

CREDIBLE SIMULATION: EVALUATING THE CREDIBILITY OF SIMULATION MODELS AND SIMULATION MODEL LIBRARIES

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ABSTRACT

This paper deals with the evaluation of the credibility of simulation models and simulation model libraries exemplarily shown on a model of the Rexroth BRSL Simulation Library.

The topic tackles the questions on how engineers know that they really can trust the results of their simulation models, how exact their simulation models are and if the accuracy of their simulation models is sufficient or not.

In industry, engineers usually validate their simulation model results against real measurement data to examine if the simulation model results match the reality. Usually there is no numerical number describing the accuracy of the simulation model. In the topic of credible simulation, numerical methods of data analysis are used to try to determine a numerical number, the so-called model form error. This then gives an idea of how plausible the simulation model is in terms of uncertainties.

At Bosch Rexroth there is a prototypically developed app that enables users estimating the model form error, performing sensitivity analyses, and simulating variations of parameter uncertainties on key model parameters. Furthermore, there is the possibility to compare the simulation results with real measurement data.

The app may help to certify credible simulation models in the future.

Keywords: Simulation credibility, model form error, sensitivity analysis, p-box, Wasserstein metric

1. INTRODUCTION

Credibility of simulations is becoming increasingly relevant. Especially in times of ever-increasing digitalization of products and workflows in the industry up to purely virtual-based product releases, simulation-based decisions become more and more important. For this purpose, it is crucial that simulation results are given the necessary confidence and that their agreement with reality can be scientifically proven. [1]

Product validation via measurements on the real product is common and state of the art. Measurement data also contain certain inaccuracies and are not always easy to obtain, which is why they are already increasingly being supplemented by computer simulations. In this way, further confidence in the product can be gained and its behavior better understood.

To save time and thus money, and provided that the simulation results can be trusted, real measurements on the product can be completely replaced by purely virtual computer simulations.

The aim of this work at the Bosch Rexroth AG is to develop methods that allow full confidence in simulation results to be obtained by simulation and that this confidence level can be suitably represented by means of a numerical key figure, the so-called model form error.

In a further step, the determination of this credibility level should also be made possible for other models of other areas within the Bosch Rexroth AG. The credibility obtained can be integrated into digital product passports as metadata of the simulation models to certify the quality of the simulation model regarding its trustworthiness. This gained credibility stamp may help answering the following questions. Can you really trust your simulation? Do the simulation results match the reality? How exact is the simulation? Is it sufficiently exact?

2. STATE OF THE ART

2.1. Credibility standards in industry

Currently there are some standards in industry developed by NASA, NAFEMS, ASME and few more [1]. Also, there is a lot of research done within the Robert Bosch GmbH trying to merge these different standards regarding their quality metrics describing simulation and model (S & M) credibility. [1] The paper in hand focusses on the calculation of a so-called model form error as a numerical key indicator describing S & M credibility. There are different methods which will be explained hereafter.

2.2. Probability box (p-box) method

The probability box (p-box) method is an interval-based method to represent mixed uncertainties of a simulation model based on epistemic and aleatory parameter uncertainties, see **Figure 1** [2].

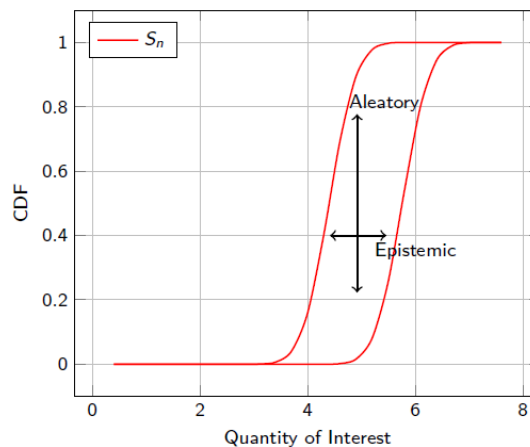


Figure 1: Representation of mixed uncertainties as p-box [2]

Epistemic parameter uncertainties are characterized to have a constant but unknown parameter value due to missing system knowledge or data. They are represented as an interval [2] [3].

Against this, aleatory parameter uncertainties do not have a constant value, e.g., the temperature in a system. Aleatory parameter uncertainties are usually represented as cumulative distribution functions (CDFs) [2] [3].

A p-box is created from the CDFs of the different simulation results of parameter combinations of epistemic and aleatory parameters including numerical uncertainties. The most left and the rightest CDF are the boundaries of the p-box representing the lower and the upper limit of trustworthiness [2], compare **Figure 2**.

In **Figure 2** the CDFs of the interesting simulation result variable, the so-called Quantity of Interest (QoI), for the different simulated parameter combinations of epistemic and aleatory parameters, named here with m_1, \dots, m_5 , are shown for the evaluated point in time while m_1 and m_5 form the lower (m_1) and the upper (m_5) boundaries of the p-box, defining the interval of trustworthiness for the simulated setup and the specific point in time.

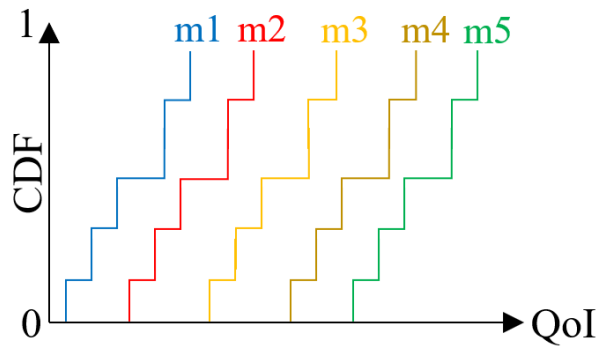


Figure 2: Exemplary CDFs for p-box generation

With the help of a p-box of a simulation model, a so-called model form error can be calculated in comparison with real measurement data, which quantifies the quality of the simulation model and its consistency with reality depending on how far the real measurement data are within or outside the p-box limits for a single point in time [2].

Therefore, the CDF of the measurement data is compared with the generated p-box of the simulation model and the model form error is calculated as area outside the p-box between the CDF of the measurement data and the lower and the upper p-box limit, as exemplarily shown in **Figure 3**.

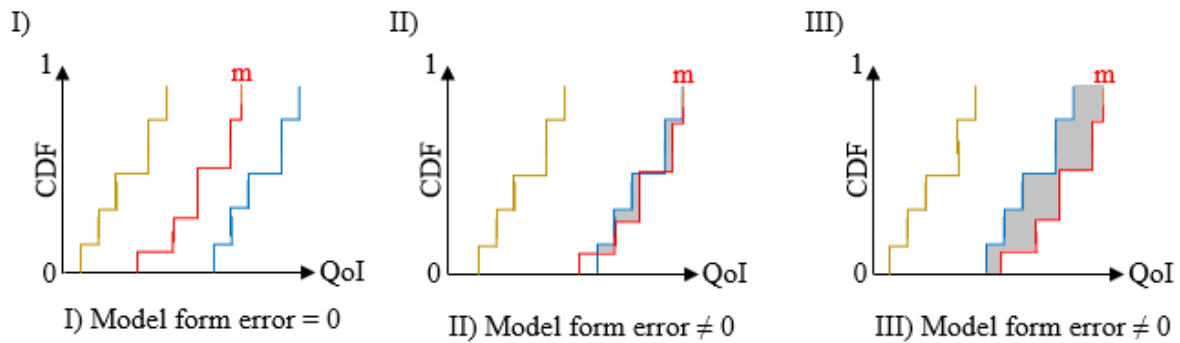


Figure 3: Calculation and graphical representation of the model form error

2.3. Wasserstein metric

The Wasserstein metric, also known as Wasserstein distance, Kantorovich-Rubinstein metric, optimal transport, or earth mover's distance, is used to calculate the numerical cost of an optimal transport problem [3]. It is used for the (statistical) time series analysis of dynamic systems and considers temporal progressions as well as spatial distances [3][4].

Since the Wasserstein metric considers not only a purely vertical point-to-point comparison between two time series, but also the horizontal distances between the curves to be compared, it is a very powerful method for time series analysis, see **Figure 4** [4].

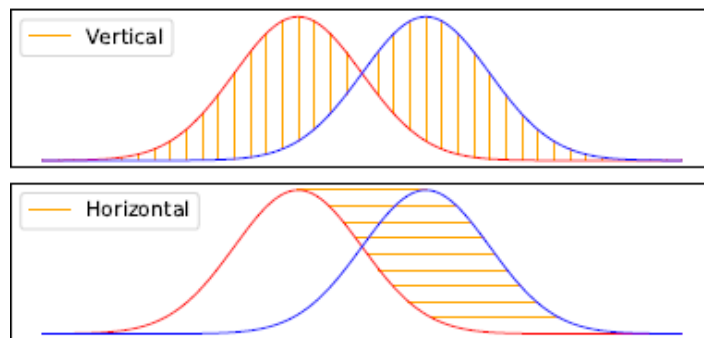


Figure 4: Comparison of two time series using the Wasserstein metric

A time series $y(t, \zeta) = x$, $x \in \mathbb{R}^m$ is modeled as a stochastic process that follows a probability measure. Calculating the empirical distribution function for the x -values that occur, one obtains a point set p with a weighting ω_p [5].

Between two time series p , $\omega_p \in \mathbb{R}^m$ and q , $\omega_q \in \mathbb{R}^n$ with probability measures γ, ν the following optimization problem, shown in (1), applies [6]:

$$W_\alpha(\gamma, \nu) = \left(\min_{(f_{ij})} \sum_{i=1}^m \sum_{j=1}^n d_{ij} f_{ij} \right)^{\frac{1}{\alpha}} \quad (1)$$

with the following properties:

- $d_{ij} = \|p_i - q_j\|^\alpha$
- $\sum_{j=1}^n f_{ij} \leq \omega_{p_i}, \forall i \in \{1, \dots, m\}$
- $\sum_{i=1}^m f_{ij} \leq \omega_{q_j}, \forall j \in \{1, \dots, n\}$
- $f_{ij} \geq 0, \forall i \in \{1, \dots, m\}, \forall j \in \{1, \dots, n\}$
- $\sum_{i=1}^m \sum_{j=1}^n f_{ij} = \min\{\sum_{i=1}^m \omega_{p_i}, \sum_{j=1}^n \omega_{q_j}\}$

This problem is then solved as a linear optimization problem.

In the Wasserstein metric, as with the p-box method, the most influential aleatory and epistemic parameters are varied, and the time series of the output variables are calculated. A Wasserstein distance is then calculated between the time series of the simulation and that of the measurement and is expressed as the model form error.

While the p-box method can be used for calculating the model form error for single points in time, the Wasserstein metric can be used to calculate the model form error for whole time series.

2.4. Sensitivity analysis

There are already numerous, different sensitivity analysis methods for different use cases as well as (commercial) sensitivity analysis tools available on the market. However, since we do not need the complete, comprehensive range of available sensitivity analysis methods (of a tool), but rather a tailored, custom-fit solution for our own investigations, we have decided to implement the required functionalities for the Rexroth BRS� Credible Simulation app ourselves.

Just as in chapter 2.3, it should be referred here mainly to the bachelor thesis of Ehsan Khademi [3] in which he has examined different sensitivity analysis methods to determine the main influencing epistemic and aleatory parameter combinations for the model form error calculation of simulation models for single points in time as well as for whole time series.

The sensitivity index of the input parameters ζ_i on the output variable $y(t_k, \zeta)$ for a point in time $t_k \in [0, T]$ is defined as $\frac{\partial}{\partial \zeta_i} y(t_k, \zeta)$. It is approximated by a finite difference method called elementary effects (EE) as follows (2):

$$EE_i = \frac{y(t_k, \zeta + \Delta_i \cdot e_i) - y(t_k, \zeta)}{\Delta_i} \quad (2)$$

Using Morris' method, the sensitivity indices are calculated at a point in time for various combinations of input values [7][8].

The result is a ranking of the parameters according to their influence on the output variable as shown in **Figure 5** [3] on the example of the well-known predator-prey model.

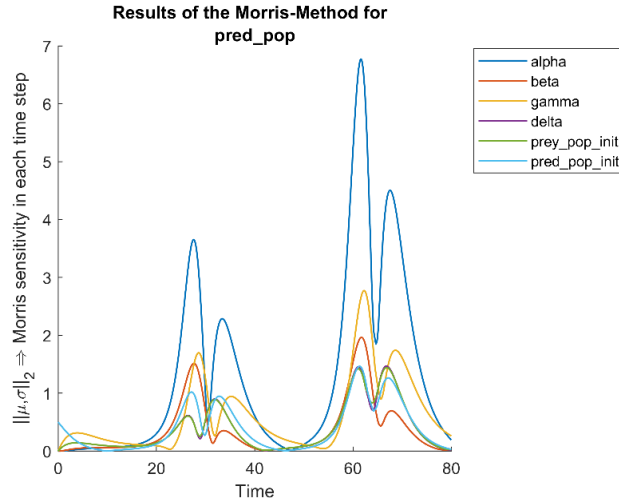


Figure 5: Sensitivity indices of the input parameters to the output variable at each time step

To check the sensitivity indices for the entire time series, the L_2 norm is calculated for the sensitivity of each individual parameter ζ_i over the entire period for all k time steps after each variation of the input values, see (3).

$$\int_0^T \frac{\partial}{\partial \zeta_i} y(t, \zeta)^2 dt \approx \sqrt{\sum_{k=0}^T (EE_i^{(k)})^2} \quad (3)$$

These values are then displayed as box plots, which provide an overview of the influence of the parameters on the time series of the output variable, exemplary see **Figure 6**.

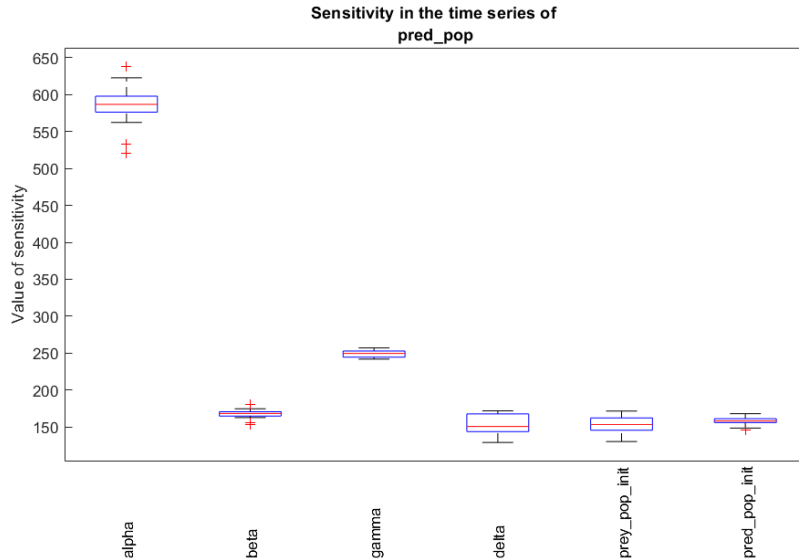


Figure 6: Sensitivity indices of the input parameters to the entire time series of the output variable

In [3], we have also investigated the use of the Wasserstein metric for a more detailed estimation of the sensitivity indices.

For calculating the model form error uncertain epistemic and aleatory parameters must be chosen by the user. To know which epistemic and aleatory parameters make sense to vary and therefore have an influence on the model result of interest, the QoI, and which do not, performing a sensitivity study

on the simulation model parameters using suitable methods before can be useful. This makes the calculation of the model form error of the simulation model for relevant QoI and therefore the credibility evaluation of the simulation model more efficient.

3. REXROTH BRSL SIMULATION LIBRARY

The Rexroth BRSL Simulation Library, shown in **Figure 7**, is a simulation model library containing simulation models, so-called digital twins, which are designed for the usage of dynamic 1D system simulations. It contains many generic components for modelling physical systems as well as specific components of Bosch Rexroth products.

The models are implemented in Modelica. Modelica is considered as a tool-independent programming language [8]. This means that the Rexroth BRSL Simulation Library models can be used in various simulation tools. The use as functional mock-up unit (FMU), a standardized exchange format, is also conceivable.

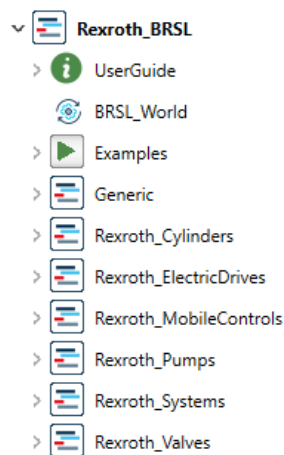


Figure 7: Rexroth BRSL Simulation Library loaded in Dymola 2023x Refresh 1

All simulation models from the Rexroth BRSL Simulation Library have been tested and validated together with experts against real measurement data as far as available or at least compared with more detailed and more complex simulation models if no measurement data have been available.

The simulation models of the Rexroth BRSL Simulation Library are pre-parameterized and designed for the generic use case of dynamic system simulations. Users can focus on system modelling using the simulation models out of the rack without the need to model single system components before on their own.

Furthermore, the generic components can be completely parameterized by the users according to their needs while the Bosch Rexroth specific components are already pre-parameterized offering the option to choose between specific products variants via their specific product type keys in a dropdown menu, see **Figure 8**.

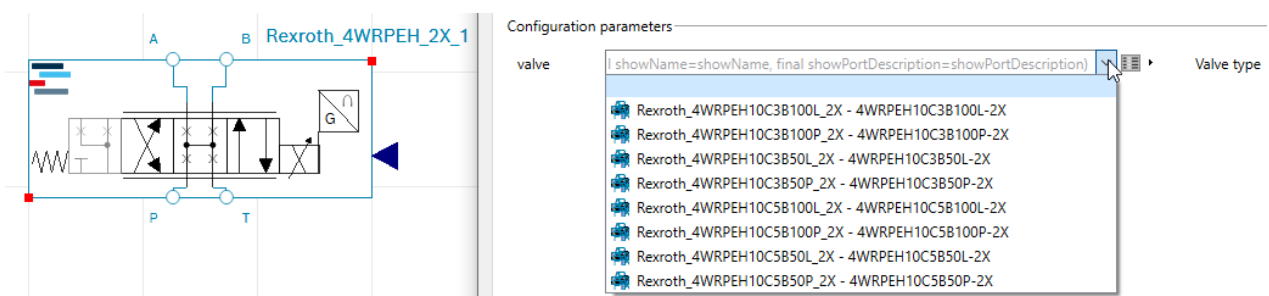


Figure 8: Pre-parameterized Bosch Rexroth_4WRPEH_2X_1 type key choices

This makes the Rexroth BRSI Simulation Library easy to use, reduces parameterization errors as well as the time needed for modelling.

The Rexroth BRSI Simulation Library can help users validating the dynamic system behavior of their systems using computer simulation models. Furthermore, it can help answering future relevant questions regarding system energy consumptions and CO₂ footprints.

Nevertheless, due to simplified modelling assumptions, the simulation models can be used differently well for different use cases. How well a simulation model fit the specific use case can be estimated using credible simulation methods.

4. REXROTH BRSI CREDIBLE SIMULATION

The Rexroth BRSI Credible Simulation app written in MATLAB can be used to calculate the model form error of an uploaded simulation model using either the p-box method or the Wasserstein metric. It is currently limited to Modelica simulation models (not only Rexroth BRSI Simulation Library models but Modelica models in general) but is to be extended for other simulation models and FMU as well.

The general workflow and the communication between the Rexroth BRSI Credible Simulation app and the simulation tool chosen by the user, pictured in **Figure 9**, is exemplary shown with Dymola as simulation tool and optional usage of the Rexroth BRSI Simulation Library.

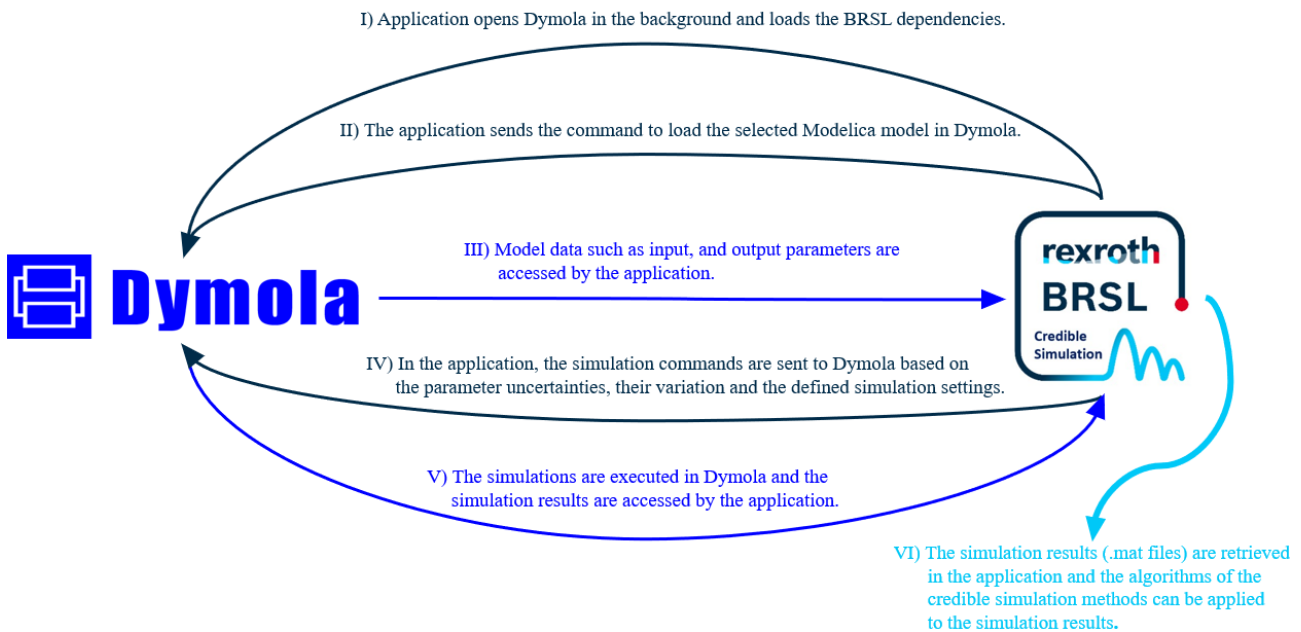


Figure 9: Communication of the Rexroth BRSI Credible Simulation app with commercial simulation tools using the example of Dymola

The users can define epistemic and aleatory parameter uncertainties on specific model parameters which can be chosen via dropdown menus (due to step III) in **Figure 9**). If only one epistemic or one aleatory parameter should be varied, for the other parameter can be set a fixed value using the manual input checkbox shown in **Figure 10**.

Moreover, the QoI can be chosen via dropdown menu as well, see **Figure 10**, and the simulation settings (*‘Start time’*, *‘Stop time’*, *‘Solver method’*, *‘Output interval’*, *‘Tolerance’*) for simulating the uploaded simulation model with varying parameter combinations of epistemic and aleatory parameters (simulating the parameter uncertainties) can be set in the app (compare step IV) in **Figure 9**).

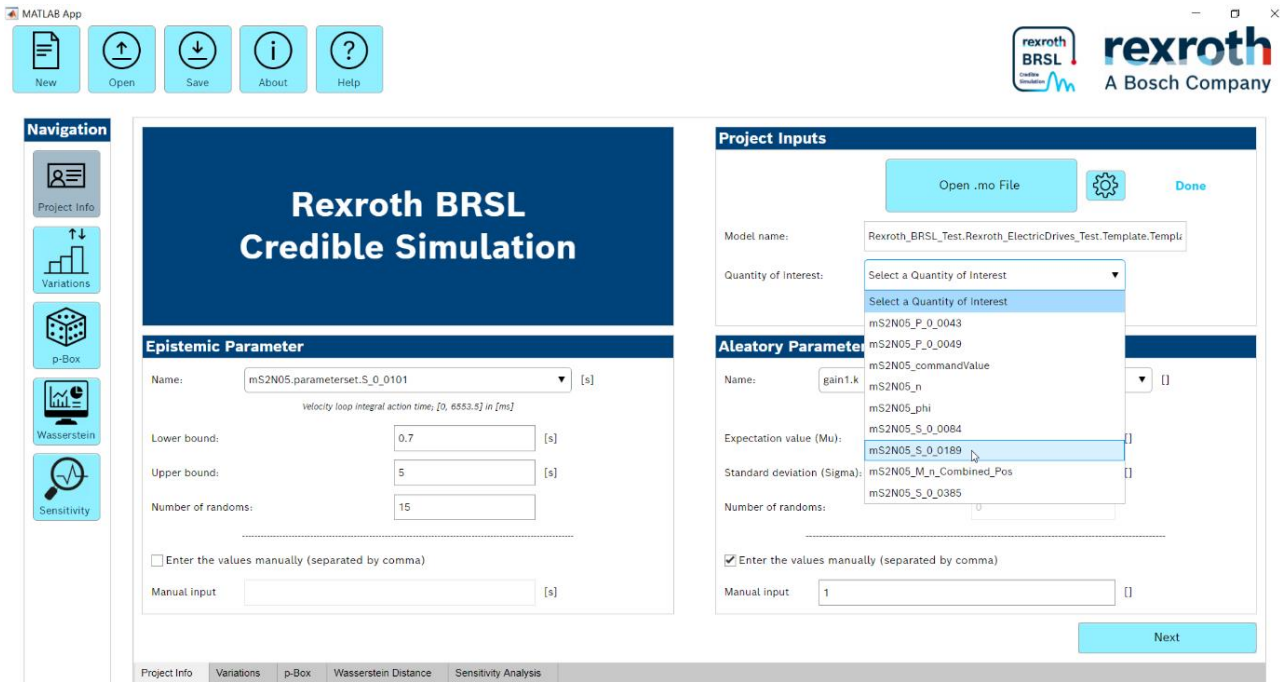


Figure 10: ‘Project Info’ tab for model and parameter settings

A tool for simulating the simulation model with each parameter combination needs to be installed locally on the user’s personal computer. Which tool should be used (currently limited to Dymola but is to be extended for other simulation tools as well) can be specified in the ‘Project info’ tab using the settings button right next to the ‘Open .mo File’ button pictured in **Figure 10**.

Starting the simulations of all variations, a pop-up window opens showing the progress of simulating all parameter combinations and plotting the result of each parameter combination in the ‘Variations’ tab. The result plot window can be maximized and curves of single (of all) parameter combinations (plotted in different colours) can be de-/selected in the results plot for better postprocessing.

The QoI can be changed without recompilation of the simulation model choosing another QoI from the ‘Changing QoI’ dropdown menu and clicking on the ‘Change’ button. Thereby, the question mark button right next to the ‘Change’ button offers further information of the chosen new QoI.

To find out which model parameters make most sense to be varied and which model parameters do not need to be varied, if they are not already known yet, there is the possibility to run a sensitivity analysis within the Rexroth BRSL Credible Simulation app (‘Sensitivity’ tab) to gain optimal simulation results.

Having defined the best suitable setup for varying epistemic and aleatory model parameters to simulate parameter uncertainties of these parameters and watching the effect on the simulation results of the specified QoI, the model form error of the uploaded user’s simulation model regarding the defined setup can be calculated using either the p-box method (‘p-Box’ tab) or the Wasserstein metric (‘Wasserstein’ tab), see step VI) in **Figure 9**.

While the Wasserstein metric exclusively provides a numerical calculation of the model form error of the whole time series, the p-box method provides a numerical calculation of the model form error as well as a graphical interpretation of specific points in time.

The points in time that are of interest for generating a p-box plot can be chosen by the user from the ‘Result variations’ plot of the specified QoI in the ‘Variations’ tab. The data can then be entered into the ‘p-Box Options’ within the ‘p-Box’ tab, see **Figure 11**.

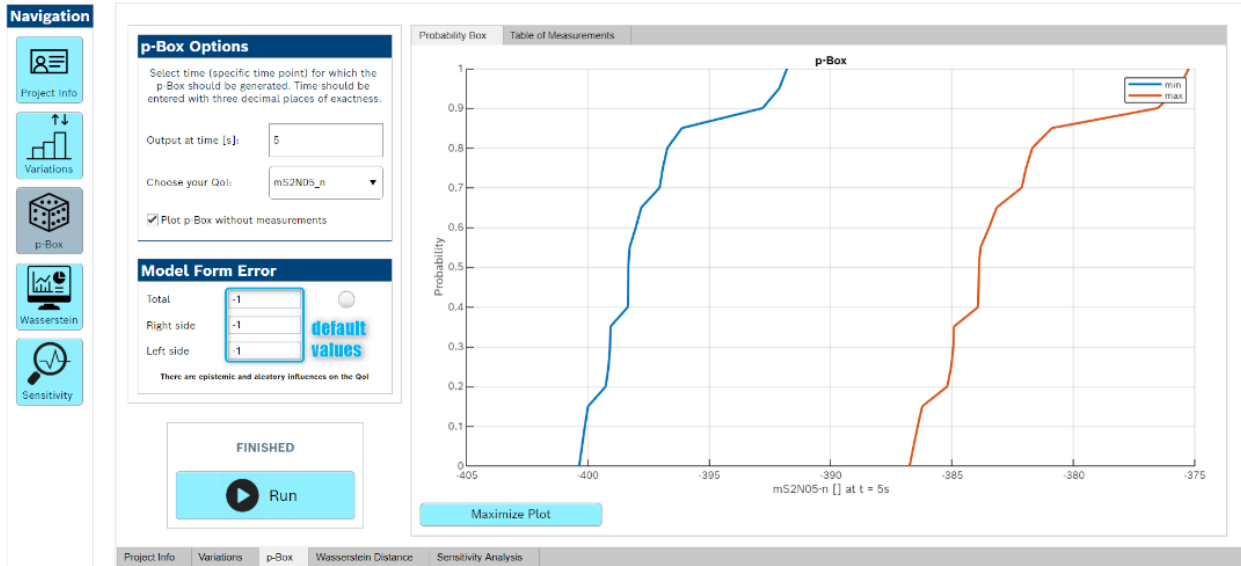


Figure 11: 'p-Box' plot without measurement data

Measurement data can be integrated into the p-box plot for the calculation of the model form error via the 'Table of Measurements' tab inside the 'p-Box' tab. This tab opens automatically unchecking the 'Plot p-Box without measurements' checkbox in the 'p-Box Options'.

Within the 'Table of Measurements' tab there is a slider choosing between manual input of measurement data or upload measurement data as .csv data into the Rexroth BRSL Credible Simulation app. The unused option is greyed out as shown in Figure 12.

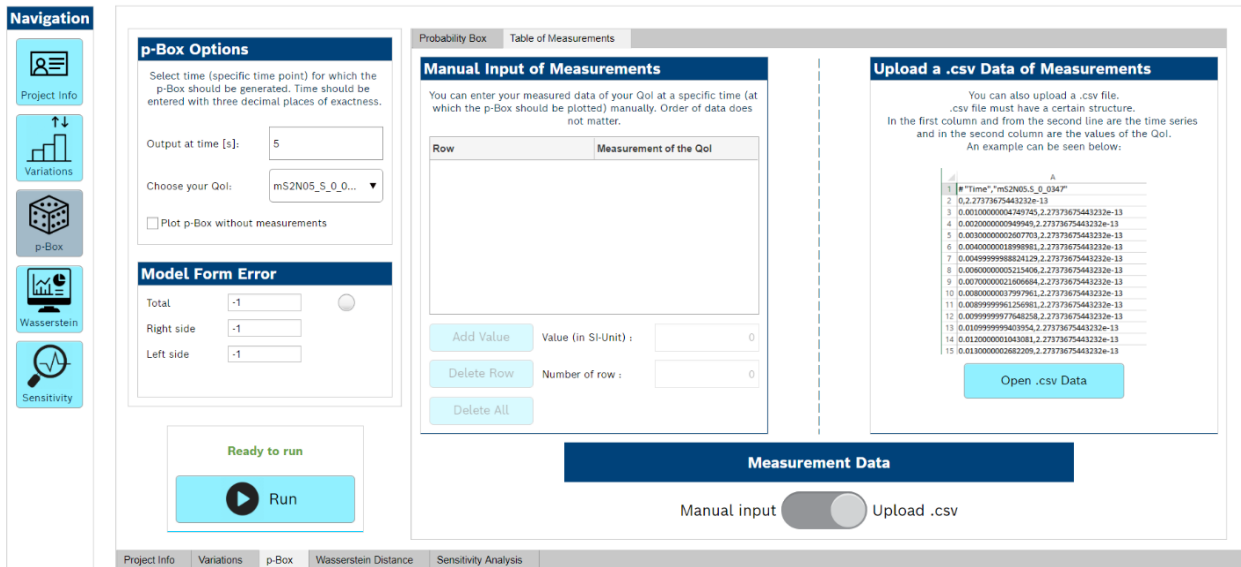


Figure 12: 'Table of Measurements' tab for the p-box

The measurement setup must match the simulation model under investigation, which is why real measurement data are often not available. Therefore, the model form error of the simulation model can not be calculated using the p-box method. Nevertheless, sensitivity studies on uncertain model parameters can be performed for a better system understanding.

As far as available, having inserted measurement data for comparison, the p-box can be plot again with the measurement data included to calculate the model form error of the uploaded simulation model for the chosen point in time.

If the uploaded simulation model totally matches the uploaded measurement data, the CDF of the measurement data will be totally inside the p-box plot and the model form error will be zero (compare **Figure 3 I**).

Otherwise, if the CDF of the measurement data is outside the p-box plot (on the left and/or on the right side), the model form error on the left side, on the right side and in total (left side + right side) will be calculated as area between the CDF of the measurement data and the left and right borders of the p-box plot of the simulation model for the specified point in time.

The circle to the right of the total model form error colours in traffic light colours depending on the calculated model form error. If the calculated model form error is zero, which means that the CDF of the measurement data is completely within the p-box limits, the circle is coloured in green. If parts of the CDF of the measurement data are outside the p-box limits the circle is coloured in orange (compare **Figure 3 II**) and if the CDF of the measurement data is completely outside the p-box limits (compare **Figure 3 III**), the circle is coloured in red.

A green circle (model form error = 0) means, that the user can trust the simulation model under the simulated conditions and for the samples used. A red circle means, that the user cannot trust the simulation model under the simulated conditions and an orange circle means, that the user can partially trust the simulation results and carefully needs to check the conditions for which the simulation model is trustworthy, and for which it is not.

The calculated model form error is use case specific. That means that the user always must check the simulation results and the calculated model form error for the specific use case the simulation model is designed for. The Rexroth BRSL Credible Simulation app therefore provides an indicator if the simulation model is (partially) credible or not.

Moreover, error handling routines are implemented for a better user experience dealing with invalid user inputs as well as errors in the used simulation tool or within the Rexroth BRSL Credible Simulation app and providing suggestions on how to solve the occurred issue.

5. USE CASE EXAMPLE

The Rexroth BRSL Credible Simulation app can be used to evaluate the simulation model credibility of simulation models such as those of the Rexroth BRSL Simulation Library, shown on a small exemplary test model of a MS2N05 electric drive of the Bosch Rexroth AG, shown in **Figure 13**.

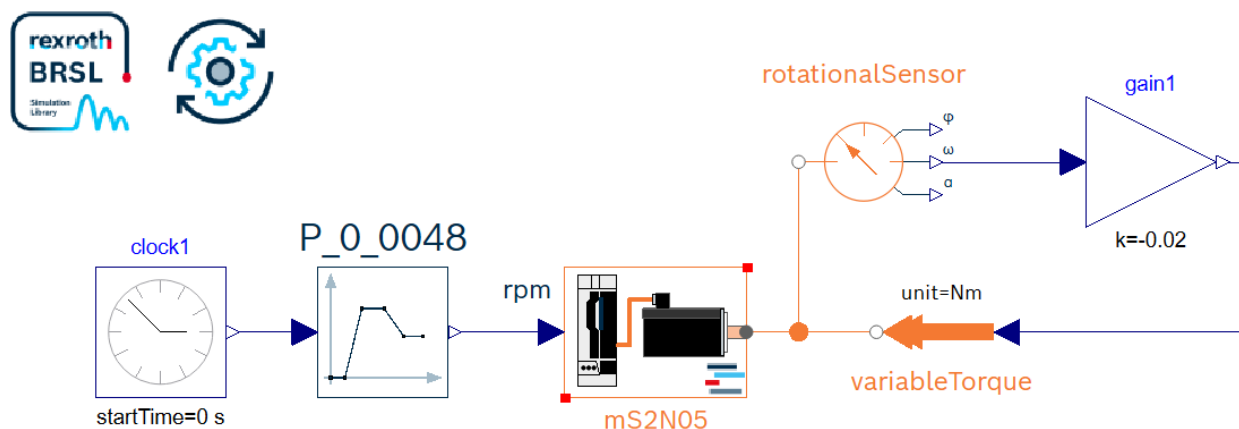


Figure 13: Simulation model of an electric drive of the Bosch Rexroth AG

In this small example a Bosch Rexroth MS2N05 electric drive is simulated in position control. It is controlled by the tabular setpoint specification of parameter ‘*P_0_0048*’ over time and runs load-free against an internal system friction modelled at the motor flange to test the idling behaviour of the electric drive.

Having talked with experts and having varied couple of different parameter combinations of epistemic and aleatory model parameters that seemed to be uncertain, it came out that for the example pictured above in **Figure 13**, the internal friction ‘*gain1.k*’ as uncertain aleatory and the motor current at standstill ‘*mS2N05.S_0_0111*’ as uncertain epistemic parameter are most suitable.

For the motor current at standstill, ‘*mS2N05.S_0_0111*’, five random epistemic parameter samples were generated within the interval of 0 – 10 Ampere, while for the internal friction ‘*gain1.k*’ twenty random aleatory parameter samples were generated with an expectation value $\mu = -0.5$ and a standard deviation of $\sigma = 0.2$ what leads to 100 simulations since each epistemic parameter value is combined with each aleatory parameter value.

The QoI are the motor speed ‘*mS2N05.n*’, shown in **Figure 14**, the torque/force feedback value ‘*mS2N05.S_0_0084*’, shown in **Figure 15**, and the velocity error ‘*mS2N05.S_0_0347*’, shown in **Figure 16**, which are interesting to look at and for which a p-box plot for the interesting point in time should be created.

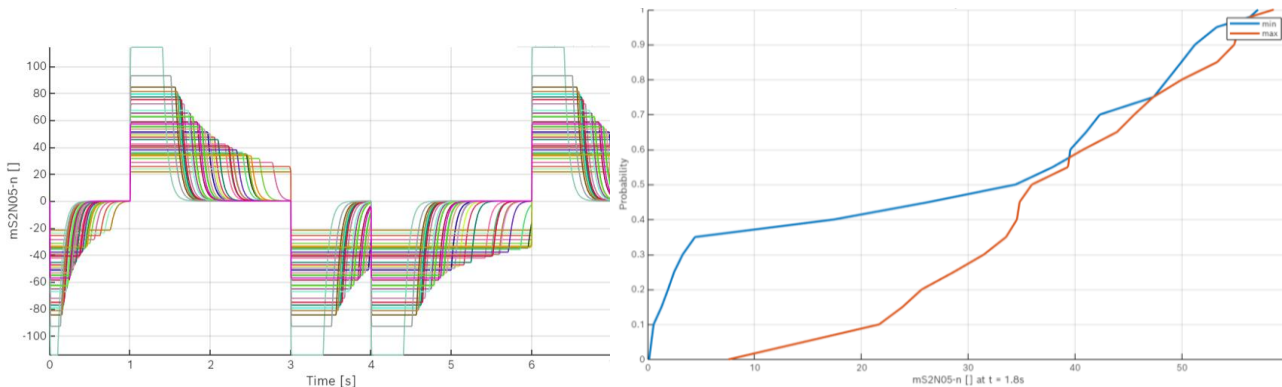


Figure 14: Variation and p-box plots of the motor speed of the MS2N05 simulation model

Figure 14 shows varying simulation results for the motor speed for the 100 simulation runs of all different parameter combinations of aleatory and epistemic parameters. The epistemic and aleatory influences on the QoI are also visible in the p-box plot. The more the CDF of the left and the right boarder of the p-box plot vary in x direction (along the time axis) the greater is the epistemic parameter influence on the QoI. The more the CDF of the left and the right boarder of the p-box plot vary in y direction (along the axis of the QoI) the greater is the aleatory parameter influence on the QoI (compare **Figure 1**).

Moreover, the permissible validity range of the real measurement data (min/max values) can be read directly from the x-axis values of the p-box limits. For example, the p-box plot of the motor speed (**Figure 14** on the right) shows that about 58% of the measurement values need to be less than 39 rpm, otherwise the measurement data will indicate a model form error. In addition, if most of the measured values are greater than 35 rpm, a model form error (outside the p-box limits) can be assumed, as most measured values are expected to be smaller than 35 rpm according to the p-box plot of the motor speed.

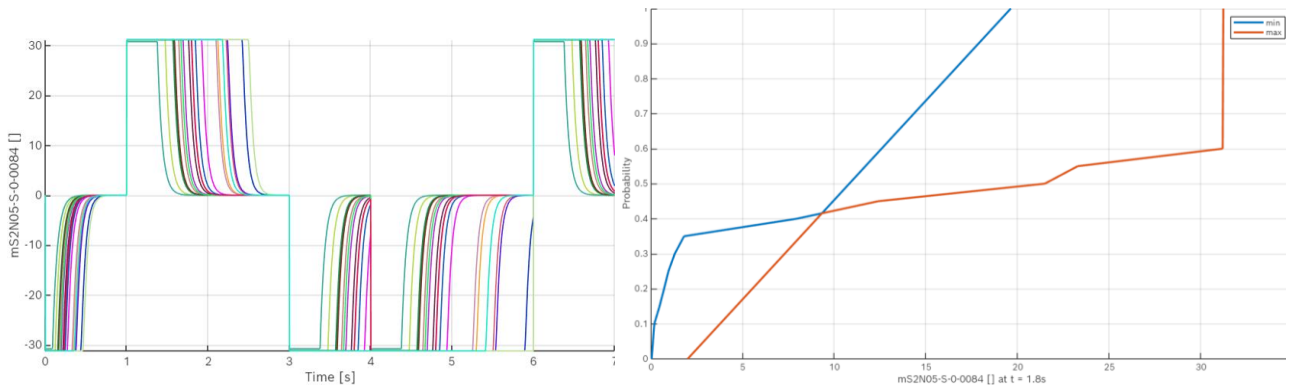


Figure 15: Variation and p-box plots of the torque/force feedback value of the MS2N05 simulation model

Figure 15 shows varying simulation results for the torque/force feedback value for the 100 simulation runs of all different parameter combinations of aleatory and epistemic parameters.

The p-box plot at $t = 1.8$ s on the right shows that measurement values greater than 31.3 N will directly lead to a model form error. Also, all values need to be greater than zero and between 19.8 N and 31.3 N. Furthermore, 41 % of the measurement values are expected to be smaller than 9 N.

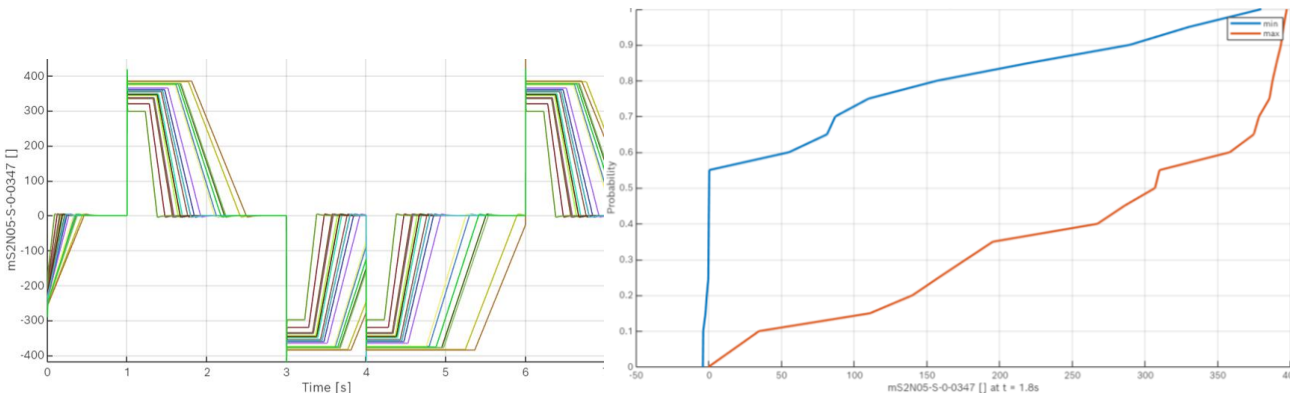


Figure 16: Variation and p-box plots of the velocity error of the MS2N05 simulation model

Figure 16 shows varying simulation results for the velocity error for the 100 simulation runs of all different parameter combinations of aleatory and epistemic parameters. The p-box shows big epistemic and aleatory parameter influences. Compared to the other two QoI, the p-box plot for the velocity error shows the biggest scope of validity.

The plotted p-boxes of the three QoI show the ranges in which the user can trust the simulation results obtained according to the aleatory and epistemic parameter combinations (values stored in ‘Variations’ plot) which is also kind of a sensitivity study for the chosen parameter combination.

Unfortunately, there are no real measurement data available for this example. If there would be measurement data available, they could be inserted in the p-box plots, and if the CDF of the measurement data would be totally within the p-box borders, the user could totally trust the simulation model for the samples used. Since there are no real measurement data available the model form error cannot be calculated via the Rexroth BRS� Credible Simulation app.

Nevertheless, there is an even more detailed simulation model for this example available which was used during development as a reference model for the validation of the MS2N05 simulation model of the Rexroth BRS� Simulation Library, so the user can trust the simulation results of the simulation model which matches reality. Only the model form error as numerical key figure describing how exact this simulation model matches the reality is missing.

6. CONCLUSION AND OUTLOOK

The paper in hand has shown how to evaluate the simulation credibility of simulation models and simulation model libraries exemplary shown on the Rexroth BRSL Simulation Library and how it is done at the Bosch Rexroth AG. It has presented useful methods such as the interval-based p-box method including a graphical interpretation as well as the Wasserstein metric for calculating the model form error of simulation models. The paper in hand has presented the inhouse Rexroth BRSL Credible Simulation app and its usage on an example simulation model of the Rexroth BRSL Simulation Library.

Nevertheless, the model form error calculation for the estimation of the simulation model credibility is use case dependent and depends on the availability and quality of measurement data for the specific use case and the epistemic and aleatory parameter samples used. The more data the better.

In the future it is planned to consecutively estimate the simulation model credibility for all simulation models of the Rexroth BRSL Simulation Library using the in chapter 2.1 mentioned framework [1] of the Robert Bosch GmbH. The result will be a simulation model credibility estimation like the one exemplary shown for the ‘*Rexroth_ElectricDrives*’ (see **Figure 7**) in **Figure 17**.

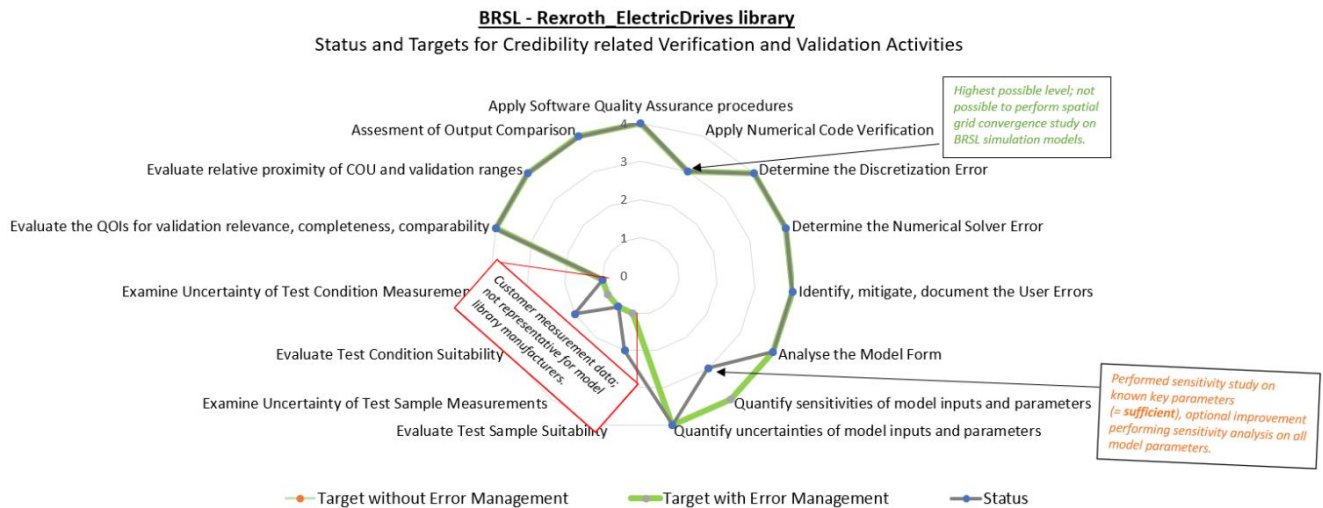


Figure 17: ‘*Rexroth_ElectricDrives*’ credibility estimation

In a next step, the credibility attributes shown in **Figure 17** can be integrated as metadata to the simulation models and thus also into the Asset Administration Shell (AAS) [9].

In the future, business models regarding certification services of the credibility of simulation models based on simulation credibility levels are also conceivable.

The further app development such as supporting further simulation tools and simulation model types or FMU in the Rexroth BRSL Credible Simulation app is constantly progressing at the Bosch Rexroth AG. Furthermore, final test validation and release for productive usage of the Rexroth BRSL Credible Simulation app is pending.

NOMENCLATURE

<i>Rexroth BRSL Simulation Library</i>	Simulation model library of the Bosch Rexroth AG containing generic 1D simulation models as well as simulation models of Bosch Rexroth specific products for system simulation
<i>Rexroth_ElectricDrives</i>	Simulation model library of the Rexroth BRSL Simulation Library containing electric drive simulation models of the Bosch Rexroth AG

<i>Rexroth BRSL Credible Simulation</i>	App developed by the Bosch Rexroth AG to estimate the credibility of simulation models	
<i>V & V</i>	Verification & Validation	
<i>NASA</i>	National Aeronautics and Space Administration	
<i>NAFEMS</i>	International Association for the Engineering Modelling, Analysis and Simulation Community	
<i>ASME</i>	American Society of Mechanical Engineers	
<i>S & M credibility</i>	Simulation & Model credibility	
<i>p-box</i>	probability box	
<i>e.g.</i>	for example	
<i>CDF</i>	Cumulative Distribution Function	
<i>QoI</i>	Quantity of Interest / Quantity of Interest	
<i>m₁, ..., m₅</i>	Exemplary CDFs	
<i>y(t, ζ)</i>	Time series as a function of time and model input parameters	
<i>t</i>	Time	s
<i>ζ</i>	Vector of model input parameters	
<i>γ, ν</i>	Probability measures of the time series	
<i>x</i>	Representation of output values of an output variable	
<i>m, n</i>	Dimension of the output variable	
<i>p, q</i>	The occurred values of the output variable	
<i>ω_p, ω_q</i>	Cumulative probability of <i>p, q</i> values	
<i>W_α</i>	Wasserstein distance of order alpha	
<i>d</i>	Cost matrix between two-point sets (Euclidean distance)	
<i>f</i>	Flow matrix between two-point sets	
<i>T</i>	Simulation time limit	s
<i>EE</i>	Elementary effects	
<i>e</i>	Unit vector	
<i>alpha</i>	Reproduction rate of the prey (Lotka-Volterra model)	
<i>beta</i>	Mortality rate of the predator per prey (Lotka-Volterra model)	
<i>gamma</i>	Mortality rate of the predator (Lotka-Volterra model)	
<i>delta</i>	Reproduction rate of the predator (Lotka-Volterra model)	
<i>prey_pop_init</i>	Initial prey population (Lotka-Volterra model)	
<i>pred_pop_init</i>	Original predator population (Lotka-Volterra model)	
<i>pred_pop</i>	Predator population (Lotka-Volterra model)	
<i>L₂ norm</i>	Euclidean norm	
<i>k</i>	Number of time steps	
<i>FMU</i>	Functional Mock-up Unit	
<i>Dymola 2023x Refresh 1</i>	Dynamic Modeling Laboratory, commercial Modelica simulation tool	
<i>Rexroth_4WRPEH_2X_1</i>	4WRPEH valve of the Bosch Rexroth AG	
<i>CO₂</i>	Carbon dioxide	
<i>MS2N05</i>	MS2N servo motor of the Bosch Rexroth AG	
<i>P_0_0048</i>	Velocity command value of controller	
<i>gain1.k</i>	Factor for friction modelling	
<i>mS2N05.S_0_0111</i>	Motor current at standstill	A
<i>mS2N05.n</i>	Motor speed	rpm
<i>mS2N05.S_0_0084</i>	Torque/force feedback value	
<i>mS2N05.S_0_0347</i>	Velocity error	rpm
<i>μ</i>	Expectation value	
<i>σ</i>	Standard deviation	
<i>AAS</i>	Asset Administration Shell	

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