Investigating the Impacts of Lateral and Angular Misalignments between Circular Filaments

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Abstract

This paper analyses the impacts of lateral and angular misalignments on the mutual inductance and the magnetic force between circular filaments which are arbitrarily positioned in space. Advanced and relevant models available in the literature are used to accomplish the aim of this paper. Using SCILAB application software, the results obtained based on the theoretical model show that as the coil misalignments increase the values of the mutual inductance and the magnetic force keep decreasing and increasing with respect to certain variable rotation angle at any point of the secondary coil. In order to further investigate the impact of coil separation distance and misalignments on the amount of voltage induced in the secondary coil, a model of air-cored transformer is constructed. The experimental results obtained show that the amount of the induced magnetic flux from the primary coil into the secondary coil becomes weaker if the coil separation distance and coil misalignments increase. As a result, a much smaller value of the mutual inductance is obtained resulting to a much smaller induced electromotive force in the secondary coil.

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In conclusion, this study shows that the full benefits of contactless inductive power transfer (CIPT) systems will not be realized if issues regarding coil separation distance and misalignments are not tackled in the model to be designed for the CIPT transformer.

Keywords: Circular filaments, coil misalignments, mutual inductance, magnetic force.

1 Introduction

In order to reduce the amount of airborne pollution caused by the transportation system, many of the big automobile companies have been compelled to move from the manufacturing of internal combustion engine vehicles (ICEVs) to hybrid electric vehicles (HEVs), hydrogen fuel cell vehicles (HFCVs) and electric vehicles (EVs) [1]. Considering high oil prices and environmental awareness, the development of EVs is considered as a healthier mode of transportation because the electricity they consume could be generated from a wide range of sources which include fossil fuel, nuclear power and renewable sources such as tidal power, solar power, wind power or any combination of those [1, 2]. Although EVs are considered as a favourable solution for a greener energy but, users and owners of EVs feel uncomfortable because EVs require sufficient battery storage onboard to provide sufficient driving autonomy.

To overcome the issue of limited driving distance per charge, conventional wired system e.g. plug-in connectors have been commonly proposed for EV battery charging [1]. Although plug-in connector is a simple and reliable solution however, it may result to safety risks (e.g. electrocution) in wet and damp conditions (see Figures 1 and 2), it is a source of inconvenience (see Fig. 2) and lastly, it only enables stationary charging which means that an EV has to be stationary during the duration of its charge replacement.

Currently, contactless inductive power transfer (CIPT) system is used to overcome the problems of plug-in connectors. CIPT systems are a novel technology used for transferring electrical energy over a relatively large air-gap via high frequency magnetic fields [3]. The potential advantages of CIPT systems over plug-in connectors include an increased driving range, immunity to dirt, dust, water, ice and chemicals, reduced cabling and risk of cable breakage, low maintenance requirement and the use of a high frequency (10 kHz – 150 kHz) magnetic fields which cannot cause electrocution. Wireless transfer of electrical power to the on-board battery storage system of EVs is accomplished

Figure 1 Plug-in Connector for EV Battery Charging

Figure 2 Plug-in Connector: Exposed Electrical Terminal

through the major component of CIPT system which is the CIPT transformer (see Figures 3 and 4).

Although CIPT transformer which consists of air-cored coils plays a major role in CIPT systems however, coil misalignments (e.g. lateral and angular) are their inherent problem. As a result, its full potential is limited because the value of the mutual inductance as well as the magnetic force exerted on the current carrying conductor depends on the relative position and orientations of the coils in the CIPT transformer [4].

Air-cored coils are widely used in various electromagnetic applications. They are preferred to iron-cored type because of their design objectives of controllability and capability of a high power transfer [5]. The authors of

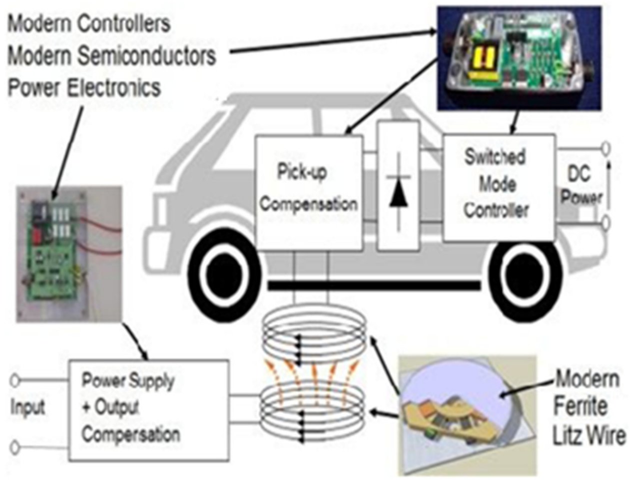

Figure 3 CIPT System for EV Battery Charging

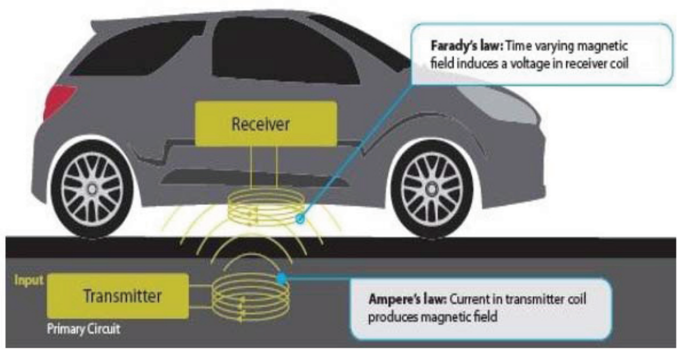

Figure 4 Fundamental Principles of CIPT Systems

this paper consider the use of circular filaments amongst other air-cored coils since in several electromagnetic applications regarding coil misalignments, the optimal magnetic coupling between circular filamentary coils is required [6, 7].

Computations of the mutual inductance and the magnetic force between coaxial circular coils have been completely solved by the authors in [7–20]. However, the present-day focus has been shifted to the computation of the mutual inductance and the magnetic force between circular coils with lateral and angular misalignments [21–30]. Furthermore, these computations can be achieved correctly and speedily by using finite element and boundary element methods [31, 32]. Notwithstanding, the authors in [22, 24] argue that analytical and semi-analytical methods can be used to achieve this task since they significantly simplify the mathematical procedures which in turn leads to a considerable reduction of the computational effort. It is also concluded that the mathematical models formulated for the mutual inductance and the magnetic force between filamentary circular coils are slightly more general and simpler to use (i.e., easy to understand, numerically suitable and easily applicable for engineers and physicists).

Based on this information, the authors of this paper analyse the impacts of lateral and angular misalignments on the mutual inductance and the magnetic force between circular filaments arbitrarily positioned in space based on the advanced and relevant models available in [24]. This task is achieved as follows. Section 2 presents the advanced and relevant models which are obtainable in the literature. In section 3, the theoretical results obtained using SCILAB application software is given as well as its discussion. Section 4 presents the model constructed and its experimental results while section 5 concludes the paper.

2 Advanced and Relevant Mathematical Model

This section presents the relevant and advanced models for computing the mutual inductance and the magnetic force between circular filaments which are arbitrarily positioned in space

2.1 Mutual Inductance Model between Circular Filaments using Magnetic Vector Potential Approach

The mutual inductance between the filamentary circular coils as shown in Fig. 5 can be computed by [24] as

$$M = \frac{\mu_0 R_S}{\pi} \int_0^{2\pi} \frac{[p_1 \cos\phi + p_2 \sin\phi + p_3]\Psi(k)}{k\sqrt{V_0}^3} d\phi \tag{1}$$

where

$$p_1 = \pm\frac{\gamma c}{l},\ p_2 = \mp\frac{\beta l^2 + \gamma ab}{lL} \quad \text{and} \quad p_3 = \frac{\alpha \mathrm{c}}{\mathrm{L}}$$

$$p_4 = \mp\frac{\beta ab + \gamma l^2 + \delta bc}{lL} \quad \text{and} \quad p_5 = \mp\frac{\beta \mathrm{c} - \delta \mathrm{a}}{\mathrm{lL}}$$
$$\alpha = \frac{R_S}{R_P}\,,\ \beta = \frac{x_C}{R_P},\ \gamma = \frac{Y_C}{R_P} \quad \text{and} \quad \delta = \frac{\mathrm{Z_C}}{\mathrm{R_P}}$$
$$L = \sqrt{a^2 + b^2 + c^2} \quad \text{and} \quad l = \sqrt{a^2 + c^2}$$
$$\Psi(k) = \left(1 - \frac{k^2}{2}\right) K(k) - E(k)$$
$$k = \sqrt{\frac{4V_0}{A_0 + 2V_0}}$$
$$V_0{}^2 = \alpha^2\left[\left(1 - \frac{b^2c^2}{l^2L^2}\right)\cos^2\phi + \frac{c^2}{l^2}\sin^2\phi + \frac{abc}{l^2L}\sin\phi\right] + \beta^2 + \gamma^2 \mp 2\alpha\frac{\beta ab - \gamma l^2}{lL}\cos\phi \mp \frac{2\alpha\beta c}{l}\sin\phi$$
$$A_0 = 1 + \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2\alpha(p_4\cos\phi + p_5\sin\phi)$$

where μ_0 is the magnetic permeability of space, R_P is the radius of primary coil, R_S is the radius of the secondary coil, α is the shape factor of the circular coil, (x_C, y_C and z_C) is centre of the secondary coil, *a, b* and *c* are the parameters defining the secondary coil plane λ, *k* is a variable and not indices, $K(k)$ and $E(k)$ are the complete integral of the first and second kind respectively [33, 34].

2.2 Magnetic Force between Circular Filaments using Mutual Inductance Approach

The magnetic force f_g between filamentary circular coils arbitrarily positioned in space can be computed by [14]

$$F_g = I_P I_S \frac{\partial M}{\partial g} \tag{2}$$

where I_P and I_S are the primary and secondary currents in the coil, M is the mutual inductance given in (1) and g = x, y, or z are the xyz components.

Finding the first derivative in (2), the magnetic force can be obtained by the following components:

$$
\begin{aligned}
F_X &= \frac{\mu_0 \alpha I_P I_S}{\pi} \int_0^{2\pi} I_X d\phi \\
F_Y &= \frac{\mu_0 \alpha I_P I_S}{\pi} \int_0^{2\pi} I_Y d\phi \\
F_Z &= \frac{\mu_0 \alpha I_P I_S}{4\pi} \int_0^{2\pi} I_Z d\phi
\end{aligned}
\tag{3}
$$

where

$$
\begin{aligned}
I_x &= q_3 \sin\phi \frac{\Psi(k)}{k\sqrt{V_0^3}} - \frac{kT_0(p_1\cos\phi + p_2\sin\phi + p_3)}{8\sqrt{V_0^9}} \times \\
&\left\{ \Psi(k) \left[\left[A_0 - 2{V_0}^2\right] + \frac{12V_0}{k_2} \right] - \Phi(k) \frac{k^2\left[A_0 - 2{V_0}^2\right]}{2} \right\} \\
I_y &= (-q_5\cos\phi + q_4\sin\phi)\frac{\Psi(k)}{k\sqrt{V_0^3}} - \frac{kS_0(p_1\cos\phi + p_2\ \sin\phi + p_3)}{8\sqrt{V_0^9}} \times \\
&\left\{ \Psi(k) \left[\left[A_0 - 2{V_0}^2\right] + \frac{12V_0}{k^2} \right] - \Phi(k) \frac{k^2\left[A_0 - 2{V_0}^2\right]}{2} \right\} \\
I_Z &= \frac{\left[p_1\cos\phi + p_2\sin\phi + p_3\right] L_0}{\sqrt{{V_0}^5}} \Theta(k)
\end{aligned}
$$

where

$$
\begin{aligned}
q_1 &= \frac{\gamma l^2 - \beta ab}{lb},\ q_2 = -\frac{\beta c}{l},\ q_3 = -\frac{l}{L},\ q_4 = \frac{ab}{lL},\ q_5 = -\frac{c}{l} \\
L &= \sqrt{a^2+b^2+c^2}\ \ and\ \ l = \sqrt{a^2+c^2} \\
\alpha &= \frac{R_S}{R_P}, \beta = \frac{x_C}{R_P},\ \gamma = \frac{y_C}{R_P},\ \delta = \frac{z_C}{R_P} \\
p_1 &= \pm\frac{\gamma^c}{l},\ p_2 = \mp\frac{\beta l^2 + \gamma ab}{lL},\ p_3 = \frac{\alpha c}{L}
\end{aligned}
$$

$$
\begin{aligned}
p_4 &= \mp\frac{\beta ab - \gamma l^2 \delta bc}{lL},\ p_5 = \mp\frac{\beta c - \delta a}{lL}\\
\Psi(k) &= \left(1 - \frac{k^2}{2}\right) K(k) - E(k)\\
\Phi(k) &= \frac{E(k)}{1-k^2} - K(k)\\
\Theta(k) &= k\left\{\Psi(k) - \frac{k^2}{2}\Phi(k)\right\}\\
k &= \sqrt{\frac{4V_0}{A_0 + 2V_0}},\\
{V_0}^2 &= \beta^2 + \gamma^2 + \alpha^2(l_1\cos^2\phi + l_2\sin^2\phi + l_3\sin 2\phi) +\\
&\quad 2\alpha(q_1\cos\phi + q_2\sin\phi)\\
A_0 &= 1 + \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2\alpha(p_4\cos\phi + p_5\sin\phi)\\
T_0 &= \beta + \alpha\left[q_4\cos\phi + q_5\sin\phi\right]\\
S_0 &= \gamma - \alpha\, q_3\cos\phi\\
L_0 &= \delta + \alpha\left[q_6\cos\phi - q_7\sin\phi\right]\\
q_6 &= -\frac{bc}{lL},\ q_7 = -\frac{a}{l}
\end{aligned}
$$

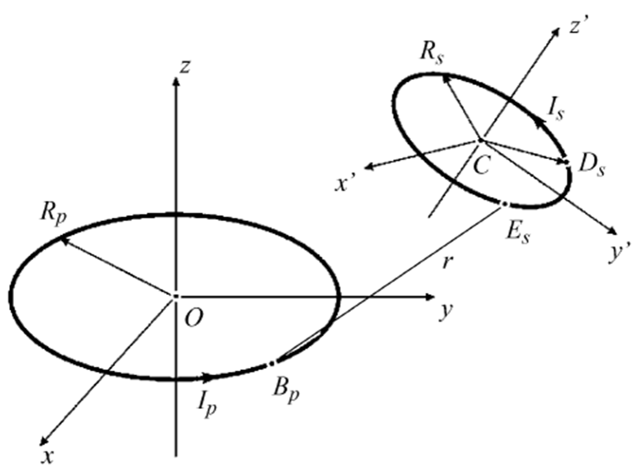

Figure 5 Circular Filaments with arbitrary misalignments [24]

3 Theoretical Result Obtained

In order to investigate the impacts of coil misalignments between circular coils arbitrarily positioned in space, the authors of this paper re-stated equations (1) and (3) in terms of the geometric configurations and common notations for circular filaments with arbitrary lateral and angular as given in [24]. Shown in Figures 6–13 are the results obtained using SCILAB application software [35] and the data used is shown in Table 1.

Table 1 Data used for Simulation [24]

Variable Parameters			
x_1 (m)	x_2 (m)	*P* (m)	θ (degree)
0.05	0.0125	0.04	30
0.10	0.0250	0.08	45
0.15	0.0375	0.14	60
0.20	0.0500	0.19	75
Constant Parameters			
R_P	0.16m		
R_S	0.10m		
μ_0	$4\,\pi \times 10^{-7} H/m$		

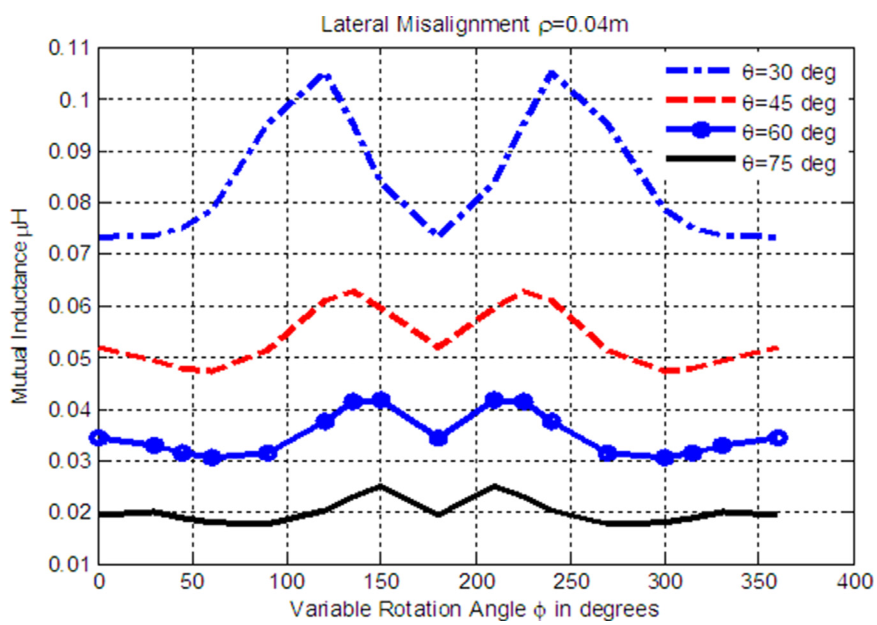

Figure 6 Impacts of Misalignments on the Mutual Inductance

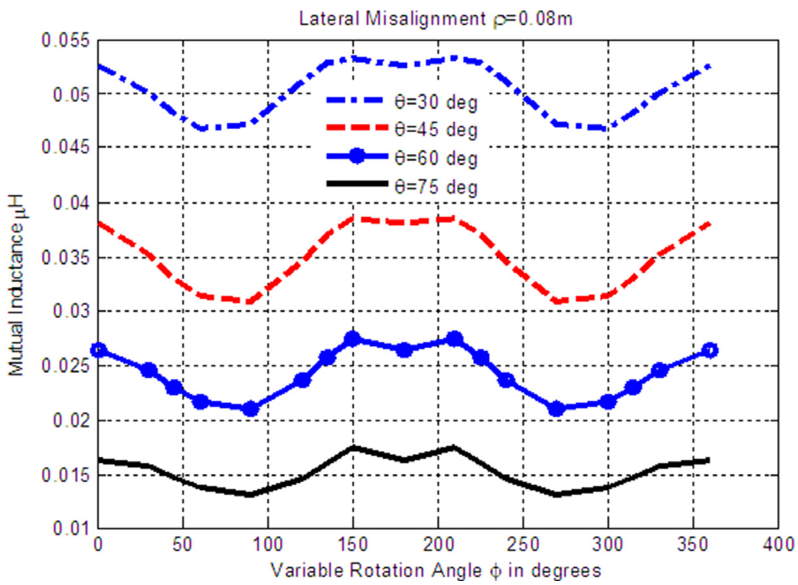

Figure 7 Impacts of Misalignments on the Mutual Inductance

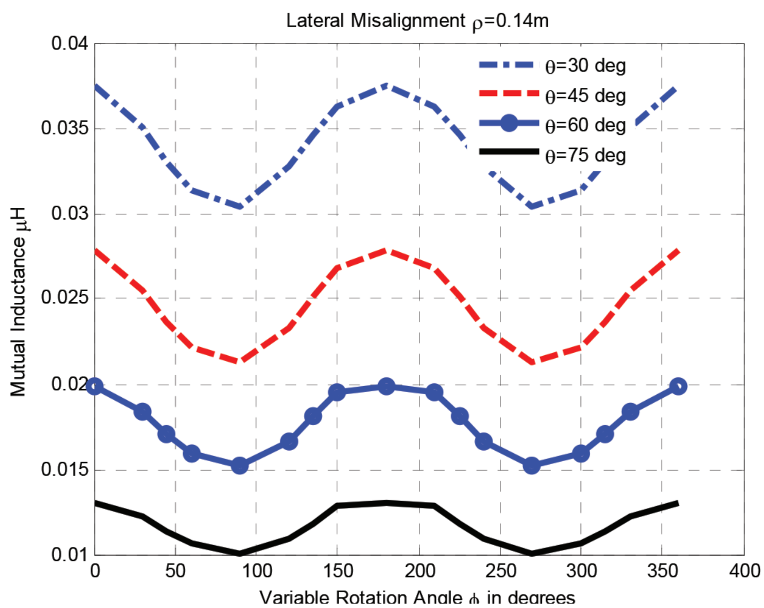

Figure 8 Impacts of Misalignments on the Mutual Inductance

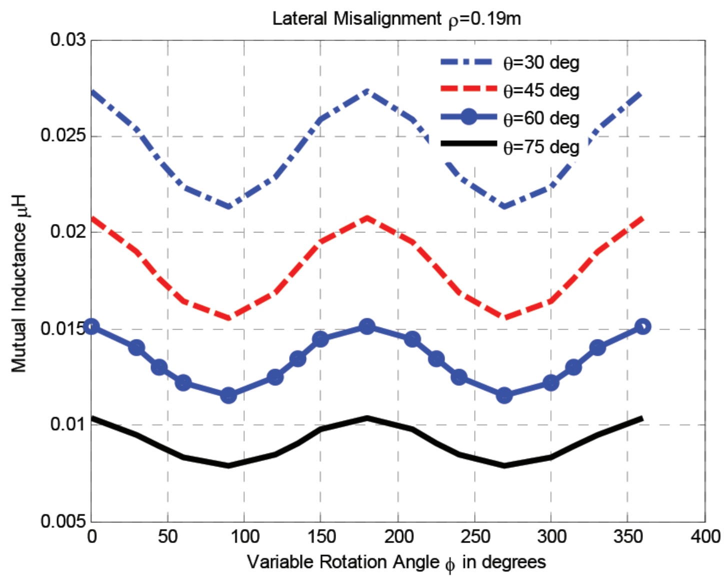

Figure 9 Impacts of Misalignments on the Mutual Inductance

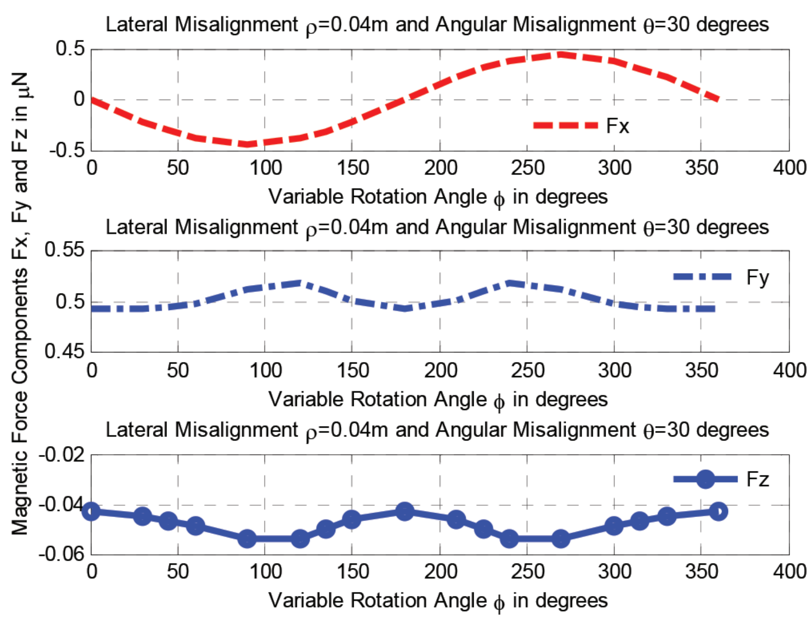

Figure 10 Impact of Misalignments on the Magnetic Force

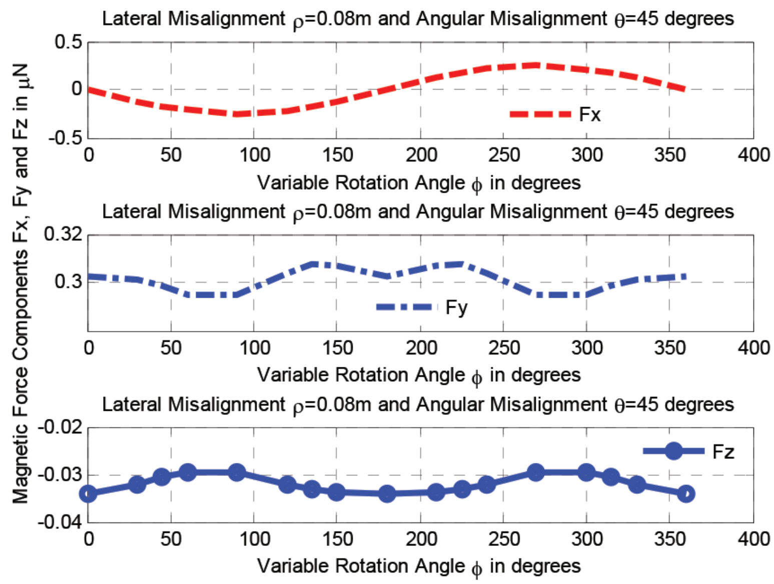

Figure 11 Impact of Misalignments on the Magnetic Force

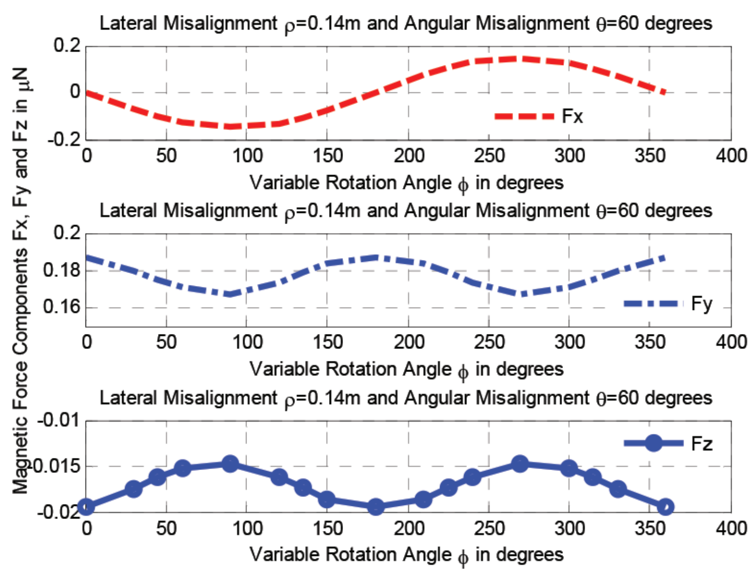

Figure 12 Impacts of Misalignments on the Magnetic Force

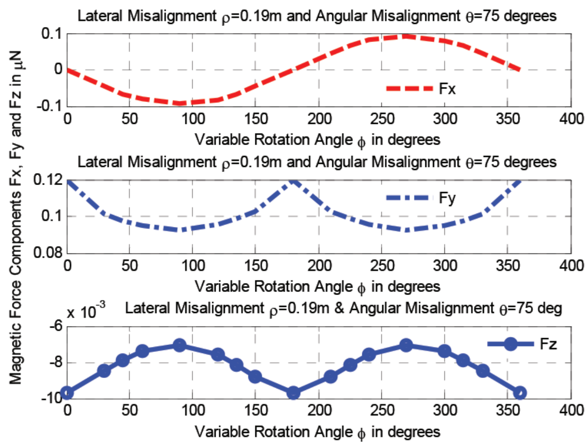

Figure 13 Impacts of Misalignments on the Magnetic Force

3.1 Discussion of the Theoretical Results

The results obtained in Figures 6–13 show the impacts of lateral and angular misalignments on the mutual inductance and the magnetic force between circular filaments arbitrarily positioned in space. These results are obtained by plotting the mutual inductance and the magnetic force against the variable rotation angle at any point of the secondary coil. Based on the data given in Table 1, it is clearly seen that the values of the mutual inductance (see Figures 6–9) and the magnetic force components (see Figures 10–13) keep decreasing and increasing at certain variable rotation angle.

4 Air-Cored Transformer and Experimental Results

The primary and secondary coils of transformers possess mutual inductance when they are magnetically linked together by a common magnetic flux. Hence, mutual inductance is an important operating property of transformers. Notwithstanding, its amount depends very much on the coil separation distance as well as lateral and angular misalignments. This implies that the amount of induced magnetic flux from the primary coil into the secondary coil is weaker if the coil separation distance and coil misalignments increase. As a result, a much smaller value of the mutual inductance is obtained resulting to a much smaller induced electromotive force (emf) in the secondary coil.

Based on this information, in order to further investigate the impact of coil separation distance as well as coil misalignments, a model of air-cored

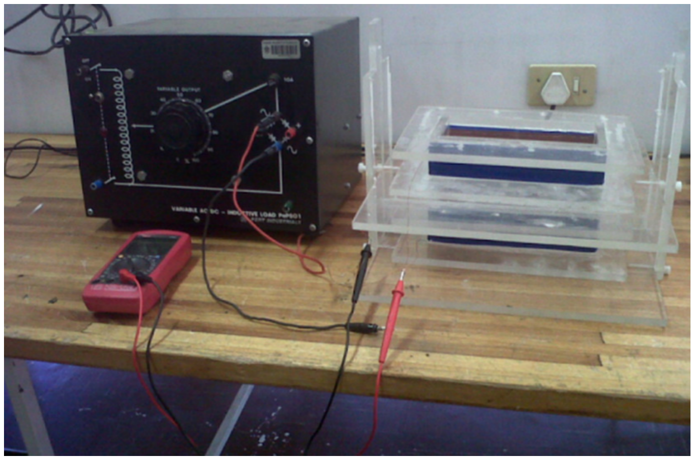

Figure 14 AC/DC Variable Power Supply, Multi-meter and Coreless Transformer

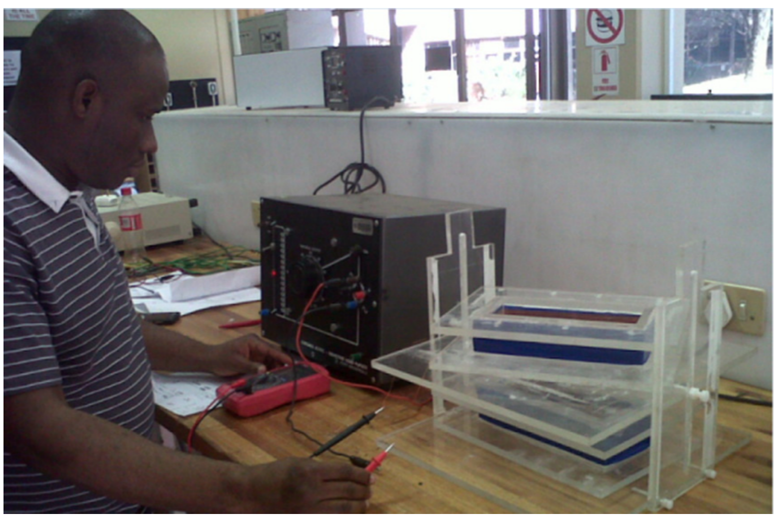

Figure 15 Author Investigating Impacts of Coil Separation Distance and Misalignments

transformer which consists of rectangular coils is constructed. Shown in Figures 14–16 is the prototype set up which consists of variable AC/DC power supply, multi-meter and the coreless transformer.

The primary and the secondary air-cored rectangular coils are of the same dimension and number of turns. In order to determine the amount of electromotive force (which is expressed in volts) induced in the secondary coil, ac

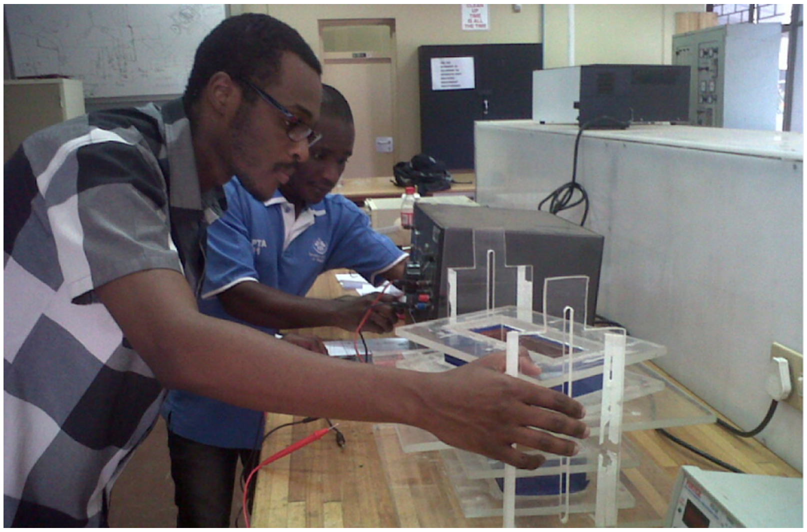

Figure 16 Colleagues during Prototype Setup

voltage of one volt is supplied to the primary coil of the coreless transformer at different coil separation distance and misalignments.

4.1 Experimental Measurements

Shown in Tables 2–4 are the experimental results obtained for the air-cored transformer which consists of rectangular coils. The experimental measurements show that as the coil separation distance as well as the lateral and angular misalignments increase the amount of the induced magnetic flux from the primary coil into the secondary coil becomes weaker. As a result, a much smaller value of the mutual inductance is obtained resulting to a much smaller induced emf (which is expressed in volts) in the secondary coil.

Table 2 Data used for Simulation

Constant Parameters		
Primary and Secondary Inductance $L_P = L_S$	3.2 mH	Primary Current I_P=0.995 $A \approx 1\ A$
$X_L = 2\pi FL$	1.00544Ω	Coil Separation Distance = 10 mm, 20 mm, 30 mm, 40 mm & 50 mm
Primary Voltage V_P	$1V$	Number of Turns $L_P = L_S$= 50

Table 3 Coil Separation Distance of 10 mm

Angular Misalignment θ (degree)	Lateral Misalignment d (mm)	Induced emf in the Secondary Coil V_S (Volts)
0	0.00	0.550
10	0.05	0.505
30	0.10	0.456
45	0.15	0.401

Table 4 Coil Separation Distance of 20 mm

Angular Misalignment θ (degree)	Lateral Misalignment d (mm)	Induced emf in the Secondary Coil V_S (Volts)
0	0.00	0.385
10	0.05	0.353
30	0.10	0.320
45	0.15	0.292

Table 5 Coil Separation Distance of 30 mm

Angular Misalignment θ (degree)	Lateral Misalignment d (mm)	Induced emf in the Secondary Coil V_S (Volts)
0	0.00	0.278
10	0.05	0.246
30	0.10	0.211
45	0.15	0.204

Table 6 Coil Separation Distance of 40 mm

Angular Misalignment θ (degree)	Lateral Misalignment d (mm)	Induced emf in the Secondary Coil V_S (Volts)
0	0.00	0.195
10	0.05	0.170
30	0.10	0.136
45	0.15	0.113

Table 7 Coil Separation Distance of 50 mm

Angular Misalignment θ (degree)	Lateral Misalignment d (mm)	Induced emf in the Secondary Coil V_S (Volts)
0	0.00	0.098
10	0.05	0.083
30	0.10	0.082
45	0.15	0.075

5 Conclusion

The authors of this paper analyse the impacts of lateral and angular misalignments on the mutual inductance and the magnetic force between circular filaments which are arbitrarily positioned in space. Advanced and relevant models available in the literature are used to achieve the theoretical results obtained in Figures 6–13. The simulations obtained show that as the coil misalignments increase the values of the mutual inductance and the magnetic force keep decreasing and increasing with respect to certain variable rotation angle at any point of the secondary coil.

In order to further investigate the impact of coil separation distance as well as lateral and angular misalignments on the amount of voltage induced in the secondary coil, a model of air-cored transformer which consists of rectangular coils is constructed. The experimental results obtained show that the amount of the induced magnetic flux from the primary coil into the secondary coil becomes weaker if the coil separation distance and coil misalignments increase. As a result, a much smaller value of the mutual inductance is obtained resulting to a much smaller induced electromotive force in the secondary coil.

In conclusion, this study shows that the full benefits of contactless inductive power transfer (CIPT) systems will not be realized if issues regarding coil separation distance and misalignments are not tackled in the model to be designed for the CIPT transformer.

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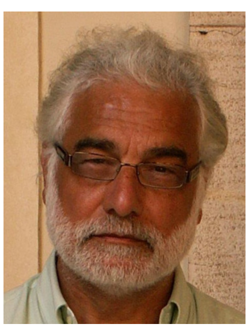

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