics center j
 Y_j : ordering cost of every time in logistics center j
 L_j : lead time in logistics center j where, $L_j \leq P/p$
 $V_j(t)$: logistics center's j inventory level start at period t , where $V_j(t = 0) = 0$ stands for inventory is zero at the initial stage of planning horizon
 And the decision variables are:
 W_{hgmt} : 1, if point h immediately go to point g on route k in period t ; 0, otherwise
 S_{ij} : 1, if customer i is allocated to logistics center j ; 0, otherwise
 A_j : 1, if logistics center j is opened; 0, otherwise
 O_{jkt} : 1, if route k is served by logistics center j in period t ; 0, otherwise
 $H_j(t)$: 1, if there exists an order for new product at logistics center j ; 0, otherwise
 $R_i(t)$: actual collection volume from customer i in period t

3 Data Analysis & Results

The analysis is divided into two sections: (1) on benchmark data for method validation and (2) on real life data for application.

3.1 Data Preparation (Benchmark Data)

Data are adapted based on [8, 9] and used based on the following assumptions. First, there are 5 candidates of logistics centers and 15 customers, the capacity of vehicles (CK) is 125 units, the unit cost of vehicles (c) is 1/unit distance and the probability of being reused after a circulation (α_j) is 0.9. In addition, we assume the expected distribution demand is randomly populated and obey the Poisson distribution with $\lambda_i = 2$ and $\delta_i = 1$. Tables 1 and 2 present the parameter values of logistics centers and customer points.

Table 1 Logistics centers

Logistics Centers	Coordinates	α_j	Setup Cost
D1	(43,49)	25	10,000
D2	(1,12)	20	12,000
D3	(41,30)	30	14,000
D4	(5,58)	10	13,000
D5	(24,19)	15	12,000

Table 2 Customer points

Customer Points	Coordinates	Demand
C1	(15,3)	42
C2	(18,24)	56
C3	(2,59)	53
C4	(9,6)	34
C5	(49,54)	48
C6	(33,10)	61
C7	(30,50)	37
C8	(24,59)	45
C9	(3,35)	48
C10	(33,21)	68
C11	(45,27)	32
C12	(46,6)	69
C13	(24,32)	45
C14	(28,33)	62
C15	(2,0)	47
Total		747

3.2 Results on Benchmark Data

In earlier study, the transshipment point is determined based on p -center. In the actual scenario, the total system cost is the summation of establishing cost of logistics centers, transportation cost and inventory cost. There are some modified costs for LRIP-T in this study.

- Logistics Center Establishment Cost – No establishment cost needed as the transshipment center will be selected among the customers.
- Transportation cost – Transportation cost covers the from the distribution centers and from the transshipment centers.
- Inventory cost – The inventory level at the transshipment center is assumed to be zero, or the center is assumed not to keep any stock.

Table 3 shows that the logistics center D1 is determined to be the best center with the lowest average travelled distance of 96.67 km when the supply is

Table 3 Results for limited supply

Logistics Center	Total Distance (km)	Average Distance (km)	Average	
			Distance/Vehicle (km)	Average Waste (unit)
D1	690.69	46.05	230.23	51
D2	725.91	48.39	241.97	51
D3	1022.31	68.15	340.77	51
D4	956.7	63.78	318.9	51
D5	749.79	49.99	249.93	51

unlimited and 46.05 km only when the supply is limited [9]. Considering the average traveled distance at 46.05 km, this means that the total cost is equal to $46.05 \times 747 = 34399.35$.

In the second phase, a transshipment point is determined based on the notion of *p*-center. The transshipment process is considered only for the case when supply is limited.

Initial sets of routes are chosen and MaxD is calculated and minimized. Each customer from C1 up till C15 has equal chance of being chosen as the transshipment point. Total transportation system cost is calculated and the best result is based on the set that produces the lowest cost.

Table 4 shows the final answer that lists the customers that being covered by D1 as the logistics center and those that being covered by C11 as the transshipment point. This also shows the total delivery cost (in km) of 19,756.8 compared to 34399.35. This means that LRIP-T model has performed tremendously well in order to save the total costs for all the three systems; location, inventory and routing.

Table 4 Coverage of logistics center and the transshipment point

D1 Coverage		C11 Coverage	
Customer Points	Cost	Customer Points	Cost
C2	2287.6	C1	775.6
C3	2276.4	C5	336.0
C4	1867.6	C9	700.0
C6	1554.0	C14	400.4
C7	2632.0	C15	532.0
C8	1920.8		
C10	1103.2		
C12	2105.6		
C13	1265.6		
Total cost = 19756.8			

3.3 Real-Life Data

In order to apply the model, a case study on delivery of frozen chicken by a local company in Selangor, Malaysia is chosen. The main distribution centre is located in Semenyih, Selangor, however, the model is applied to solve its delivery problem to its 36 customers in another state, Pahang, Malaysia. Figure 1 shows a map of chicken order for a month in Pahang, Malaysia and the customers are clustered in seven (7) areas, as some (for example C6–C28) customers are quite close to each other. There are a lot of independent agents who operate from grocery shops or homes in each

Figure 1 Map of customers location.

area which lead to the unavailability of the data. Therefore, data cleaning is done by combining all of the customers in different locations to be within the same clustered area. For the present case study, the analyses are divided into two phases. The first phase is having all chickens delivered from the main distribution centre and described as One Depot. In the analysis the best planning route for delivery is chosen while minimizing the total travel distance and at the same time determining the number of lorry use. The second phase is when the transshipment point is identified using p -centre and to be the second distribution centre and described as Two Depot. Similarly, the best planning route for delivery is chosen while minimizing the total travel distance and at the same time determining the number of lorry use.

Table 5 shows the vehicle type which is the size of lorry such as small lorry and large lorry, capacity of the lorry and cost per delivery. Each customer is served by one or more facilities with specified transportation links. Moreover, each outsource transportation has a fixed capacity to deliver customers demand. Furthermore, the total chicken out per delivery takes about maximum 1000 of chickens when customers request for the small lorry and maximum 2300 of chickens when customers request for the large lorry, based on customers' demands. In addition, each delivery is charged about RM 500 per delivery for small lorry otherwise each delivery is charged about RM 800 per delivery for large lorry.

3.3.1 Result for Phase 1 – One Depot

Tables 6 and 7 show the results when all chickens are delivered from the main distribution centre or one depot only.

3.3.2 Result for Phase 2 – Two Depots

The transshipment point C1 is chosen using p -centre and the chickens are delivered from both the main distribution centre (0) and from C1. The best route is determined using Excel Solver and 15 customers get their delivery directly from the main distributor while the balance two (12) customers get it from the transshipment point C1. Note that C1 also gets its supply from the main distributor. Total distance travel by all chickens are summarised in Tables 8 and 9.

Table 5 Vehicle type, capacity of lorry and cost per delivery

Vehicle Type (Size of Lorry)	Capacity of the Lorry	Fixed Cost Per Delivery
Small	1000 chickens per delivery	RM 500
Large	2300 chickens per delivery	RM 800

Table 6 Average traveled distance using small lorry

Number of Lorry	Routes	No. of Chickens	Distance Traveled
First	Depot-C13	1000	326700
Second	Depot-C13	1000	326700
Third	Depot-C6-C7-C10-C11-C12-C14-C17-C18-C19-C21-C23	740	241758
Forth	Depot-C13-C30-C32-C1	850	326647
Fifth	Depot-C29-C36	130	19686
Total		3720	1241491
Average travel distance per chicken			333.73

Table 7 Average traveled distance using large lorry

Number of Lorry	Routes	No. of Chickens Per Delivery	
First	Depot-C13	2300	751410
Second	Depot-C6-C7-C10-C11-C12-C13-C14-C17-C18-C19-C21-C23-C30-C32-C1-C29-C36	1420	484049
Total		3720	1235459
Average travel distance per chicken			332.11

Table 8 Delivery from transshipment point C1

No. of Lorry	Routes	No. of Chickens	Total Distance
First	Depot-C1-C13	2640	121440
Second	Depot-C1-C6-C7-C10-C11-C12-C13-C14-C17-C18-C19-C21-C23-C30-C32	950	165550
Total		3590	286990
Average travel distance by each chicken			79.94

Table 9 Delivery from main depot

Number of Lorry	Routes	No. of Chickens	Total Distance
First	Depot-C36-C29	130	4316
Total		130	4316
Average travel distance by each chicken			33.2

In Section 3.3.1, the results show that the size of lorry does not contribute to lower distance traveled by each unit of the chicken. However, comparing the results in Sections 3.3.1 and 3.3.2, we can see that by having the transshipment point the average traveled distance can be reduced tremendously from 333.73 (using small lorry) and 332.11 (using large lorry) to only 79.94 from the transshipment point and only 33.2 from the main depot.

4 Conclusion

In this paper, we have proposed a new way of determining the transshipment point among the customers for location routing inventory problem. The selection of routes in this analysis are randomly picked and tested in order to achieve the maximum distance that has the minimum total system cost. The total system cost has considered the new technique of choosing the transshipment point using p -center. This technique has excluded some costs and only has the total distance travelled in the formulation and applied into the real life problem and see tremendous improvement. In future research, we will further consider using the heuristic method called genetic algorithm for a better route's selection. Another area of possible research involves LRIP-T together with development of new algorithm is the stochastic demand. For real life problem, fluctuation of customer's demand also needs to be considered.

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