

DEVELOPMENT OF A GENERIC TEST RIG FOR THE DETERMINATION OF THE INFLUENCE OF NON-NEWTONIAN FLUID PROPERTIES ON THE LEAKAGE CHARACTERISTIC OF ROTATING DISPLACEMENT PUMPS

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ABSTRACT

The efficiency of rotating displacement pumps is mainly influenced by internal leakage. When designing the pumps for Newtonian fluids, the manufacturers report satisfactory quality of predictive models. For the non-Newtonian fluids however, there are no suitable models that allow the manufacturers to predict the pumps behaviour or efficiency. Many of the manufacturers try to estimate the efficiency for non-Newtonian applications with their well-established models, which have however been developed specifically for Newtonian fluids. The efficiency is therefore overestimated by a significant amount. Especially non-Newtonian fluids with pseudoplastic and viscoelastic properties are of interest since the overlap of these properties has not been well researched. The most important blind spots in literature are the impact and interdependency of the moving boundary within the sealing gaps and the contraction flow for this type of fluid. The Chair of Fluid Systems addresses these blind spots with the development of a test rig for generic studies of the gap flow representative of the characteristic sealing gaps within pumps. The test rig is characterised by its modular construction which allows the efficient investigation of geometric gap parameters and fluid properties.

Keywords: non-Newtonian leakage, pump efficiency, rotating displacement pump, gap flow, contraction flow

1. INTRODUCTION

In order to improve their sustainability, both plant operators as well as component and system manufacturers seek to meet the desired function with best availability and acceptance while minimising efforts, specifically life cycle cost. The desired function for pumps is the supply of a required volume flow at defined pressure. In addition to the manufacturing costs, the life cycle costs for pumps depend mainly on operating costs, which depend on the efficiency of the pump in the specific application.

Rotating displacement pumps, e.g. rotary lobe pumps, gear pumps or screw pumps, are used throughout many industrial applications especially for pumping high viscosity fluids or satisfying high pressure demand. The fluids cover a range from gaseous to low-viscosity fluids, e.g. in the field of oil hydraulics, as well as high-viscosity media, e.g. for the food industry [1].

The achievable efficiency of displacement pumps is limited by internal leakage. The internal leakage mechanisms are inherent to the specific pump types. The clearance gaps between the components represent leakage paths [2]. A detailed prediction of the volumetric efficiency requires the knowledge of the leakage characteristics of a given fluid when passing through these tight sealing gaps. **Figure 1** shows characteristic sealing gaps for rotating displacement pumps.

While the behaviour of the leakage flow when pumping Newtonian fluids has been well understood [3, 4], the assessment of leakage flow of non-Newtonian fluids still presents a challenge. This is due to the complex rheological behaviour of the fluids.

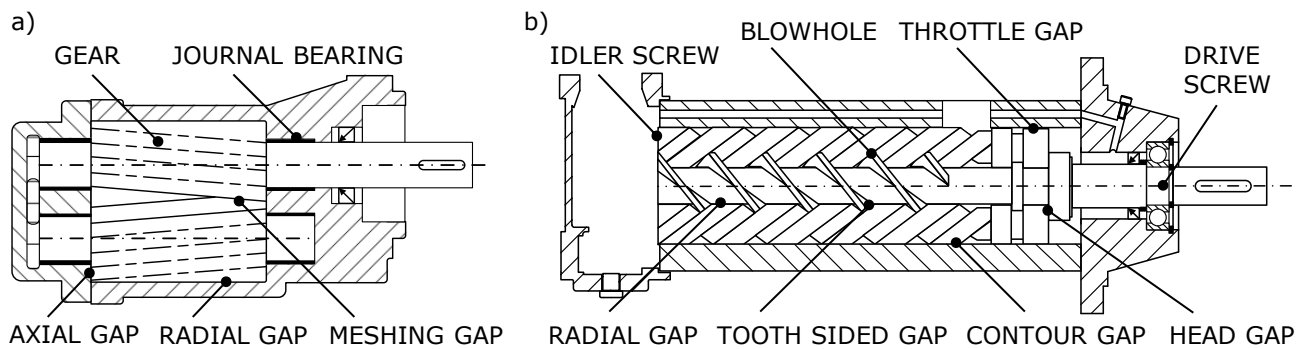


Figure 1: Gaps in a) an external gear pump, b) triple screw pump

The non-Newtonian fluids which are most relevant for the assessment of the gap flows can be categorised as shear-thinning fluids with viscoelastic properties. When experiencing shear stresses, the viscosity decreases due to the shear-thinning properties, which leads to an increase in leakage flow compared to Newtonian fluids with a viscosity similar to the zero-shear viscosity. Therefore, engineering the pumps according to the models for Newtonian fluids significantly underestimates the leakage flow. This effect may be superimposed by viscoelastic effects, which are known to be of importance e.g. for polymer solutions or melts. For the flow through small orifices or piping components, an increase in the pressure difference is documented that is linked to the normal stress effects. Similar effects can be expected for the flow across the contraction from the fluid chamber into the sealing gap, thus reducing the leakage. The superposition of this so-called extensional flow in the entrance to the sealing gap with the Couette-flow and the Poiseuille-flow that are relevant within the sealing gap, has not yet been subject to research.

Due to the lack of suitable models for the estimation of the leakage flow for non-Newtonian fluids, the prediction of the volumetric efficiency is subject to great uncertainty. Many pumps that are operated in many industrial applications day by day are therefore a major contributor for not reaching sustainability goals, especially when considering life cycle cost and emissions. Some manufacturers report cases, of newly developed pumps, being unable to satisfy the pumping requirements. This leads to leads to great financial losses both for the manufacturer as well as the operator.

The few empirical models that are already in use by some of the manufacturers are only validated for specific fluid properties. Slight deviations of the fluid properties or the specifications of the pumps from the validated limits, cannot be handled dependently by these black-box models. Thus, in order to find the optimal specifications for their pumps, the manufacturers are forced to resort to experimental analysis of their specific pumps for the specific applications. This approach allows little carry-over of the findings to future projects, while taking a lot of time and generating great costs. Many of the relevant process fluids do not allow such dedicated experimental work at the pump manufacturers facilities e.g., polymer melts.

In short, there is a great need for (i) precise and dependable models for the prediction of non-Newtonian leakage flow and (ii) a viable database for validating future models. In order to address this issue, the Chair of Fluid Systems proposes to approach the different geometrically complex sealing gaps that are characteristic for the rotating displacement pumps with a generic experiment that also includes the relative motion of the boundaries. Due to the modular structure of the proposed test rig, models for the different gap leakage flows can be derived, thus upgrading the modelling of the pumps for non-Newtonian flows.

2. LEAKAGE OF NON-NEWTONIAN FLUIDS

The variety of fluids, collectively known as non-Newtonian fluids is vast. All of these fluids diverge from the linear and purely viscous behaviour that defines Newtonian fluids [5]. For the research of gap flow in the context of rotating displacement pumps, the pseudoplastic fluids with viscoelastic behaviour and without time-dependencies are of most relevance, since they cover the most frequent applications with non-Newtonian fluids. This kind of behaviour is documented for example for polymer melts or solutions. In order to achieve consistent fluid properties, silicone oils (Polydimethylsiloxane PDMS) are chosen for the experimental work. Silicone oils can be described as polymer melts that are liquid at room temperature. The rheological behaviour of silicone oils is well documented. [6–8]

2.1. Research on leakage in rotating displacement pumps

A literature review shows multiple analytical, numerical and experimental works that cover leakage flows in rotating displacement pumps. These however cover only applications with Newtonian fluids [2, 9–11]. The few works that cover the gap flow of non-Newtonian fluids range from findings in valves and piping components [12] to findings in pumping applications of centrifugal [13] and displacement pumps [14]. Rather than focusing on the modelling of non-Newtonian fluids, these works however focus on deriving effective viscosities for the specific applications. This approach does not allow for a carry-over of the results to different applications, fluids, pump sizes or pump types. Such field studies are therefore too close to the application to allow for extensive, yet cost effective parameter studies which would allow, to derive leakage mechanisms and models. They can be considered as black-box models and do not resolve the isolated gap flows.

Rituraj and Vacca [15] model the non-Newtonian leakage flow within gear pumps in their numerical simulation. The authors differentiate between the leakage paths within these pumps. Even pump specific behaviour such as the deflection of the gears due to the journal bearings can be modelled. The authors show good agreement of the simulation and experimental findings. Viscoelastic properties of the fluids are considered; however, the model fluids and their parameters are subject to confidentiality. The results can therefore not be evaluated or reused.

Research activities at the Chair of Fluid Systems in Darmstadt in the context of developing a type-independent efficiency definition for positive displacement pumps have shown that for Newtonian fluids, the internal leakage for screw pumps is independent of the rotational speed of the pump [2]. Imamoglu [16] reports similar observations for gear pumps, both based on analytical considerations and numerical simulation efforts. These findings are however not expected to be applicable to non-Newtonian fluids, which is one of the research questions to be addressed with help of the experimental test rig that is covered by this paper.

2.2. Research on generic gap flows

Extensive research on generic gap flows of non-Newtonian fluids can be found. These allow for a more differentiated view of the gap flow types that might be of relevance for the characteristic gaps in displacement pumps. A specific gap flow can thereby be further divided into the zones of characteristic flows as shown in **Figure 2**. The entrance and exit of the gap show an extensional flow, while the flow through the gap length with constant gap height can be modelled as a laminar flow.

The most relevant properties for the non-Newtonian fluids in scope for the research project are the shear viscosity and the extensional viscosity. For the type of flow relevant for the research activities, the planar extensional flow is assumed to be representative [17]. For Newtonian fluids the shear viscosity and the extensional viscosity for planar extensional flow are coupled by the following Trouton ratio, for non-Newtonian flow this generally is not the case [18].

$$\mu_{E,planar} \sim 4 \mu \quad (1)$$

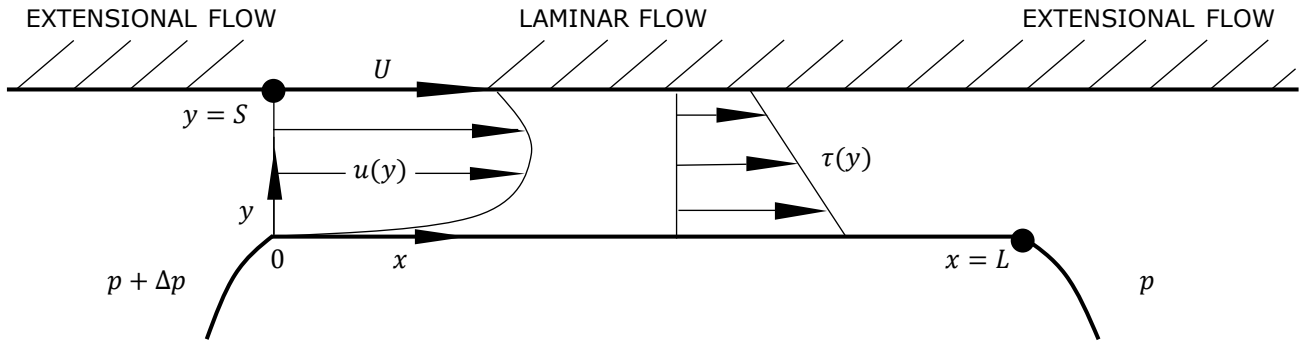


Figure 2: Characteristic gap for the leakage flow in displacement pumps

The shear and strain rates for the flow along sealing gaps are location dependent. In order to determine the actual viscosities, a fully resolved analysis of the non-Newtonian flows would be required. For the complex gap geometries and flow states in displacement pumps this is however not feasible. It is therefore not possible to estimate the local Reynolds number. The transition from laminar to turbulent flow is shifted to lower Reynolds numbers due to the shear-thinning fluid properties, which further increases the difficulties in predicting the leakage behaviour. [19] Therefore, modified or generalised Reynolds numbers are often used. These however are specific to the underlying fluid models. Rütten et al. [19] report the local Reynolds number to be multiple orders of magnitude higher compared to the global Reynolds number. Lacking the detailed knowledge for the superposition for all relevant effects, an apparent viscosity μ_{app} is often used. [18]

Apart from the extensional flow in the entrance and exit of the gap, the flow along the gap length L with constant gap height is also of interest. The laminar flow is expected to be represented by a Couette-Poiseuille flow due to the pressure difference across the sealing gap and the relative motion of the boundaries represented by the pump components surfaces. [9] The interaction of Couette and Poiseuille flow are well researched and documented both for Newtonian and non-Newtonian fluids. For non-Newtonian fluids the flow profiles may not simply be superimposed, since they are not independent from one another. [20] The resulting flow is known as the generalised Couette-Flow. The modelling of Couette and Poiseuille flow is based on the assumption of fully developed flow and may therefore not include all relevant effects, especially for viscoelastic fluids.

The viscoelastic fluid properties have been subject to research especially in the context of orifice flow. Rheological models often describe an orifice of capillary length L and radius R with an effective capillary length $L + n_{corr}R$ for Newtonian fluids. The approach allows for the consideration of viscous resistance due to the velocity gradient at the entrance of the orifice. Bagley [21] shows that the correction coefficient n_{corr} is a function only of L/R even for non-Newtonian fluids with viscoelastic properties. The Bagley correction thus allows for the characterisation of flows entering a capillary, as may be the case at nozzles or measuring orifices. More than 50 years after the works of Bagley, square edged orifices are still the subject of research activities, which highlights the complexity and relevance of detailed models for small channel flows. [22, 23]

Apart from the works of Bagley [21], the works cited within this chapter do not consider fluids with elastic properties; apart from the simulative work of Rituraj and Vacca on gear pumps (see 2.1), the research centre is not aware of any work that investigates similar contraction flows in the context of moving boundaries.

In addition to the literature research, initial simulative work on the leakage of non-Newtonian fluids has already been carried out during the preliminary project at the Institute of Fluid Systems [24, 25]. Initial results confirm the assumption that the wall movement of non-Newtonian fluids must not be neglected. The test rig presented in this paper is therefore intended to offer data to be used in the validation of simulative efforts.

3. GAP FLOW TEST RIG DESIGN

Even though there has been extensive research on the fundamentals of gap flows of non-Newtonian fluids, the possibility of a transfer to the gap flows within the sealing gaps of rotating displacement pumps is not possible. Therefore, the design process of such pumps is far from straight forward and often requires very cost intensive iterative approaches. Evaluating the finalised pumps in their specific applications is rarely possible and places a big burden on the customers, yet the lessons learned are rarely, if at all transferable to future projects in different applications. With such approaches there is no practical way of isolating the different gap flows and performing a structured optimisation of characteristic geometric and operating parameters. In order to address these issues, the Chair of Fluid Systems at TU Darmstadt has set out to design a generic experiment which will allow the isolated consideration of the different characteristic sealing gaps. The experiment is to be situated in between the application specific considerations based on selected pump types for their specific applications and the research on standard gap flows, where the transfer to real engineering tasks is, as of now, not yet feasible. Based on the experimental results, a model is to be developed which allows the transfer to real engineering tasks. The model is expected to vastly improve the efficiency that is achieved in practical operation as well as the design process for non-Newtonian applications.

3.1. Hypotheses

For the experimental considerations of non-Newtonian gap leakage, a modular generic test rig has been designed. The test rig will aid the research on the following hypotheses:

1. When pumping non-Newtonian fluids, the internal leakage within the pumps can no longer be considered to be independent of the rotational speed. Therefore, the experiments on the generic gap flow should not show that the drag flow for the generic gaps with superimposed contraction flow is of negligible influence.
2. Upon entering into the gap and exiting out of the gap, due to the contraction flow of the non-Newtonian fluid, dissipative losses are present. By considering different characteristic gap shapes, the influence of the geometric parameters on the dissipative losses can be resolved.
3. Based on the results from hypotheses 1 and 2, guidelines for the design of sealing gaps and selection of pump operating parameters can be derived. These guidelines allow the manufacturers to improve their pump portfolio and allows them proactive design for new applications and markets with non-Newtonian fluids.

The results will be the basis for the development of an extensively validated, empirical model which allows the description of the leakage behaviour of non-Newtonian fluids across characteristic gaps within rotating displacement pumps. Furthermore, the experimental results are to be published in a reusable database in order to encourage further research.

3.2. Requirements

In order to be representative of the continuous pumping cycles that are most relevant for applications of the pumps, and in order to ensure quasi-steady flow, the test rig is required to offer continuous operation, batch operation is not desired. Thus, the measuring section needs to provide a continuously moving wall. The hydraulic test field needs to provide a continuous supply of conditioned fluid.

Further requirements for the test rig are a modular construction, ease and unambiguity of the assembly process as well as ease of adaptability for future, more detailed research projects. With these requirements the test rig is expected to provide reliable and repeatable test results even after major disassembly and reconstruction with minimum effort for validation and setup.

For the design of the test rig, the research parameters are specified. The range of zero shear viscosity extends from 0.1 Pas to 1 Pas, the pressure drop at the gap to be investigated will be up to 50 bar. Due to sealing components, the temperature is limited to 120°C.

3.3. Concept for the modular measuring section

In order to achieve a continuously moving wall with constant, yet adjustable surface speed, the measuring section is equipped with a cylindrical surface that is in motion relative to the gap module. In order to reduce the number of dynamic seals and maximise adaptability as well as ease of maintenance, a concept with stationary gap module and rotating convex cylindrical surface was chosen. Hereby care must be taken to keep the geometric similarity to the relevant leakage gaps within the pumps. The result of a conceptual design process is shown in **Figure 3** with an exemplary gap module highlighted in yellow.

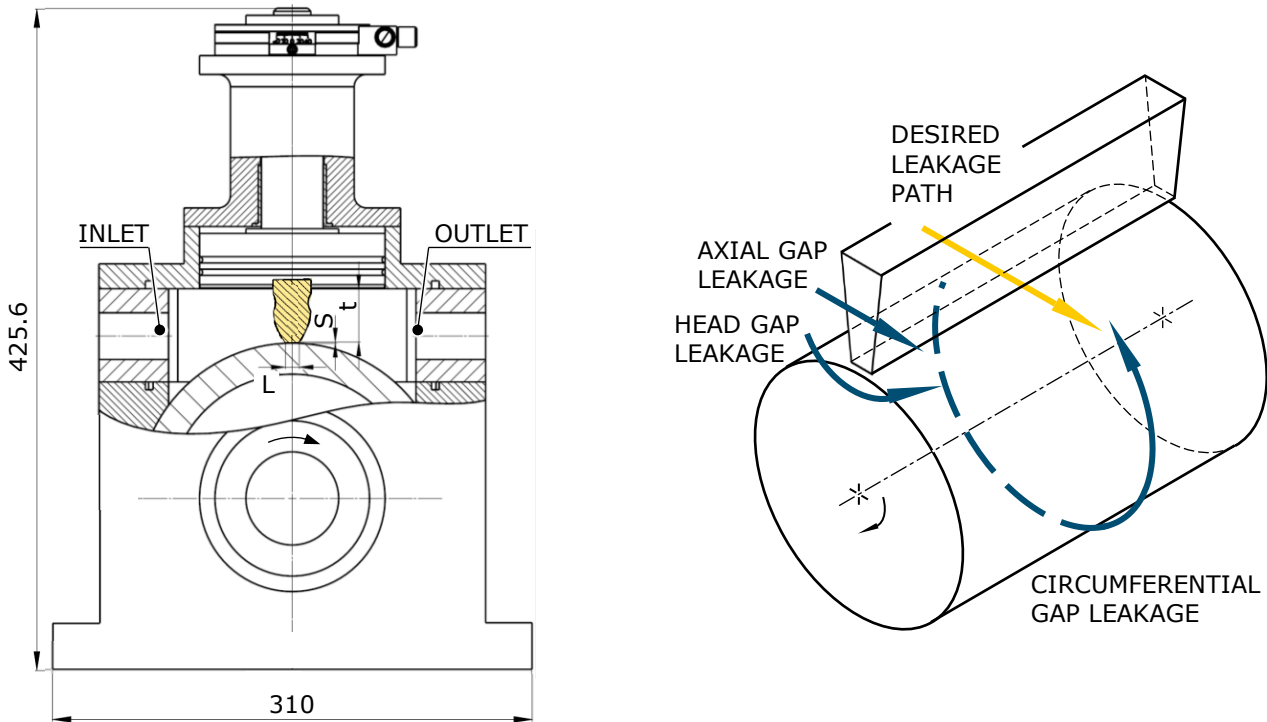


Figure 3: Modular measuring section of the gap leakage test rig (left) and schematic sketch of the secondary leakage paths within the measuring section (right)

The rotating shaft is hollow in order to reduce the rotating masses and thus decrease energy consumption as well as the forces exerted on the bearings of the test rig. By adjusting the rotational speed, the Couette-flow can be set. The pressure differential from inlet to outlet can be provided by a positive displacement pump, thus allowing for a Poiseuille-flow to be imposed.

The housing is made up of three parts and can be split axially. The bearings (not shown in **Figure 3**) are preloaded in order to minimise radial play. Thus, very tight tolerances between rotor and shaft can be achieved, while allowing secondary leakage flows to exit the test rig via drainage holes.

The sealing gaps within the displacement pumps that are the focus of the research activities often have a gap height of less than $10\ \mu\text{m}$. With the minimum achievable gap height of the secondary leakage paths in the same order of magnitude, the feasible gap heights to be assumed for the experiments are higher than the gap heights within the pumps. The gap module therefore needs to be scaled accordingly in order to be geometrically representative of the sealing gaps within the pumps, while the gap height for the secondary leakage paths is to be minimised, see **Figure 3**.

The head gap leakage in between the inlet and outlet side along the front side of the rotor is to be considered the most critical leakage path for secondary leakage, since it is a dynamic seal with comparatively short gap length. Therefore, tight tolerances are required.

For the analysis of different parameters, the experimental gap height needs to be adjustable. The mechanism for adjusting the height of the gap module is housed in the top plate. The gap module

which is affixed to a piston can be retracted from the rotating shaft using a spindle nut. The spindle nut is in turn affixed to a precision rotary table, thus allowing precise adjustment of the gap height. The piston seals the measurement section and is keyed in order to avoid rotation of the gap module. By variation of the dimensionless geometric and operating parameters, the measuring section allows to characterise the behaviour of the leakage flow as described in the sections before. The inlet and outlet ports are located in an intermediate plate between the three-part housing and the top plate.

In order to achieve visual accessibility for flow visualisation, the top plate with the adjusting mechanism as well as the intermediate plate can be substituted by an intermediate plate and a fixed-height top plate made from a clear plastic (e.g., PMMA). This allows for flow visualisation and even Particle Image Velocimetry (PIV) measurements to be performed [26].

For the characterisation of the operating conditions and fluid properties provided, measurement of the pressure and temperature is required. The sensors can be integrated into the intermediate plate. The temperature of the fluid is not only important to characterise the fluid properties but also for the controls of the fluid conditioning unit which is set to provide an adjustable constant fluid temperature for the experiments.

In order to minimise calibration efforts, the gap module is to include a distance sensor, which can be calibrated prior to being inserted into the measuring section. Thereby the actual gap height for each experiment can be logged. The direct measurement by laser micrometer e.g., is not feasible due to the very low gap height (0.1 ... 0.6 mm). Capacitive distance sensors are sensitive to the fluid properties, thus complicating their application to the measuring task at hand. Therefore, inductive sensors e.g., eddy current sensors are favourable [27].

In order to be able to adjust the fluid parameters without impacting the general fluid properties, a suitable test fluid must be chosen. The choice of the test fluid also impacts the applicability of the results to future research, as can be seen by the works of Rituraj and Vacca [15] where the type of fluid cannot be disclosed due to confidentiality, thus drastically hindering future research. The fluid needs to have shear-thinning behaviour, show viscoelastic properties, while not presenting a health and safety risk or being subject to aging effects. A literature review shows a multitude of test fluids within the scope of shear thinning-fluids. The most prominent fluids are silicone oils [19, 6–8], aqueous Xanthan-solutions [12, 18] and Sodium Carboxymethyl Cellulose (CMC) solutions [17]. However, apart from silicone oils, none of the test fluids are able to satisfy the requirements mentioned above. Xanthan-solutions e.g., show changes in their properties in between batches and are subject to ageing effects. For CMC-solutions the literature review shows a shear thickening behaviour at low shear rate and thixotropic behaviour. Therefore, silicone oils have been chosen for the experimental tests. Silicone oils are also often quoted in the relevant technical literature. Silicone oils can be understood as viscoelastic polymer melts with a melting point below room temperature [28]. By a variation of the molecular weight, the zero-shear viscosity can be set. For low molecular mass and low shear rate there may be an area with Newtonian properties, however this is not expected to be of relevance for the experiments.

A characterisation of the leakage behaviour relies on the measurement of the leakage flow. Due to the low electric conductivity of silicone oils, magnetic-inductive flow meters are not applicable, therefore gear or spindle type flow meters can be applied.

The frictional torque as well as the rotational speed at the hollow shaft are measured by a torque measuring shaft in order to characterise the operating parameters.

The hydraulic test environment is required to provide an adjustable, constant volume flow of the non-Newtonian fluid with constant fluid properties. The concept depicted in **Figure 4** is able to provide the required volume flow for continuous operation of the test rig by pumping the fluid from a tank to the measuring section. In order to ensure consistent experiment parameters, a special focus is to be placed on the conditioning of the fluid. Compared to Newtonian fluids this is even more relevant for the non-Newtonian fluids, since, for example the temperature-induced change in viscosity also

impacts the shear stresses and the distribution of these, thus drastically changing the local flow characteristics. The conditioning of the fluids is realised by a heating and a cooling and filtering unit that are designed to control the desired fluid temperature in the tank. The fluid is pumped by a positive displacement pump in order to achieve the desired pressure drop between inlet and outlet.

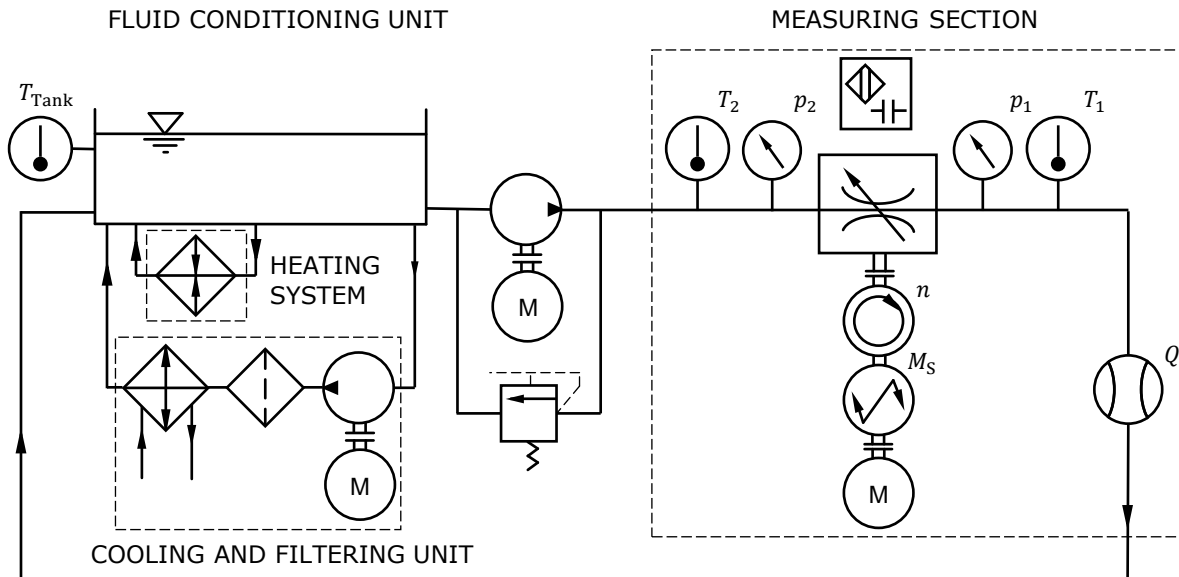


Figure 4: Schematic plan of the hydraulic test field for non-Newtonian fluids

In order to characterise the test fluids prior to the experiments, the parameters, such as shear storage modulus G' and shear loss modulus G'' , can be determined by oscillatory rheometry. In accordance with the Cox-Merz-rule, the shear-rate dependent viscosity can then be derived [25]. Utilising the time-temperature-shift, the experimental effort can be reduced [29]. This objective characterisation of the fluid properties allows the reusability of the data obtained by the experiments and is of fundamental importance for the transferability of the findings.

4. CONCLUSION

As could be shown, the literature contains relevant blind spots when trying to develop a systematic model for the design of displacement pumps for non-Newtonian applications. The proposed setup of the test rig is expected to (i) provide important insight into the gap leakage, allowing for the (ii) development a model that will enable manufacturers to design their pumps according to the complex applications. Based on an extensive literature review, the test rig was designed to be a modular platform for future experimental works on the leakage of non-Newtonian fluids. The setup allows for extensive parameter studies to be performed, varying fluid parameters and operation parameters as well as the geometric parameters of the gap modules, in order to resolve the influence of the extensional flow.

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NOMENCLATURE

A	area	m^2
G'	shear storage modulus	N/m^2
G''	shear loss modulus	N/m^2
L	gap/capillary length	m
M_S	shaft torque	Nm
n	rotational speed	$1/\text{min}$
n_{corr}	correction coefficient	-
p	pressure	bar
Δp	pressure differential	bar
p_1	outlet pressure	bar
p_2	inlet pressure	bar
Q	volume flow	m^3/s
R	radius	m
S	gap height	m
T_1	outlet temperature	$^\circ\text{C}$
T_2	inlet temperature	$^\circ\text{C}$
T_{Tank}	tank temperature	$^\circ\text{C}$
u	velocity	m/s
U	speed of the boundary wall	m/s
μ	viscosity	kg m/s
μ_{app}	apparent viscosity	kg m/s
μ_0	zero-shear viscosity	kg m/s
$\mu_{\text{E,planar}}$	extensional viscosity for the planar case	kg m/s
ν	kinematic viscosity	mm^2/s
τ	shear stress	N/m^2

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