REMAINING USEFUL LIFE ESTIMATION FOR RUBBER O-RING UNDER STORAGE CONDITIONS CONSIDERING DEPENDENT PERFORMANCE INDICATORS

Rentong Chen[1], Shaoping Wang[1], Chao Zhang[1,2*], Boyu Shen[1], Zhouhe Xie[1]

[1]*School of Automation Science and Electrical Engineering, Beihang University, Beijing, 100191, P.R. China*
[2]*Research Institute of Frontier Science, Beihang University, Beijing, 100191, P.R. China*

* Corresponding author: Tel.: +86 10 82338365; E-mail address: cz@buaa.edu.cn

ABSTRACT

Rubber O-ring seals have been extensively used in various types of hydraulic actuators. If rubber O-ring seals are exposed to a heat environment during storage, it will result in material aging and mechanical properties change. Therefore, it is necessary to obtain an accurate reliability model for rubber O-ring under storage conditions. This paper develops a bivariate-dependent remaining useful life prediction model based on inverse Gaussian (IG) process and Frank Copula. The nonlinear explainable IG process considering unit-to-unit variability is utilized to describe the degradation process of two performance indicators, namely as compression set and compressive stress relaxation. The time scale function in IG process model is determined by material aging model. The Frank Copula is employed to capture the dependent relationship between these indicators. The two-stage parameter estimation method is adopted to estimate parameters in IG process and copula, separately. Bayesian and Expectation-Maximum algorithm are jointly utilized to update parameters in IG process degradation model under exponential family distribution framework. Maximum likelihood estimation is used to update parameter in Frank Copula. To validate the proposed method, aging degradation tests for O-ring seals are conducted. The results demonstrate that the proposed real-time parameter updating method also improves the accuracy of online RUL assessments.

Keywords: Remaining useful life prediction, Rubber O-Ring, Nonlinear inverse Gaussian process, Copula.

1. INSTRUCTIONS

Rubber O-rings play a significant role in preventing the leakage and facilitating the efficient transmission of fluid power into mechanical power. There are a number of stored seals in factories for immediate replacement. However, rubber O-rings are susceptible to various forms of degradation, including chemical reactions or temperature changing, which will lead to premature failure. Therefore, assessing the reliability of rubber O-rings under storage conditions is a critical aspect of ensuring the safe and effective operation of various complex hydraulic systems.

Mahankar et al. [1] provided a comprehensive review on mechanisms of hydraulic seal failure and the effect of medium to high temperatures on it. Morrell et al. [2] conducted the accelerated aging test for rubber O-ring, and it indicates that the predominant reaction contributing to compression set in these studies is oxidative degradation. Previous studies mainly focused on material mechanical property change under storage conditions and constructed reliability model based on ageing mechanism. Kömmling et al. [3] investigated the influence of compression during aging by conducting five years aging test. However, physics of failure model cannot reflect dynamic characteristics during degradation. In addition, it cannot incorporate multiple uncertainties, such as time invariant and unit-to-unit variability [4].

In response to these limitations, data-driven methods have gained prominence. Stochastic process models have a great potential for capturing stochastic dynamic and they have been developed to conduct reliability assessment [5]. Sun et al. [6] used Gamma process model to describe the ageing process of rubber O-Ring. Arrhenius model is used to represent the shape parameter of Gamma degradation model. Sun et al. [7] used the Wiener process model with time scale transformation to describe the ageing degradation process. However, the degradation for mechanical components can be reflected by various dependent performance indicators Their dependence relationship may arise from shared failure modes or working operations. Wahab et al. [8] supported that there are two dependent performance indicators, namely as compression set and compressive stress relaxation, can reflect rubber O-Ring ageing degradation process. These two performance indicators are caused by the thermo-oxidative aging of the polymer. Therefore, it is necessary to consider the interdependence of the two PCs. Pan et al. [9] used bivariate time-varying copula to describe two dependent performance indicators of rubber O-Ring. However, to the best of our knowledge, few studies constructed a bivariate reliability model considering multiple uncertainties. Remaining useful life (RUL) model considering two dependent performance indicators are seldom developed. In addition, most of studies assume that the time scale function in stochastic process model to be linear or specific nonlinear function without physics of failure explanations [10].

The main contribution of this paper is constructing a bivariate reliability model based on nonlinear explainable IG process considering unit-to-unit variability and Frank copula. RUL prediction is conducted by updating the parameters in the reliability model utilizing Bayesian and Expectation-Maximum (EM) algorithm together. The rest of this paper is organized as follows. Section 2 develops bivariate reliability model based on nonlinear explainable IG process and Frank Copula. Section 3 shows and on-line RUL evaluation with new degradation data. Section 4 uses rubber O-ring ageing degradation data to show the accuracy and effectiveness of the proposed RUL prediction model. The whole paper concludes with Section 5.

2. MODEL DEVELOPMENT

2.1. Inverse Gaussian process model considering unit-to-unit variability

IG process is widely used to model monotonic degradation for mechanical component. IG process model is utilized to construct degradation model for performance indicators. The IG process $\{X(t), t \geq 0\}$ has the following properties:

- The IG process model with initial degradation value $X(0) \equiv 0$;
- The IG process model has independent degradation increments;
- The degradation increments for IG process model, $\Delta X(t) = X(t+\Delta t) - X(t)$ follows IG distribution:

$$\Delta X(t) \sim IG\left(\eta \Delta\Lambda(t;\gamma), \lambda \Delta\Lambda^2(t;\gamma)\right) \tag{1}$$

where $\eta, \lambda > 0$. η denotes the degradation rate. $\Lambda(t)$ denotes the time scale function denoting the degradation trajectory. In this study, $\Lambda(t;\gamma)$ is determined by physical law function according to the failure mechanism, γ is the corresponding parameters in time scale function. IG distribution is one of the typical exponential family of distributions, whose standard PDF equation can be expressed as

$$p(x \mid \beta) = h(x)\exp\left(\beta^T \cdot T(x) - A(\beta)\right) \tag{2}$$

where x is the random variable, β is parameters in exponential family distribution, $T(x)$ denotes the sufficient statistic, $h(x)$ denotes the base measure, $A(\beta)$ is log partition function. Let $x_t = X(t)$, its PDF under the framework of exponential family distribution is expressed as:

$$f(x_t)=\frac{1}{\sqrt{2\pi x_t^3}}\exp\left\{\begin{bmatrix}-\frac{\lambda}{2\eta^2} & \frac{\lambda\Lambda^2(t;\gamma)}{2}\end{bmatrix}\begin{bmatrix}x_t\\ \frac{1}{x_t}\end{bmatrix}-\left[-\frac{\lambda\Lambda(t)}{\eta}-\frac{1}{2}\log\lambda\Lambda^2(t;\gamma)\right]\right\} \tag{3}$$

where $\beta=\begin{bmatrix}\beta_1 & \beta_2\end{bmatrix}^T=\begin{bmatrix}-\frac{\lambda}{2\eta^2} & \frac{\lambda\Lambda^2(t;\gamma)}{2}\end{bmatrix}^T$, and $A_{IG}(\beta)$ can be expressed as $A_{IG}(\beta)=2\sqrt{\beta_1\beta_2}-\frac{1}{2}\log(-2\beta_2)$. According to the properties of exponential family distribution, the expectation and variance of $X(t)$ can be obtained by

$$\mathrm{E}[X(t)]=\frac{\partial}{\partial\beta_1}A_{IG}(\beta)=\eta\Lambda(t;\gamma) \tag{4}$$

$$\mathrm{Var}[X(t)]=\frac{\partial^2}{\partial\beta_1}A_{IG}(\beta)=\frac{\eta^3}{\lambda}\Lambda(t;\gamma) \tag{5}$$

The failure time T represents the point in time at which the degradation path first reaches the predefined threshold d. The PDF and CDF of failure time T can be found in Eq. (6) and Eq.(7), respectively.

$$F_T(t)=\Phi\left(\sqrt{\frac{\lambda}{d}}\left(\Lambda(t;\gamma)-\frac{d}{\eta}\right)\right)-\exp\left(\frac{2\lambda}{\eta}\Lambda(t;\gamma)\right)\Phi\left(-\sqrt{\frac{\lambda}{d}}\left(\Lambda(t;\gamma)+\frac{d}{\eta}\right)\right) \tag{6}$$

$$\begin{aligned}f_T(t)=&\sqrt{\frac{\lambda}{d}}\Phi\left(\sqrt{\frac{\lambda}{d}}\left(\Lambda(t;\gamma)-\frac{d}{\eta}\right)\right)-\frac{2\lambda}{\eta}\exp\left(\frac{2\lambda}{\eta}\Lambda(t;\gamma)\right)\Phi\left(-\sqrt{\frac{\lambda}{d}}\left(\Lambda(t;\gamma)+\frac{d}{\eta}\right)\right)\\&+\sqrt{\frac{\lambda}{d}}\exp\left(\frac{2\lambda}{\eta}\Lambda(t;\gamma)\right)\Phi\left(-\sqrt{\frac{\lambda}{d}}\left(\Lambda(t;\gamma)+\frac{d}{\eta}\right)\right)\end{aligned} \tag{7}$$

From the meaning of failure life time, the RUL L_j can be seen as the failure time of IG process $\{X'(l_j), l_j>0\}$ with corresponding threshold $d-X(t_j)$, where $X'(l_j)=X(l_j+t_j)-X(t_j)$. Similarly, the corresponding CDF and PDF for RUL L_j can be also obtained.

Considering the design errors, processing techniques and different working conditions, unit-to-unit variability can be represented in degradation paths. It is more suitable to include unit-to-unit variability in the reliability model. The expectation of degradation data is used to represent unit-to-unit variability in the IG process model. We can find that the degradation rate η is showed as the form of $\frac{1}{\eta}$ or $\frac{1}{\eta^2}$ in Eq.(3). For the convenience of following modeling and computation. We assume that $\frac{1}{\eta}$ follows normal distribution as $\frac{1}{\eta}\sim N\left(\mu_\eta,\frac{1}{\sigma_\eta^2}\right)$, and there will be

$$g_\eta\left(\eta \mid \mu_\eta, \frac{1}{\sigma_\eta^2}\right) = \frac{\sigma_\eta}{\sqrt{2\pi \cdot \eta^2}} \exp\left\{-\frac{1}{2}\sigma_\eta^2\left(\frac{1}{\eta} - \mu_\eta\right)^2\right\} \tag{8}$$

The, the CDF and PDF for failure lifetime T can be expressed as Eq. (9) and Eq.(10), respectively.

$$\begin{aligned} F_T(t) &= \Phi\left(\sqrt{\frac{\lambda}{d}} \cdot \frac{\sigma_\eta \Lambda(t;\gamma) - \mu_\eta \sigma_\eta d}{\sqrt{\sigma_\eta^2 + \lambda d}}\right) \\ &- \exp\left(2\lambda\mu_\eta \Lambda(t;\gamma) + \frac{2\lambda^2 \Lambda^2(t;\gamma)}{\sigma_\eta^2}\right) \Phi\left(-\sqrt{\frac{\lambda}{d}} \cdot \frac{\left(\sigma_\eta^2 + 2\lambda d\right)\Lambda(t;\gamma) + \mu_\eta \sigma_\eta^2 d}{\sqrt{\sigma_\eta^4 + \lambda d \sigma_\eta^2}}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} f_T(t) &= \sqrt{\frac{\lambda}{d}} \frac{\sigma_\eta}{\sqrt{\sigma_\eta^2 + \lambda d}} \Phi\left(\sqrt{\frac{\lambda}{d}} \cdot \frac{\sigma_\eta \Lambda(t;\gamma) - \mu_\eta \sigma_\eta d}{\sqrt{\sigma_\eta^2 + \lambda d}}\right) \\ &- \exp\left(2\lambda\mu_\eta \Lambda(t) + \frac{2\lambda^2 \Lambda^2(t;\gamma)}{\sigma_\eta^2}\right) \exp\left(2\lambda\mu_\eta + \frac{4\lambda^2 \Lambda(t;\gamma)}{\sigma_\eta^2}\right) \Phi\left(-\sqrt{\frac{\lambda}{d}} \cdot \frac{\left(\sigma_\eta^2 + 2\lambda d\right)\Lambda(t;\gamma) + \mu_\eta \sigma_\eta^2 d}{\sqrt{\sigma_\eta^4 + \lambda d \sigma_\eta^2}}\right) \\ &+ \sqrt{\frac{\lambda}{d}} \cdot \frac{\sigma_\eta^2 + 2\lambda d}{\sqrt{\sigma_\eta^4 + \lambda d \sigma_\eta^2}} \cdot \exp\left(2\lambda\mu_\eta \Lambda(t;\gamma) + \frac{2\lambda^2 t^2}{\sigma_\eta^2}\right) \cdot \Phi\left(-\sqrt{\frac{\lambda}{d}} \cdot \frac{\left(\sigma_\eta^2 + 2\lambda d\right)\Lambda(t;\gamma) + \mu_\eta \sigma_\eta^2 d}{\sqrt{\sigma_\eta^4 + \lambda d \sigma_\eta^2}}\right) \end{aligned} \tag{10}$$

2.2. Bivariate Reliability Model

The copula function is an effective tool in reliability engineering for describing the dependence between two degradation performance indicators. Frank copula, one of the Archimedean copulas, has symmetric structure. Frank copula can capture both positive and negative correlations between variables, so it has been widely used in bivariate reliability analysis The CDF and PDF of Frank Copula can be found in Chen et al. [11]. Considering two dependent performance indicators $X_1(t)$ and $X_2(t)$ with threshold d_1 and d_2, the reliability function $R(t)$ and failure lifetime T can be given by:

$$R(t) = \Pr\left\{X_1(t) < d_1, X_2(t) < d_2\right\} \tag{11}$$

$$T = \min\left\{T_1, T_2\right\} \tag{12}$$

According to Chen et al. [11], when the unit-to-unit variability is not taken into account, Eq. (11) can be furthered calculated by:

$$R(t) = C\left(R_1(t), R_2(t)\right) \tag{13}$$

In addition, RUL L_j at degradation time t_j is obtained by:

$$F_{L_j}\left(l_j\right) = C\left(F_{L_j,1}\left(l_j\right), F_{L_j,2}\left(l_j\right)\right) \tag{14}$$

Considering the unit-to-unit variability, the reliability and RUL function can be obtained by:

$$R^v(t) = \iint R\left(t \mid \eta_1, \eta_2\right) g_\eta\left(\eta_1\right) g_\eta\left(\eta_2\right) \mathrm{d}\eta_1 \mathrm{d}\eta_2 \tag{15}$$

$$F_{L_j}^v\left(l_j\right) = \iint F_{L_j}\left(l_j \mid \eta_1, \eta_2\right) g_\eta\left(\eta_1\right) g_\eta\left(\eta_2\right) \mathrm{d}\eta_1 \mathrm{d}\eta_2 \tag{16}$$

3. STATISTICAL INFERENCE

We assume that N samples with $M_i\left(i=1,2,\cdots,N\right)$ measurement times are conducted in the degradation test. Let $X_k\left(t_{ij}\right)$ denote the degradation data for kth performance indicator of the ith test sample at jth measurement $\left(k=1,2;i=1,2,\cdots,N;\ j=1,2,\cdots,M_i\right)$. Assuming that $\boldsymbol{\Theta}$ denotes the unknown parameters in the proposed model, and it will be $\boldsymbol{\Theta}=\left(\mu_{\eta1},\sigma_{\eta1}^2,\lambda_1,\gamma_1,\mu_{\eta2},\sigma_{\eta2}^2,\lambda_2,\gamma_2,\theta\right)$. Let $\boldsymbol{\Theta}_k^{\mathrm{IG}}$ denote the unknown parameters in IG process, that is $\boldsymbol{\Theta}_k^{\mathrm{IG}}=\left(\mu_{\eta k},\sigma_{\eta k}^2,\lambda_k,\gamma_k\right)$. Assuming that $\boldsymbol{D}=\left(\boldsymbol{D}_1,\boldsymbol{D}_2\right)$ denotes the degradation data for two dependent performance indicators. We will discuss the unknown parameter estimation methods both in initial value determination stage (off-line stage) and on-line stage.

3.1. Initial Value Determination

Initial values for unknown parameters $\boldsymbol{\Theta}=\left(\mu_{\eta1},\sigma_{\eta1}^2,\lambda_1,\gamma_1,\mu_{\eta2},\sigma_{\eta2}^2,\lambda_2,\gamma_2,\theta\right)$ are determined by the off-line degradation data. The reliability model based on bivariate dependent performance indicators is given by:

$$\begin{aligned}
&\Delta X_k\left(t_{ij}\right)=X_k\left(t_{i,j+1}\right)-X_k\left(t_{ij}\right)\\
&\Delta X_k\left(t_{ij}\right)\sim IG\left(\eta_k\Delta t_{ij},\lambda_k\left(\Delta\Delta t_{ij}\right)^2\right),\frac{1}{\eta_k}\sim N\left(\mu_{\eta k},\frac{1}{\sigma_{\eta k}^2}\right)\\
&f\left(\Delta X_1\left(t_{ij}\right),\Delta X_2\left(t_{ij}\right)\right)=\prod_{k=1}^{2}\prod_{i=1}^{N}\prod_{j=1}^{M_i}f_k\left(\Delta X_k(t_{ij});\boldsymbol{\Theta}_k^{\mathrm{IG}}\right)\prod_{i=1}^{N}\prod_{j=1}^{M_i}c\left(F_1\left(\Delta X_k(t_{ij})\right),F_2\left(\Delta X_k(t_{ij})\right);\theta\right)\\
&\left(k=1,2;i=1,2,\cdots,N;\ j=1,2,\cdots,M_i\right)
\end{aligned}\tag{17}$$

Then, the log-likelihood function based on complete data $\left(\boldsymbol{D},\eta_1,\eta_2\right)$ is given by:

$$\begin{aligned}
\ln L\left(\boldsymbol{\Theta}\mid\boldsymbol{D},\eta_1,\eta_2\right)=&\sum_{i=1}^{N}\sum_{j=1}^{M_i}\ln c\left(F_1\left(\Delta X_k(t_{ij})\mid\eta_{1,i}\right),F_2\left(\Delta X_k(t_{ij})\mid\eta_{2,i}\right);\theta\right)\\
&+\sum_{k=1}^{2}\sum_{i=1}^{N}\sum_{j=1}^{M_i}\ln\left[f_k\left(\Delta X_k(t_{ij})\mid\eta_{k,i}\right);\boldsymbol{\Theta}_k^{\mathrm{IG}}\right]+\sum_{k=1}^{2}\sum_{i=1}^{N}\ln g_\eta\left(\eta_{k,i}\right)
\end{aligned}\tag{18}$$

Bayesian MCMC is employed to estimate unknown parameters:

$$\pi\left(\boldsymbol{\Theta}\mid\boldsymbol{D}\right)\propto\ln L\left(\boldsymbol{\Theta}\mid\boldsymbol{D},\eta_1,\eta_2\right)\cdot\pi\left(\boldsymbol{\Theta}\right)\tag{19}$$

where $\pi\left(\boldsymbol{\Theta}\right)$ is the prior distribution, $\pi\left(\boldsymbol{\Theta}\mid\boldsymbol{D},\boldsymbol{T}\right)$ is the posterior distribution. In our study, non-informative distribution is used to estimate $\boldsymbol{\Theta}$. The estimation results are chosen as the initial value in the on-line parameter update process. Then, the reliability function given the posterior distribution of $\boldsymbol{\Theta}$ can be expressed as:

$$R^v\left(t\mid\boldsymbol{D}\right)=\int_{\boldsymbol{\Theta}}R^v\left(t\mid\boldsymbol{\Theta}\right)\pi\left(\boldsymbol{\Theta}\mid\boldsymbol{D}\right)\mathrm{d}\boldsymbol{\Theta}=\mathrm{E}_{\pi(\boldsymbol{\Theta}\mid\boldsymbol{D},\boldsymbol{T})}\left[R^v\left(t\mid\boldsymbol{\Theta}\right)\right]\tag{20}$$

3.2. On-line Parameter estimation

In this section, the on-line parameter and RUL update algorithm based on on-line degradation data is discussed. Note that the time scale parameter $\boldsymbol{\gamma}$ reflecting the degradation trajectory, there are not large fluctuations for $\boldsymbol{\gamma}$ during the whole service time. Therefore, $\boldsymbol{\gamma}$ will not be updated at the on-line stage, and it will be beneficial to reduce computational burden. As for other parameters in $\boldsymbol{\Theta}$, two-stage estimation method, inference function for the margins (IFM) is utilized to estimate the unknown parameters in IG process and dependence parameter in Frank copula, separately [12]. The two-stage estimation method has significant computational advantages in terms of computational efficiency.

Stage 1. Unknown Parameter Update in IG process

Assuming that there is degradation data $\boldsymbol{D}_{k,1:m} = \left[\Delta X_k(t_1), \Delta X_k(t_2), \cdots, \Delta X_k(t_m)\right] (k=1,2)$ for both performance indicators at time t_m. Using the IG distribution under exponential family framework, we can get the likelihood function:

$$p\left(\boldsymbol{D}_{k,1:m};\beta\right) = \prod_{j=1}^{m} h\left(\Delta X_k\left(t_j\right)\right)\exp\left(\beta^T \cdot T\left(\Delta X_k\left(t_j\right)\right) - A(\beta)\right) \tag{21}$$

Based on the Bayesian theory, there is:

$$p\left(\frac{1}{\eta_k} \mid \boldsymbol{D}_{k,1:m}\right) \propto p\left(\boldsymbol{D}_{k,1:m} \mid \frac{1}{\eta_k}\right) \cdot p\left(\frac{1}{\eta_k}\right) \tag{22}$$

where $p\left(\frac{1}{\eta_k}\right)$ is the prior information for $\frac{1}{\eta_k}$. It follows the normal distribution, which can be represented under exponential family framework. $p\left(\frac{1}{\eta_k} \mid \boldsymbol{D}_{k,1:m}\right)$ is the posterior distribution given the real time degradation $\boldsymbol{D}_{k,1:m}$, and it also follows normal distribution, that means $p\left(\frac{1}{\eta_k} \mid \boldsymbol{D}_{k,1:m}\right) \sim N\left(\mu_{\eta k,m}, \frac{1}{\sigma^2_{\eta k,m}}\right)$. Regardless of the terms without $\frac{1}{\eta_k}$, Eq. (22) can be furthered expressed as

$$p\left(\frac{1}{\eta_k} \mid \boldsymbol{D}_{k,1:m}\right) \propto \left\{\prod_{j=1}^{m} \exp\left\{-\frac{\lambda}{2\eta^2}\Delta X_k\left(t_j\right) + \frac{\lambda \cdot \Delta\Lambda\left(t_j;\hat{\gamma}\right)}{\eta}\right\}\right\} \cdot \exp\left\{-\mu_{\eta k}\sigma^2_{\eta k}\frac{1}{\eta_k} - \frac{1}{2}\sigma^2_{\eta k}\frac{1}{\eta_k^2}\right\} \tag{23}$$

Then, we can obtain the mean and variance of posterior distribution of $\frac{1}{\eta_k}$:

$$\begin{cases} \mu_{\eta k,m}^{\text{posterior}} = \dfrac{\mu_{\eta k}\sigma^2_{\eta k} + \lambda\Lambda\left(t_m;\hat{\gamma}\right)}{\sigma^2_{\eta k} + \lambda X_k\left(t_m;\gamma\right)} \\ \left(\sigma^2_{\eta k,m}\right)^{\text{posterior}} = \sigma^2_{\eta k} + \lambda X_k\left(t_m;\hat{\gamma}\right) \end{cases} \tag{24}$$

Since $\frac{1}{\eta_k}$ follows normal distribution, it cannot be observed directly. Expectation-Maximum (EM) algorithm can be used to estimate unobserved latent variables. The log-likelihood function for complete data can be expressed as:

$$\log p\left(\boldsymbol{D}_{k,1:m},\frac{1}{\eta_k}|\boldsymbol{\Theta}\right)=\log p\left(\boldsymbol{D}_{k,1:m}\,|\,\frac{1}{\eta_k},\boldsymbol{\Theta}\right)+\log p\left(\frac{1}{\eta_k}\,|\,\boldsymbol{\Theta}\right) \tag{25}$$

Regardless of the base measure, Eq. (25) can be furthered calculated as:

$$\log p\left(\boldsymbol{D}_{k,1:m},\frac{1}{\eta_k}|\boldsymbol{\Theta}\right)\Leftrightarrow\sum_{j=1}^{m}\left\{\beta^T T\left(\Delta X_k\left(t_j\right)\right)-A_{IG}\left(\beta\right)\right\}+\left\{\alpha^T T\left(\frac{1}{\eta_k}\right)-A_N\left(\alpha\right)\right\} \tag{26}$$

Given the degradation data $\boldsymbol{D}_{k,1:m}$, the estimated value $\hat{\boldsymbol{\Theta}}_{k,m}^{IG(g)}=\left[\mu_{\eta k}^{(g)},\sigma_{\eta k}^{2(g)},\lambda_k^{(g)}\right]$ at g th iteration step in the EM algorithm, we can get the posterior value $\hat{\boldsymbol{\Theta}}_{k,m}^{IG\text{posterior}(g)}$ through Eq. (22). As for the EM algorithm under exponential family framework, $T\left(\frac{1}{\eta_k}\right)$ is replaced by $\mathrm{E}_{p\left(\frac{1}{\eta_k}|\boldsymbol{D}_{k,1:m},\hat{\boldsymbol{\Theta}}_{k,m}^{IG\text{posterior}(g)}\right)}\left[T\left(\frac{1}{\eta_k}\right)\right]$ to complete E-Step. Note that $\frac{1}{\eta_k}$ is also involved in β, which also needs to be replaced. Let $q\left(\frac{1}{\eta_k}\right)=p\left(\frac{1}{\eta_k}\,|\,\boldsymbol{D}_{k,1:m},\hat{\boldsymbol{\Theta}}_{k,m}^{IG\text{posterior}(g)}\right)$, and the Q function in E-Step is given by:

$$\begin{aligned}Q\left(\boldsymbol{\Theta}_k^{IG}\,|\,\boldsymbol{D}_{k,1:m},\hat{\boldsymbol{\Theta}}_{k,m}^{IG\text{posterior}(g)}\right)=&\sum_{j=1}^{m}\left\{\left\{\mathrm{E}_{q\left(\frac{1}{\eta_k}\right)}\left[\beta\right]\right\}^T T\left(\Delta X_k\left(t_j\right)\right)-A_{IG}\left(\mathrm{E}_{q\left(\frac{1}{\eta_k}\right)}\left[\beta\right]\right)\right\}\\&+\left\{\alpha^T\cdot\mathrm{E}_{q\left(\frac{1}{\eta_k}\right)}\left[T\left(\frac{1}{\eta_k}\right)\right]-A_N\left(\alpha\right)\right\}\end{aligned} \tag{27}$$

Let $\frac{\partial}{\partial\boldsymbol{\Theta}_k^{IG}}Q\left(\boldsymbol{\Theta}_k^{IG}\,|\,\boldsymbol{D}_{k,1:m},\hat{\boldsymbol{\Theta}}_{k,m}^{IG\text{posterior}(g)}\right)=0$, then we can get:

$$\mu_{\eta k,m}^{(g+1)}=\mu_{\eta k,m}^{\text{posterior}(g)} \tag{28}$$

$$\sigma_{\eta k,m}^{2(g+1)}=\left(\sigma_{\eta k,m}^{2}\right)^{\text{posterior}(g)} \tag{29}$$

$$\lambda_{k,m}^{(g+1)}=\frac{m}{\sum_{j=1}^{m}\left\{\left(\mu_{\eta k,m}^{2(g+1)}+\frac{1}{\sigma_{\eta k,m}^{2(g+1)}}\right)\Delta X_k\left(t_j\right)-2\mu_{\eta k,m}^{2(g+1)}\Delta\Lambda\left(t_j;\hat{\gamma}\right)+\frac{\left(\Delta\Lambda\left(t_j;\hat{\gamma}\right)\right)^2}{\Delta X_k\left(t_j\right)}\right\}} \tag{30}$$

That completes Stage 1.

Stage 2. Unknown Parameter Update in Frank Copula

The second stage is to update the unknown parameter θ in Frank copula. The estimated value in stage 1 can be used to calculated marginal distribution, and then we can get the θ by maximum likelihood estimation (MLE):

$$\hat{\theta}_m=\arg\max_{\theta}\ln L^c\left(\theta\,|\,\boldsymbol{D}_{k,1:m}\right) \tag{31}$$

That completes Stage 2. Then, we can also update RUL when the real-time degradation data comes. Algorithm 1 shows the whole procedure of on-line parameter update.

Algorithm 1: On-line parameter estimation procedure

Input: $\mu_{\eta,0}$, $\sigma^2_{\eta,0}$, $\lambda_{k,0}$, θ_0 and $\hat{\gamma}$ (Obtained from the off-line reliability analysis), On-line degradation data $\boldsymbol{D}_{k,1:m}$, iteration times L, and EM algorithm iteration threshold ε

Output: $\mu_{\eta k,m}$, $\sigma^2_{\eta k,m}$, $\lambda_{k,m}$ and θ_m

Step 1: Initialize: $\left\{\mu_{\eta k}^{(0)}, \sigma_{\eta k}^{2(0)}, \lambda_k^{(0)}\right\}$

For $g = 1:L$ or $\left|\hat{\boldsymbol{\Theta}}_k^{IG(g+1)} - \hat{\boldsymbol{\Theta}}_k^{IG(g)}\right| \geq \varepsilon$

  Calculate $\left\{\mu_{\eta k,m}^{\text{posterior}(g)}, \left(\sigma^2_{\eta k,m}\right)^{\text{posterior}(g)}\right\}$ based on (24);

  Calculate $\left\{\mu_{\eta k}^{(g+1)}, \sigma_{\eta k}^{2(g+1)}, \lambda_k^{(g+1)}\right\}$ based on (28), (29) and (30);

End for

Step 2: Calculate $\hat{\theta}_m$ based on (31)

Step 3: Update RUL based on (14).

When getting the new degradation date D_{m+1}, repeat Step 1 ~ Step 3

4. CASE STUDY: RUBBER O-RING STORAGE DEGRADATION TEST

In this section, we conduct the rubber O-Ring storage degradation test to illustrate the effectiveness and advantages of the proposed RUL prediction method.

4.1. Rubber-O Ring Storage Degradation Test

The rubber O-ring is widely used both in static applications and dynamic applications. However, during the long-term storage, large number of spare parts will age due to its storage conditions. It will result in the reducing their sealing mechanical properties, eventually losing their use value or affecting the reliability of the mechanical system. The degradation performance indicator, compression set and compressive stress relaxation, can both describe the degradation process during the storage stage. The compression set reflects the sign of elasticity and deformation resistance of rubber materials. The compressive stress relaxation represents the change value of elastic pressure before and after aging. The smaller the compression permanent deformation, the better the resilience of the material and the stronger the deformation resistance. Compression set tends to increase with aging time. Wahab et al. [8] supported that these two performance indicators are statistically dependent with each other. Therefore, it is reasonable to assume that both two performance indicators could be described by IG process, and their dependence could be described by Frank Copula.

The degradation test rig is shown in **Figure 1**. Compression set ε (Performance Indicator 1, PI1) and compressive stress relaxation σ_t/σ_0 (Performance Indicator 2, PI2), were tested under storage environment. The specific test methods can be found in Pan et al. [9]. There are ten test samples in the degradation test. The degradation paths after linear transformation for compression set $X_1(t) = -\ln(1-\varepsilon)$ and compressive stress relaxation $X_2(t) = -\ln(\sigma_t/\sigma_0)$ are shown in **Figure 2**. Based on the engineering experience, the failure thresholds of PI1 and PI2 are set to be 0.4 and 0.7, respectively. Thus, the thresholds after linear transformation are $d_1 = 0.511$ and $d_2 = 0.3567$ .The aging model commonly used to describe the seal materials can be expressed as:

$$P = A\exp\left(-Kt^{\gamma}\right) \tag{32}$$

where P denotes the material properties for seal, t denotes the service time, K denotes the coefficient related to aging, A and γ are related constants. To determine the specific expression of $\Lambda(t)$ in IG process, Eq. (32) also needs to be linearized. Then, as for materials aging, $\Lambda(t)$ should be expressed as:

$$\Lambda(t) = t^{\gamma} \tag{33}$$

According to the principle of hold-out method, the degradation data from the first 8 samples are utilized to determine the initial values, while the remaining 2 samples are utilized for verifying the proposed model.

Figure 1: The degradation test rig.

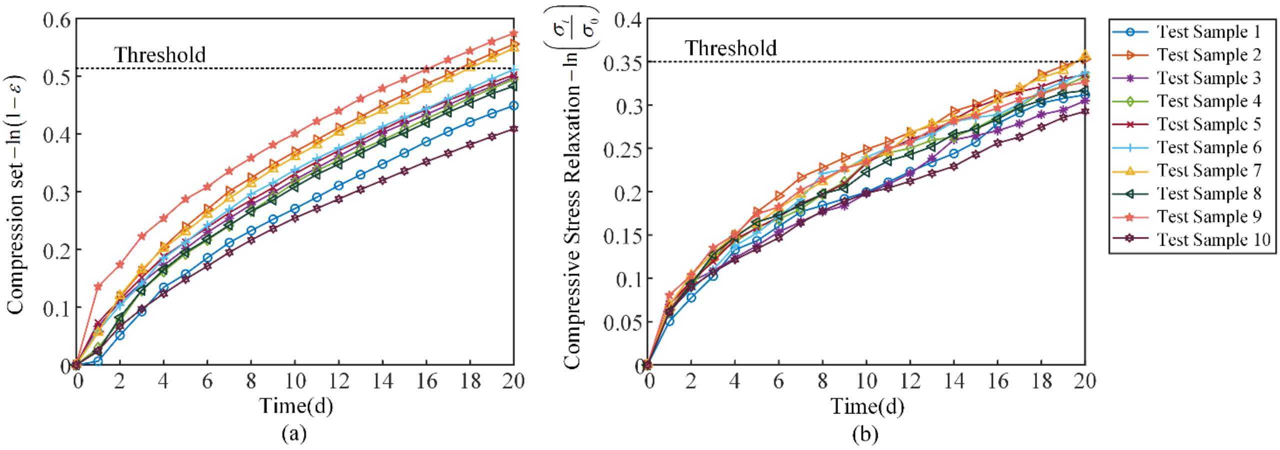


Figure 2: Linear transformation degradation paths (a) Performance Indicator 1, PI1: Compression set (b) Performance Indicator 2, PI2: Compressive stress relaxation

4.2. RUL prediction

The initial values are determined by Bayesian MCMC method proposed in Section 3.1. The estimation results are shown in **Table 1**. Note that the parameters in time scale functions γ_1 and γ_2 will not be updated at the online stage.

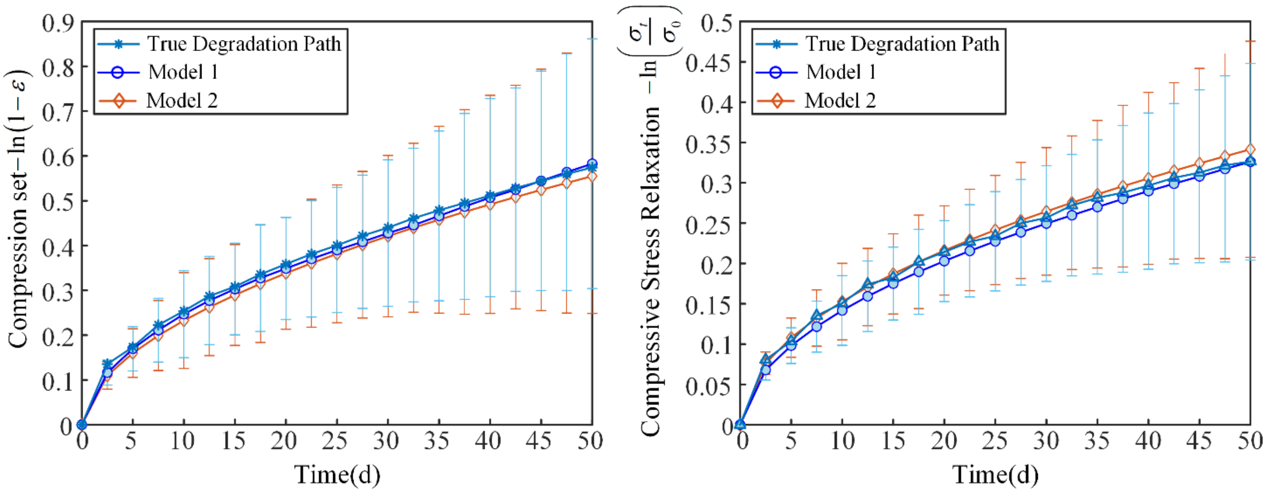
Table 1: Parameters Estimation Results

Unknow parameter	Estimation Results		
	Medium	2.5pc	97.5pc
$\mu_{\eta 1}$	0.9212	0.9008	0.9432
$\sigma_{\eta 1}^2$	79.83	77.62	81.93
λ_1	95.8	90.32	100.44
γ_1	0.5412	0.5311	0.5509
$\mu_{\eta 2}$	0.5312	0.5176	0.5542
$\sigma_{\eta 2}^2$	58.16	56.19	60.44
λ_2	149.6	145.63	154.37
γ_2	0.5321	0.5296	0.5398
θ	20.69	18.43	22.64

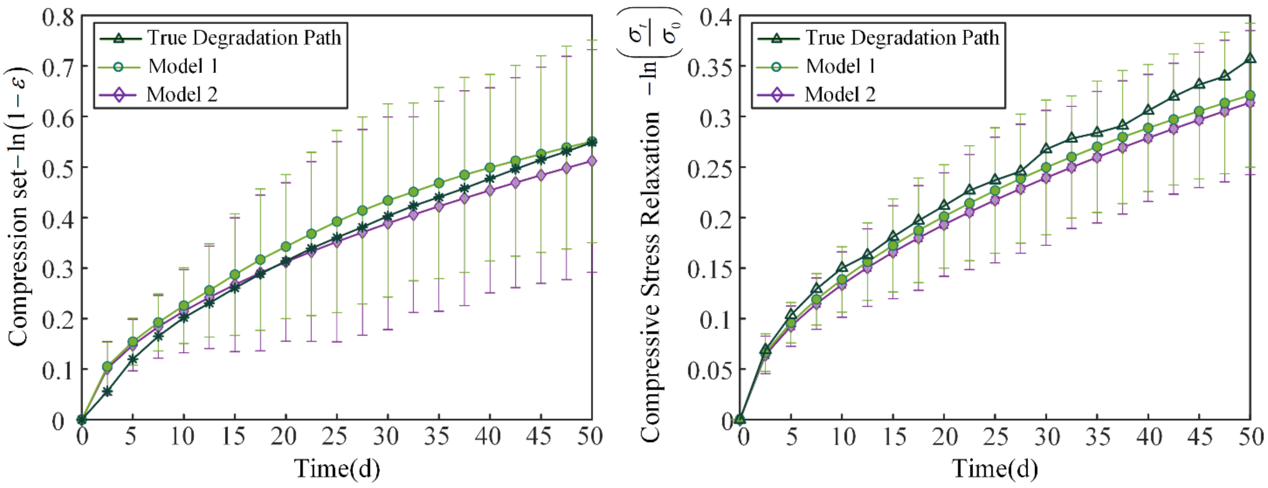
The 9th and 10th seal sample are used to conduct on-line parameters update and RUL prediction. The Bayesian-EM update algorithm is shown based on the method proposed in Section 3.2. We also compare two models in order to show the accuracy and effectiveness of the on-line reliability model, e.g.:

- Model 1 (The on-line RUL update algorithm proposed in our study): Considering PI1 and PI2, using Bayesian and EM algorithm to update unknown parameters in IG process
- Model 2: Considering PI1 and PI2, only using EM algorithm to update parameters in IG process

During the parameter update process, we can find that parameter estimation may lack accuracy during the early stages due to limited data. However, as more degradation data accumulates, the estimated values tend to stabilize and improve accuracy. **Figure 3** shows the estimated degradation paths and corresponding 95% confidence interval obtained based on the parameter update results. It can be found that predicted degradation values at different measurement time are all within the confidence intervals. As more degradation data accumulates, the predicted degradation paths become increasingly accurate. The estimated degradation values for 9th test sample are much more accurate than the estimated degradation values for 10th test sample. The estimated degradation values for 9th test sample almost consistent with the real degradation trajectory.



(a) Estimated degradation paths for 9th test sample



(b) Estimated degradation paths for 10th test sample

Figure 3: Estimated degradation paths

Figure 4 shows the reliability curves obtained by off-line parameter estimation results, 9th and 10th sample on-line parameter estimation results. We can find that differences between the off-line reliability evaluation and individual reliability evaluation due to the unit-to-unit variability. If the reliability of individual component is assessed, it is necessary to update the parameters in the reliability analysis model. Or else, there will be mistakes when using population parameters to conduct individual reliability.

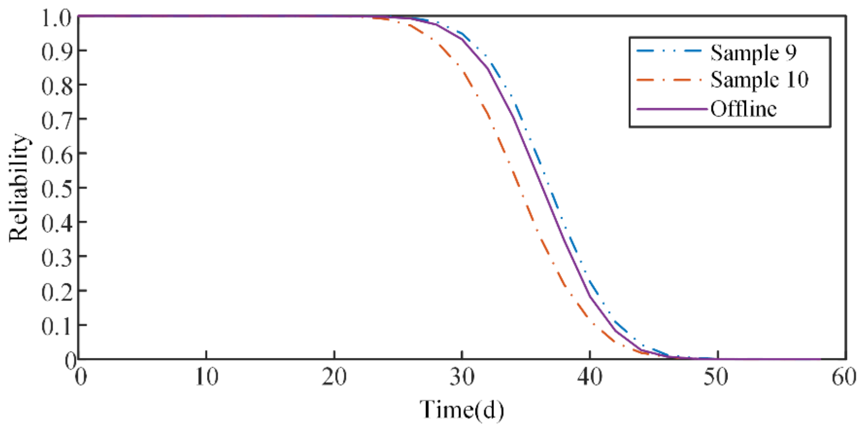


Figure 4: Estimated degradation paths

Figure 5 shows the PDF of estimated RUL. **Table 2** compares the mean square errors (MSE) between true RUL and estimated RUL. We can find that the estimated RUL by Model 1 is more accurate that the estimated RUL by Model 2. From **Figure 5**, it is noticeable that the PDF of RUL becomes progressively narrower as more degradation data accumulates. It indicates that the RUL prediction is getting more accurate. The PDF of RUL based on Model 1 is taller than the PDF of RUL based on Model 2. It means that Model 1 is more accurate than Model 2. In addition, the prediction accuracy of 9th test sample is higher than 10th test sample, which is also consistent with the prediction accuracy of degradation trajectories. Based on the above discussion, it can be concluded that employing Bayesian and EM algorithms together to update unknown parameters leads to more accurate RUL evaluation results.

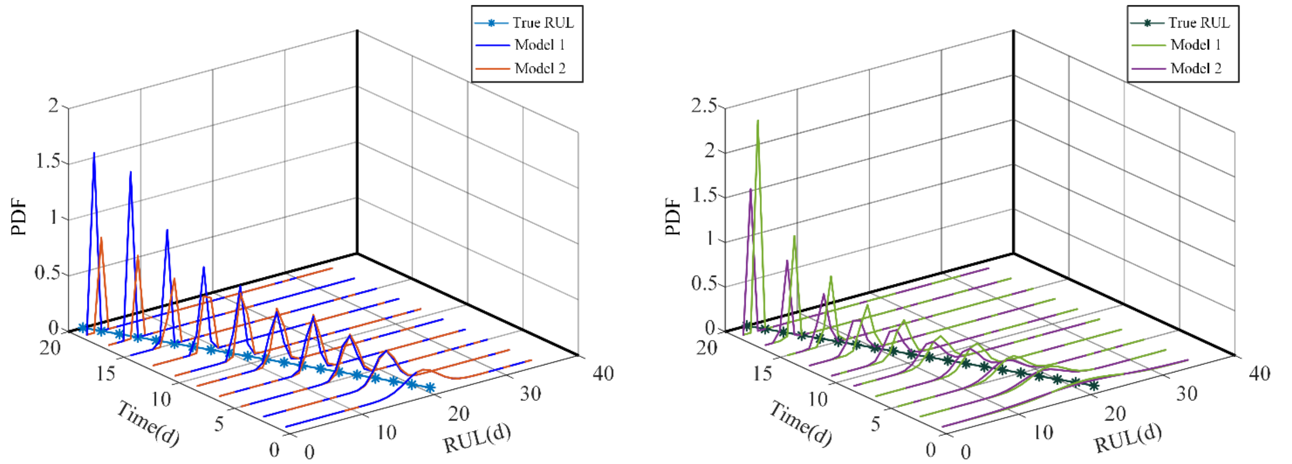


Figure 5: The PDF for RUL estimated by Model 3 (a) 9th test sample (b) 10th test sample

Table 2: MSE between true RUL and estimated RUL

Test Sample/MSE	Model 1	Model 2
9th Sample	0.3142	0.7962
10th Sample	0.9063	3.1562

5. CONCLUSION AND FUTURE WORK

This study utilizes nonlinear explainable IG process model considering the individual variability to describe two performance indicators. The time scale function is determined by failure mechanism model. The Frank Copula is employed to establish the reliability model, considering the correlation between the two performance degradation indicators. The initial values are determined by Bayesian MCMC method based on the degradation data. As for online RUL prediction, the two-stage parameter estimation approach is employed. The unknown parameter updates in the IG process model are achieved by combining the Bayesian and Expectation-Maximization algorithms, while the unknown parameter in the Frank Copula is estimated using maximum likelihood estimation. The effectiveness and accuracy of the proposed reliability assessment model are validated through an aging test of rubber O-ring seals conducted under the storage environment. The results indicate that the proposed model and algorithm significantly enhance the accuracy of remaining useful life prediction. Future work will concentrate on integrating the failure mechanism model and machine learning model to achieve more comprehensive and accurate reliability evaluation.

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