METHODOLOGY OF SYSTEM PARAMETER OPTIMIZATION FOR PARALLEL ELECTRIC HYDRAULIC HYBRID MOBILE MACHINE VIA CONVEX PROGRAMMING

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ABSTRACT

Hydraulic hybrid powertrain is widely investigated for its high power density of hydraulic power system. In designing a hybrid vehicle, finding the combined optimality of component sizing and energy management is essential for minimizing vehicle costs and maximizing energy efficiency. Simultaneous optimization framework is an effective and important method due to its computational efficiency and resolution. In this paper a convex programming-based system parameter optimization framework is proposed for hydraulic hybrid vehicle. This technique allows simultaneous optimization of component sizing and energy management by converting it into a convex problem. To illustrate this, the system optimization problem in a parallel electric hydraulic hybrid wheel loader is posed over a fixed loading cycle. The Pareto front of PEHH system parameter optimization problem is obtained. The HM size is the main factor of system performance trade-off between battery aging and energy consumption. PEHH can reduce the battery capacity loss in a loading cycle by 26.4% compared to the pure electric drive with a 13.0% increase of energy consumption. With the same grid number of 7, the CP-based simultaneous method consumes 99% less computing time than DP-based bi-level method and provides the optimal solution with 1.5% less battery capacity loss.

Keywords: System parameter optimization, wheel loader, electric hydraulic hybrid system, convex programming

1. INTRODUCTION

The electrification of mobile machines is becoming a well-established trend. The electric construction equipment market is expected to grow to $19.9 billion in 2027 at a compound annual growth rate of more than 21% according to The Business Research Company. In 2023, China's sales of electric loaders reached 3595 units, accounting for 3.5% of the total sales of wheel loaders.

Parallel electric hydraulic hybrid powertrain (PEHH) is a promising substitute for pure electric drive and has been widely investigated [1]. The electric hydraulic hybrid powertrain provides hydraulic launching and braking to release battery current stress. For on-road vehicles like city bus, the energy saving of PEHH can reach up to 50% in the best scenario [2]. PEHH has also been applied on mobile machines such as wheel loaders [3], excavators [4] and forklifts [5], improving their energy efficiency and battery lifetime.

System parameter optimization has been an important topic in the field of hybrid vehicle [6]. Unlike the non-hybrid vehicles, system parameter optimization of hybrid powertrain should include the optimization of energy management strategy (EMS). There are four methods for the combined plant and controller optimization problem and global optimum can only be guaranteed with bi-level and simultaneous strategies.

In the bi-level optimization method, the possible component sizes are enumerated in the outer loop, and the optimal energy management is assessed in the inner loop [7]. By comparison, simultaneous method is an effective and important method due to its computational efficiency and resolution. A simultaneous optimization method is using an adjustable rule-based strategy and optimize the strategy design parameters and system parameters simultaneously [8]. However, for complex problems, the rule-based strategy is not close enough to the global optimum.

Another simultaneous optimization method is the convex programming. By modelling the combined plant and controller optimization problem as a convex programming problem, the optimal control variable sequence and the optimal system parameters are obtained simultaneously with a convex optimization solver [9]. This method is adopted extensively in electric hybrid systems, but has not been performed on hydraulic hybrid systems [10]

In this paper, a system parameter optimization method based on convex programming is proposed for PEHH mobile machines. The design problem of a PEHH wheel loader is used to demonstrate the optimization method. The rest of this paper is organized as follows. Section 2 introduces the PEHH wheel loader. Section 3 illustrates the optimization problem formulation. Section 4 introduces the framework of the proposed optimization method. Section 5 shows the results of the optimization study. The last section is the conclusion and outlook.

2. PARALLEL ELECTRIC HYDRAULIC HYBRID WHEEL LOADER

The PEHH wheel loader schematic is shown in **Figure 1**. The working and other functions are not considered here for simplicity. In the electric wheel loader, these functions are electrically decoupled from drivetrain and are not of the interest of this study. The main energy source is a battery pack, powering the electric motor through an inverter. High-pressure and low-pressure hydraulic accumulators and a variable hydraulic motor are added to provide hydraulic auxiliary power. The electric motor (EM) and hydraulic motor (HM) are mechanically coupled via gears.

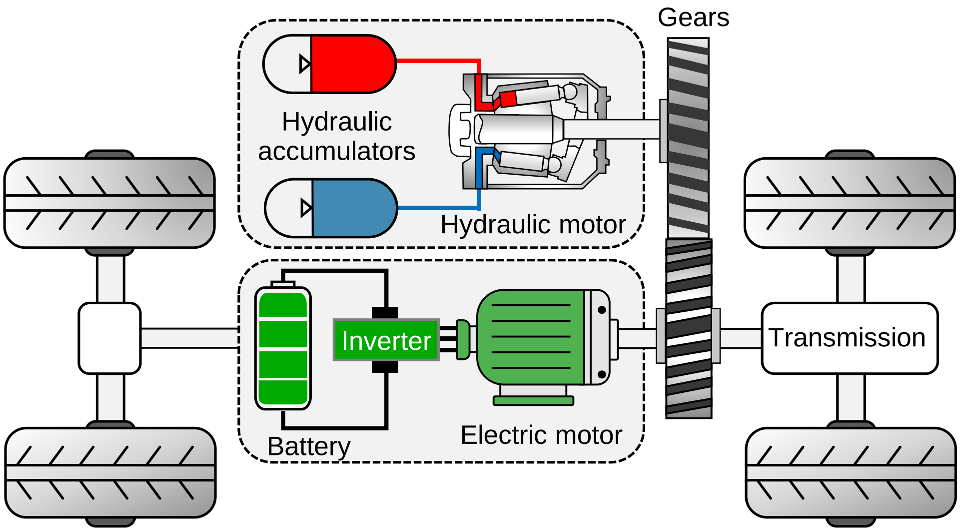

Figure 1: PEHH wheel loader schematic

Generally, the system parameters to be optimized in PEHH wheel loader includes: gear ratio, EM size, hydraulic motor geometric displacement, accumulator size and pre-charge pressure. The transmission ratio is not changed to make use of the original drive train. The main wheel loader parameters are shown in **Table 1**.

Table 1: Parameters of the wheel loader

Wheel loader parameter	Value	Unit
Operating weight	19000	kg
Load weight	5500	kg
Maximum speed	40	km/h

3. PROBLEM FORMULATION

In this section, the problem formulation and modelling details are introduced. The PEHH is expected to provide the same driving performance as the pure electric drive with smaller EM, less energy consumption and longer battery lifetime. The problem constitutes an objective function and constrains. The variables to be optimized are the state vector and the sizing factor for components. The control input variable u includes the control inputs of length N. These variables are explained in more detail at the end of this section.

3.1. Driving cycle

In the optimization problem, a given driving pattern is required to find the optimal component sizes and the energy management variables. The V loading cycle is a typical driving cycle for wheel loaders that can reflect real-life driving. In this study, the EM propel torque $T_{load}(k)$ and speed ω_{EM} of an electric wheel loader during a V loading cycle are used as the input of the optimization problem. The driving cycle is divided into N discrete instants with a time difference of ΔT.

3.2. Modelling

In this section the models of the powertrain and its components are presented. Since the models are used for convex programming, they all guarantee convexity. Quasi-static models are therefore approximated with nonlinear convex functions and some variable change is also used.

Powertrain

The EM and HM are mechanically linked to the drive train in the studied PEHH and can propel the wheels. Having the required EM speed $\omega_{EM}(k)$ and propel torque $T_{load}(k)$, the powertrain model is described by torque balance equations, given as

$$T_{EM}(k)\omega_{EM}(k) + T_{HM}(k)\omega_{HM}(k) = T_{load}(k)\omega_{EM}(k) \quad (1)$$

where $T_p(k)$ and $\omega_{EM}(k)$ is given by loading cycle data, T_{EM}(k) is EM torque, $T_{HM}(k)$ is HM torque and $\omega_{HM}(k)$ is HM speed calculated as $\omega_{HM} = i_G\omega_{EM}$, where i_G is gear ratio.

Battery

The battery capacity is determined by required operation time and is not optimized in this study. It's modelled as an open circuit voltage V_{oc} in series with a constant internal resistance R. The open circuit voltage is approximated to be constant in the allowed state of charge operating region. The terminal power, P_{bat}, and the battery current, I_{bat}, are calculated as

$$P_{bat}(k) = I_{bat}(k)V_{OC}(k) - I_{bat}^2(k)R \quad (2)$$

The battery current I_{bat} is chosen to be positive when charging. The electric energy consumption of battery E_{cons} is calculated by integrating battery power.

The battery cycle aging capacity loss model is adopted from [11]. It is related to the battery full equivalent cycle FEC, the battery state of charge SOC and the battery current I_{bat}. During a loading cycle, FEC and SOC are approximated to be constant. The battery cycle aging capacity loss Q_{ag} in percentage is given by

$$\begin{aligned} Q_{ag} &= \sum_{k=1}^{N} \Delta FEC(k) \left(a(\frac{|I_{bat}(k)|}{Q_b}) + b \right) (c(0.4 - SOC)^3 + d) \frac{FEC^{-0.5}}{2} \\ \Delta FEC(k) &= \frac{1}{3600} \frac{1}{2Q_b} \frac{I_{bat}(k-1) + I_{bat}(k)}{2} \Delta T \end{aligned} \tag{3}$$

where Q_b is the battery capacity in Ah, a, b, c, d is constant model coefficients.

EM

The EM model with the inverter is described by a power loss map, $P_{EM,loss,base}$, where the losses are measured at steady-state for different torque-speed combinations. The power losses for each EM speed are approximated by a second-order polynomial in torque. To vary the size of the EM, the torque limits and losses are scaled linearly by scaling factor s_{EM}. In this way the losses of the scaled EM are calculated at each time instant as

$$P_{EM,loss}(k) = c_1(k) \frac{T_{EM}^2(k)}{s_{EM}} + c_2(k)|T_{EM}(k)| + c_3(k) s_{EM} \tag{4}$$

where the coefficients $c_1 \geq 0$, c_2 and c_3 are functions of $\omega_{EM}(k)$ and are calculated using least squares method for a number of grid points of ω_{EM}. For speed values not belonging to the grid nodes, the coefficients are obtained by linear interpolation. The accuracy of the approximation are high and are discussed in detail in [9]. The maximum EM torque at each time instant $T_{max}(k)$ is obtained by interpolation and considered as constrains to EM torque T_e.

The electric power load of EM, P_{EM}, is given by

$$P_{EM}(k) = \omega_{EM}(k) T_{EM}(k) + P_{EM,loss}(k) \tag{5}$$

The electric power load of hydraulic working functions P_{HW} is not considered here and can be easily included in the problem by considering $P_{bat}(k) = P_{EM}(k) + P_{HW}(k)$.

HM

The HM torque, T_{HM}, and the HM flow rate Q_{HM} are functions of geometric displacement D, displacement fraction x_{HM}, pressure differential p_{HM} and angular speed ω_{HM} of HM. A modified version of Wilson's model [12] for variable-displacement pump-motor is used here.

To preserve the problem convexity[1], the variable change $T_{xpD}(k) = x_{HM}(k) p_{HM}(k) D$ and $T_{pD}(k) = p_{HM}(k) D$ are introduced. The equations (6) and (7) are rewritten as

$$T_{HM}(k) = T_{xpD}(k) + K_\omega \omega_{HM}(k) D + K_p T_{pD}(k) + T_0 D \tag{6}$$

$$P_{HM,Hydr}(k) = p_{HM}(k) Q_{HM}(k) = T_{xpD}(k) \omega_{HM}(k) - k_s(k) \frac{T_{pD}^2(k)}{D} - q_0 T_{pD}(k) \tag{7}$$

where $P_{HM,Hydr}$ is the hydraulic power of HM, k_s is calculated by $k_s = a + b\omega_{HM} + c\omega_{HM}^2$, The coefficients $a, b, c,\ K_w, K_p, T_0, q_0$ are calculated using least squares method with a number of grid points of ω_{HM}, p_{HM} and x_{HM}. The accuracy of the approximation is discussed in the simulation section. The HM displacement x_{HM} is chosen to be positive when motoring.

It is worth noting that even when HM has no power output, there are still losses due to leakage and

[1] $f(x, y) = x * y$ is neither convex or concave.

friction. For example, the volumetric efficiency of HM is negative if the pump leakage is greater than the theoretical pumping flow rate.

Hydraulic accumulator

The low-pressure hydraulic accumulator pressure is constant, equal to atmospheric pressure. For the high-pressure accumulator, the accumulator pressure p_{HA} is approximated as an affine function to simplify the problem and preserve problem convexity, given by:

$$p_{HA}(k) = k_{HA}E_{HA}(k) + p_0 \tag{8}$$

$$T_{pD}(k) = p_{HM}(k)D = K_{HA}E_{HA}(k) + B \tag{9}$$

The coefficient k_{HA}, $K_{HA} = k_{HA}D$ and $B = p_0 D$ are related to the accumulator parameter p_0 and V_0 and used as a sizing factor. The accuracy of the approximation is discussed in the simulation section.

The energy stored in the high-pressure accumulator E_{HA} are given by

$$E_{HA}(k) = E_{HA}(k-1) + \frac{P_{HA,Hydr}(k-1) + P_{HA,Hydr}(k)}{2}\Delta T \tag{10}$$

For charge sustaining of hydraulic accumulator, the initial E_{HA} is required to be no higher than the final E_{HA}.

4. OPTIMIZATION FRAMEWORK

The framework schematic of the proposed optimization method is shown in **Figure 2**. With some approximation and variable change, some component sizing factors can be integrated in the convex problem including HM geometric displacement D, EM scaling factor s_{EM} and accumulator pre-charge pressure p_0. Other sizing factors such as gear ratio i_g and accumulator volume V_0, are optimized in the outer loop.

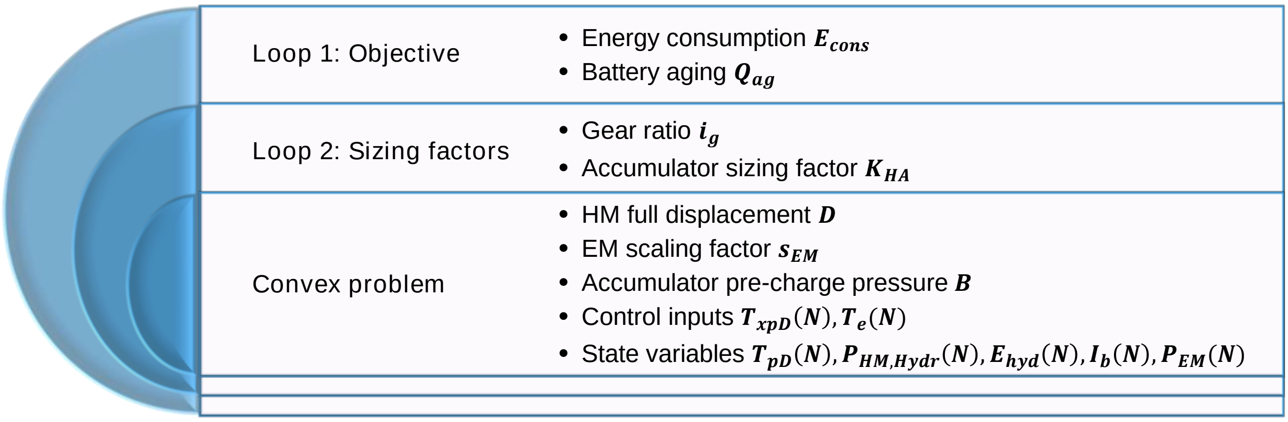

Figure 2: Framework schematic of the proposed optimization method

4.1. Objective function

The objective of the optimization contains two indexes, energy consumption and battery aging. Pareto front stands for a set of solutions where one of the objectives cannot be improved without sacrificing another one. In this paper, the Pareto optimal solutions are obtained by minimizing battery capacity loss under different maximum energy consumption constrains.

4.2. Sizing factors

Some sizing factors are hard to be integrated in the convex programming problem. To address this, an outer loop is added to enumerate the possible solution of these factors. In this study, the sizing factor loop include two index, Gear ratio i_G and Accumulator sizing factor K_{HA}. A number of 2-dimension vectors are generated and enumerated. The convex programming problem is solved for each sizing factor.

4.3. Convex optimization

Combing eq. (1)-(10). Most of the equations preserve the problem convexity since they include only affine functions, except (2), (3), (4) and (7). The original equalities are relaxed to inequations (10a) - (10d) to preserve the problem convexity. Because these inequities include convex non-linear term, including absolute value function and quadratic-over-linear function. The optimal solution will satisfy (10a) -(10d) with equality as it is optimal for energy consumption and capacity loss. When the equality of these inequality does not hold, there are some extra energy loss or battery capacity loss in the system [9]. Therefore, it will be optimal for the non-relaxed problem as well. The convex programming problem is formulated as follow, the variables to be optimized are in bold font:

$$
\begin{aligned}
& min\ \boldsymbol{Q_{ag}} \\
& \boldsymbol{subject\ to}: \\
& T_{load}(k)\omega_{EM}(k) = \boldsymbol{T_{EM}}(k)\omega_{EM}(k) + \boldsymbol{T_{HM}}(k)\omega_{HM}(k) \\
& \boldsymbol{T_{HM}}(k) = \boldsymbol{T_{xpD}}(k) + K_\omega \omega_{HM}(k)\boldsymbol{D} + K_p \boldsymbol{T_{pD}}(k) + T_0\boldsymbol{D} \\
& \boldsymbol{P_{HM,Hydr}}(k) \geq \boldsymbol{T_{xpD}}(k)\omega_{HM}(k) - k_s(k)\frac{T_{pD}^2(k)}{D} - q_0 T_{pD}(k) && \text{(a)} \\
& \boldsymbol{E_{HA}}(k) = \boldsymbol{E_{HA}}(k-1) + \frac{\boldsymbol{P_{HA,Hydr}}(k-1) + \boldsymbol{P_{HA,Hydr}}(k)}{2}\Delta T \\
& \boldsymbol{T_{pD}}(k) = K_{HA}\boldsymbol{E_{HA}}(k) + B \\
& \boldsymbol{P_{EM}}(k) \geq c_1(k)\frac{\boldsymbol{T_{EM}^2}(k)}{\boldsymbol{s_{EM}}} + c_2(k)|\boldsymbol{T_{EM}}(k)| + c_3(k)\boldsymbol{s_{EM}} + \omega_{EM}(k)\boldsymbol{T_{EM}}(k) && \text{(b)} \\
& \boldsymbol{P_{EM}}(k) \leq \boldsymbol{I_{bat}}(k)V_{OC}(k) - \boldsymbol{I_{bat}^2}(k)R && \text{(c)} \qquad (11) \\
& \boldsymbol{E_{cons}} = \sum_{k=1}^{N} \boldsymbol{I_{bat}}(k)V_{OC}(k)\Delta T \\
& \boldsymbol{Q_{ag}} \geq \sum_{k=1}^{N} \boldsymbol{\Delta FEC}(k)\left(a\left(\frac{|\boldsymbol{I_{bat}}(k)|}{Q_b}\right) + b\right)\left(c(0.4 - SOC)^3 + d\right)\frac{FEC^{-0.5}}{2} && \text{(d)} \\
& \boldsymbol{\Delta FEC}(k) = \frac{1}{3600}\frac{1}{2Q_b}\frac{\boldsymbol{I_{bat}}(k-1) + \boldsymbol{I_{bat}}(k)}{2}\Delta T \\
& \boldsymbol{E_{cons}}(k) \leq E_{max} \\
& |\boldsymbol{T_e}(k)| \leq T_{max}(k)\boldsymbol{s_{EM}} \\
& \boldsymbol{s_{EM}} \in [0, s_{EM,max}] \\
& \boldsymbol{T_{pD}}(k) \in [p_0 D, p_{max} D] \\
& \boldsymbol{D} \in [D_{min}, D_{max}]
\end{aligned}
$$

5. OPTIMIZATION STUDY

In this section, the accuracy of model approximation is first discussed. The Pareto front is then obtained with given optimization framework. The influence of component sizing factors on powertrain performance and the optimal energy management under different optimization objective are also analysed. Comparison between the computing time of the proposed convex optimization method and the bi-level method is carried out to check its computational efficiency.

5.1. Driving cycle and model parameters

The loading cycle used in the study is obtained via field test as shown in **Figure 3**. The loading time is 30.7 seconds long and the maximum vehicle speed is about 12 km/h. The EM propel torque is provided by EM and HM in the PEHH powertrain.

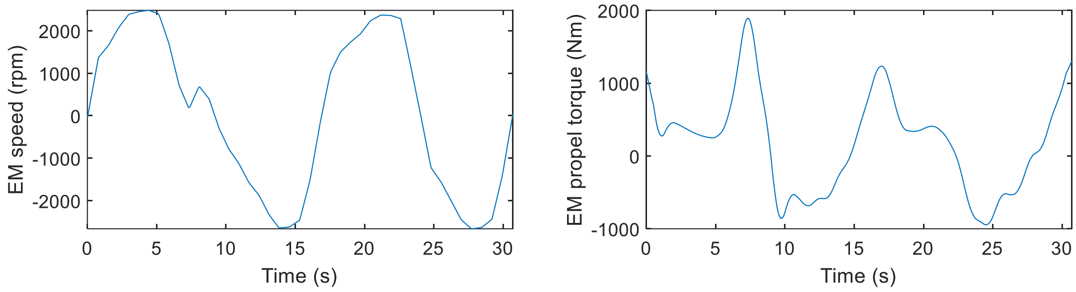

Figure 3: EM speed and propel torque during a loading cycle

The HM efficiency data includes mechanical and volumetric efficiency. The hydraulic pump/motor model coefficients are obtained by fitting the efficiency test data of a variable-displacement pump. The efficiency map obtained after model fitting and the original efficiency map are shown in **Figure 4**. The efficiency test data is measured with displacement fraction of $x = -1, -0.8, -0.6, -0.4, -0.2$, and only the data of $x = -1$ and $x = -0.2$ is shown here due to page limit. As shown, the fitted efficiency map is a set of ellipse curve and is close to the original test data.

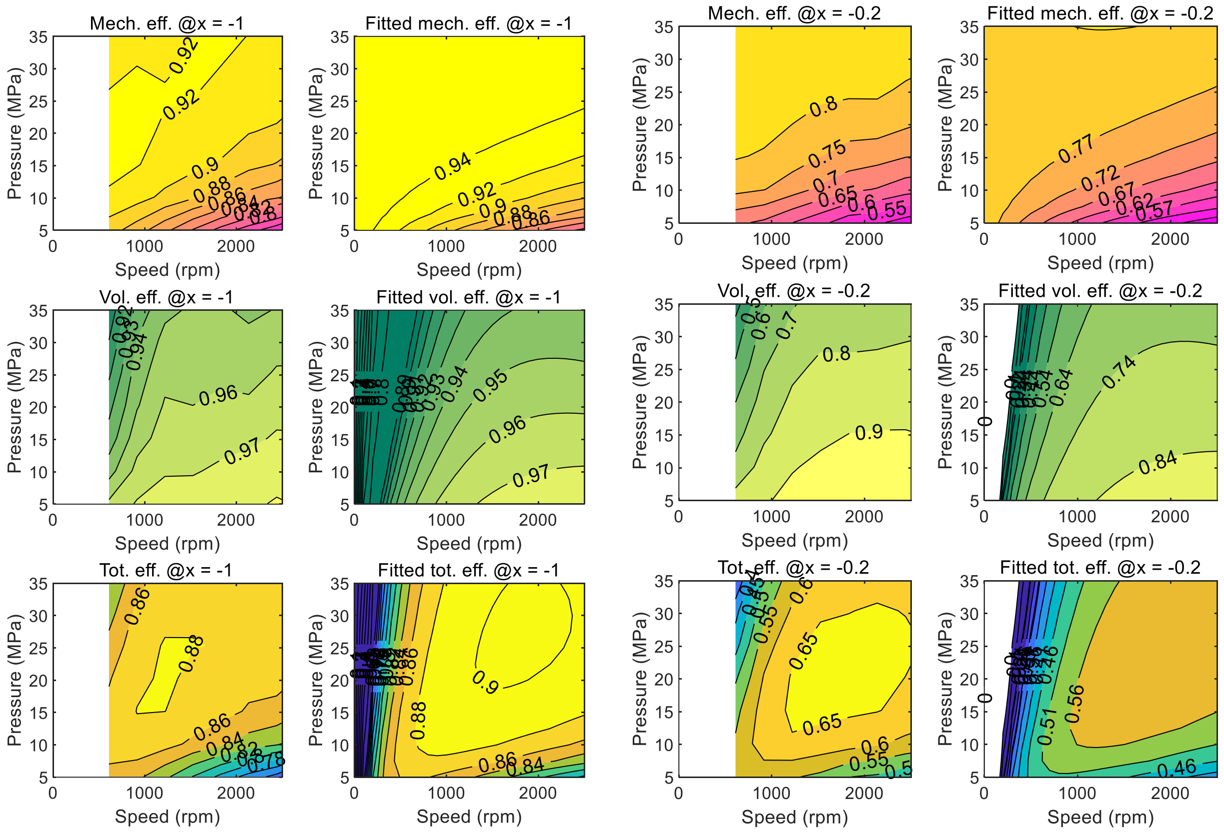

Figure 4: Original and fitted HM mechanical, volumetric and total efficiency map. (a) $x = -1$. (b) $x = -0.2$

The HM efficiency in pumping and motoring mode is shown in **Figure 5**. The maximum efficiency of HM is about 90%.

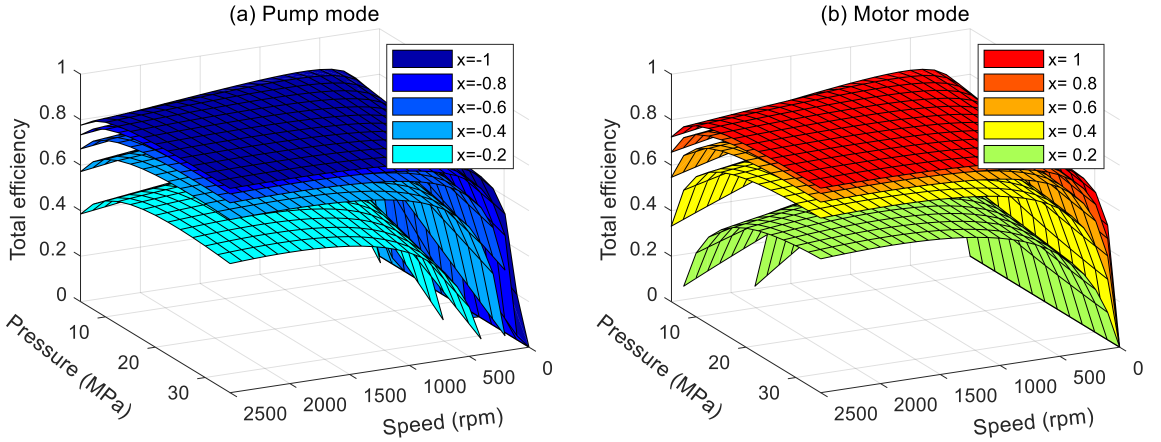

Figure 5: Fitted HM efficiency map with different displacement fraction. (a) pump mode. (b) motor mode

Under the hydraulic accumulator isothermal model, the curve of gas pressure versus volume is an inverse proportional function curve. Within a limited pressure range, the inverse proportional function curve can be approximated with an affine function. The error of pressure of model approximation can affect the maximum torque and power loss of HM, but the error is smaller than 1.0 MPa and its impact on system performance can be ignored.

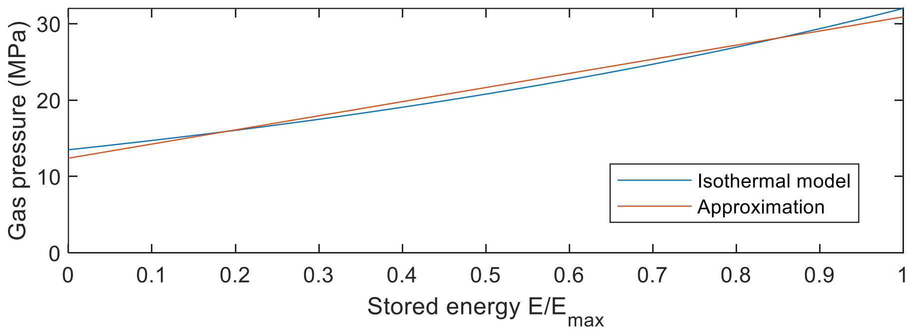

Figure 6: Hydraulic accumulator isothermal model and approximation

The EM and HM model coefficients are obtained by fitting actual machine test data. The EM used in this study is an 80-kW permanent magnet synchronous motor with a maximum speed of 3300 rpm. The efficiency and power loss map obtained after model fitting are shown in **Figure 7**.

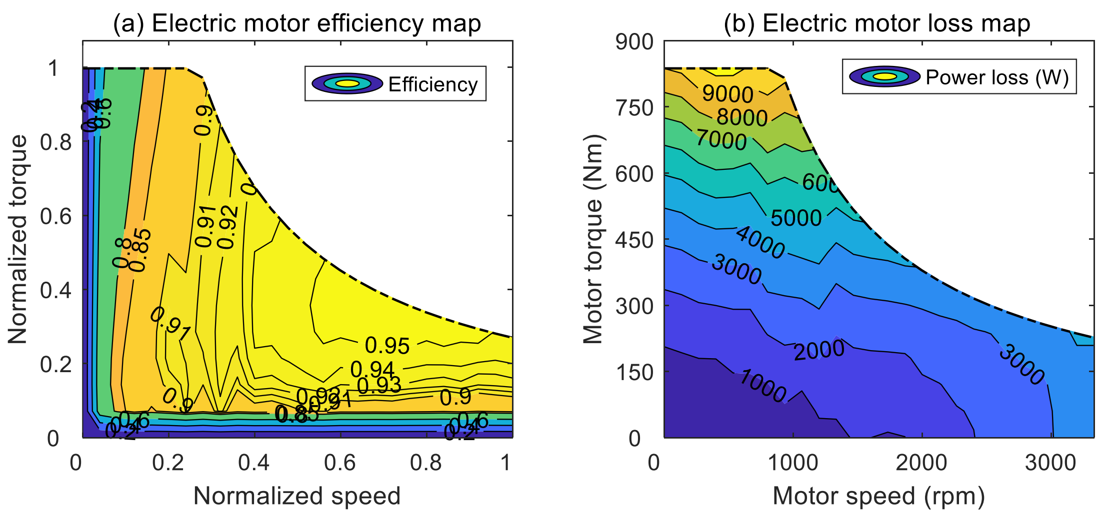

Figure 7: EM efficiency map and loss map.

5.2. System parameter optimization results

The Pareto front and Pareto optimal points for the PEHH parameter design solutions is shown in **Figure 8**. In the $E_{cons} - Q_{ag}$ space, each point corresponds to one system parameter solution. The left boundary in the dashed line of all possible points is the Pareto front consist of different Pareto optimal points with different HM displacement. Among six selected designs, solution with largest displacement $D = 149\ cm^3/rev$ has the lowest battery capacity loss and highest energy consumption, 1.93×10^{-6} % and 1721.3 kJ, respectively. Solution with lowest HM displacement $D = 20\ cm^3/rev$ has the highest battery capacity loss and lowest energy consumption, 2.52×10^{-6} % and 1549.2 kJ, respectively.

Compared to the electric drive solution (2.62×10^{-6} % and 1522.9 kJ), the energy consumption of PEHH is always higher while the battery capacity loss can be reduced by 26.4%. The reason is that HM leads to more leakage and friction loss but the battery charging and discharging is reduced with hydraulic power assist. Solution #3 provides a relatively good trade-off between battery aging and energy consumption, reducing the battery capacity loss by 17% with only 5% more energy consumption. More precise trade-off requires weighted calculation of capacity losses and energy consumption targets, considering battery price and electricity price.

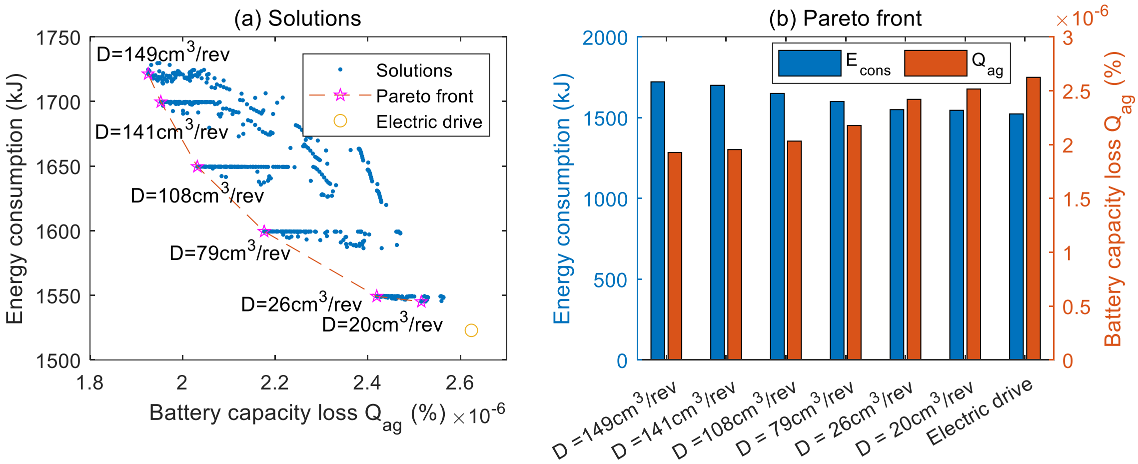

Figure 8: Pareto front and Pareto optimal points for the system design solutions. (a) Solution distribution. (b) System performances of Pareto front

The component energy losses and component sizing factors of Pareto optimal solutions with different energy consumption is shown in **Table 2**. The EM energy loss is reduced while the HM loss increases in PEHH. Firstly, when the weight of battery aging in the objective increases, the optimal HM geometric displacement increases accordingly and the EM size decreases. Therefore, the hydraulic hybrid ratio is the main factor of system performance trade-off.

Table 2: Performances and system parameters of the Pareto optimal solutions

Solution	Energy consumption (kJ)	Battery capacity loss (%)	HM displacement (cm^3/rev)	EM maximum torque (Nm)	Accumulator volume (L)	Gear ratio	Pre-charge pressure (MPa)	Initial pressure (MPa)
#1	1721.3	1.93×10^6	149	1554	19.9	0.9	13.5	32
#2	1699.6	1.95×10^6	141	1558	20.9	0.9	13.5	32
#3	1649.5	2.03×10^6	108	1656	16.0	0.9	13.5	32
#4	1599.4	2.18×10^6	79	1735	10.6	0.8	13.5	32
#5	1549.2	2.42×10^6	26	1827	4.4	0.9	13.5	32
#6	1545.1	2.52×10^6	20	1848	6.4	0.7	13.5	27
Electric Drive	1522.9	2.62×10^6	N/A	1900	N/A	N/A	N/A	N/A

The total optimization time of the proposed method varies with the number of sizing factors. As shown in **Figure 9**, for the typical bi-level optimization method based on dynamic programming (DP), the sizing factors are enumerated with six loops. With a grid number of N, the DP problem needs to be solved for N^6 times. While the convex programming problem is solved for N^2 times with two loops. This provides smaller computational burden and high discrete resolution.

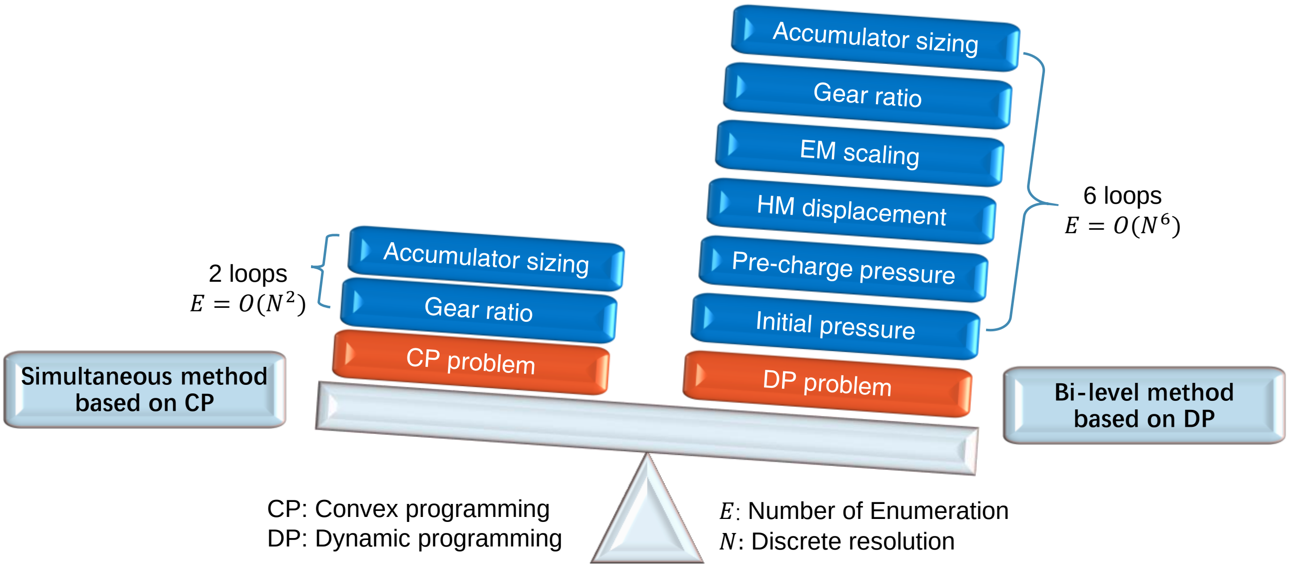

Figure 9: Comparison between the proposed simultaneous and DP-based bi-level optimization methods.

The DP-based bi-level method and CP-based simultaneous method were implemented and were run on a computer equipped with an i5-6500 CPU. Both the CP-based and DP-based methods adopts Latin hypercube sampling for the system parameter selection loop. For CP problem (11), we used CVX, a package for specifying and solving convex programs [13, 14]. For DP problem, a standard DP solver, DynaProg, is used to solve the optimal energy management problem [15].

The mean computing time is 3.5 seconds for each convex programming problem and 0.14 second for each DP problem. Considering the enumeration number, when $N \geq 3$, the convex programming method shows computational superiority. As shown in **Figure 10**, The computing time of DP-based bi-level method increases much faster than CP-based simultaneous method. With the same grid number of 7, the CP-based simultaneous method consumes 99% less computing time than DP-based bi-level method and provides the optimal solution with 1.5% less battery capacity loss (1.93×10^6% and 1.96×10^6%).

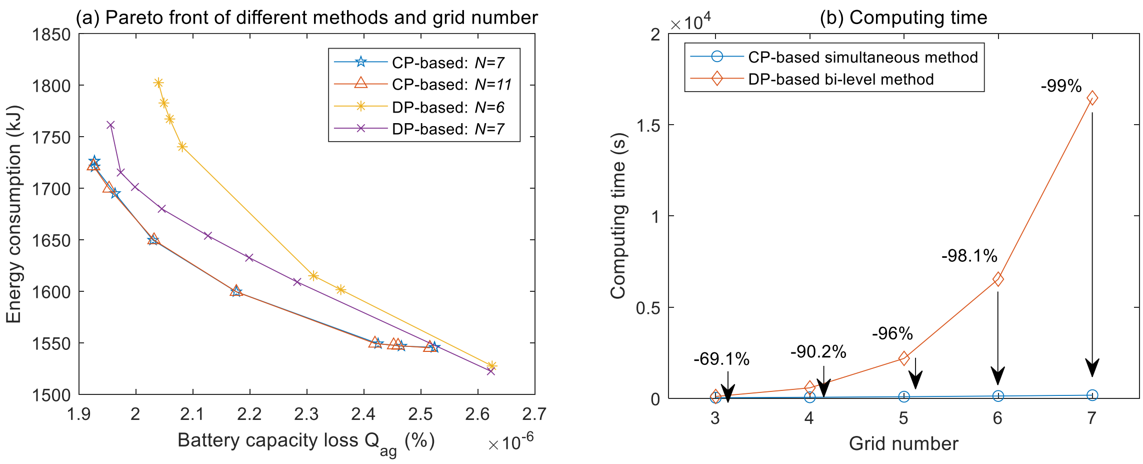

Figure 10: Comparison between CP-based simultaneous method and DP-based bi-level method (a) Pareto optimal solutions (b) Computing time.

6. CONCLUSION AND OUTLOOK

In this paper a system parameter optimization framework based on convex programming is proposed. It is applied on a parallel electric hydraulic hybrid wheel loader for the simultaneous optimization of its component sizing and energy management.

(1) PEHH extends battery life at the cost of energy consumption. The Pareto front of PEHH system parameter optimization problem is obtained. The HM size is the main factor of system performance trade-off between battery aging and energy consumption.

(2) PEHH can reduce the battery capacity loss in a loading cycle by 26.4% compared to the pure electric drive with a 13.0% increase of energy consumption.

(3) The convex programming method have superiority in terms of computational burden and digital resolution over the bi-level method. With the same grid number of 7, the CP-based simultaneous method consumes 99% less computing time than DP-based bi-level method and provides the optimal solution with 1.5% less battery capacity loss.

In the future, several aspects need to be studied. First, more application of this method can be studied such as series hydraulic hybrid powertrain and diesel-hydraulic hybrid powertrain. Secondly, an attractive topic is to incorporate the optimal vehicle speed and life cycle cost into the optimization framework, which will generate more valuable insights.

ACKNOWLEDGEMENT

This work was supported in part by the National Key R&D Program of China under Grant 2022YFE0128900, in part by the Zhejiang Provincial Natural Science Foundation under Grant LR22E050003, and in part by the National Natural Science Foundation of China under Grant 51875509.

REFERENCES

[1] Yang J, Liu B, Zhang T, et al (2022) Application of energy conversion and integration technologies based on electro-hydraulic hybrid power systems: A review. Energy Convers Manag 272:116372. https://doi.org/10.1016/j.enconman.2022.116372

[2] Niu G, Shang F, Krishnamurthy M, Garcia JM (2017) Design and Analysis of an Electric Hydraulic Hybrid Powertrain in Electric Vehicles. IEEE Trans Transp Electrif 3:48–57. https://doi.org/10.1109/TTE.2016.2628792

[3] Zhang H, Wang F, Xu B, Fiebig W (2022) Extending battery lifetime for electric wheel loaders with electric-hydraulic hybrid powertrain. Energy 261:125190. https://doi.org/10.1016/j.energy.2022.125190

[4] Qin T, Quan L, Li Y, Ge L (2021) A Dual-Power Coordinated Control for Swing System of Hydraulic-Electric Hybrid Excavator. In: 2021 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM). IEEE, pp 55–60

[5] Tong Z, Jiang Y, Tong S, et al (2023) Hybrid drivetrain with dual energy regeneration and collaborative control of driving and lifting for construction machinery. Autom Constr 150:104806. https://doi.org/10.1016/j.autcon.2023.104806

[6] Huang Y, Wang H, Khajepour A, et al (2018) A review of power management strategies and component sizing methods for hybrid vehicles. Renew Sustain Energy Rev 96:132–144. https://doi.org/10.1016/j.rser.2018.07.020

[7] Frank B (2016) Using optimal control in concept evaluation and system optimization of diesel-electric hybrid construction machines. 2016 Int Conf Electr Syst Aircraft, Railw Sh Propuls Road Veh Int Transp Electrif Conf ESARS-ITEC 2016. https://doi.org/10.1109/ESARS-ITEC.2016.7841323

[8] Uebel K, Raduenz H, Krus P, De Negri VJ (2018) Design optimisation strategies for a hydraulic hybrid wheel loader. BATH/ASME 2018 Symp Fluid Power Motion Control FPMC 2018 1–11. https://doi.org/10.1115/FPMC2018-8802

[9] Murgovski N, Johannesson L, Sjöberg J, Egardt B (2012) Component sizing of a plug-in hybrid electric powertrain via convex optimization. Mechatronics 22:106–120. https://doi.org/10.1016/j.mechatronics.2011.12.001

[10] Li Y, Tang X, Lin X, et al (2022) The role and application of convex modeling and optimization in electrified vehicles. Renew Sustain Energy Rev 153:111796. https://doi.org/10.1016/j.rser.2021.111796

[11] Zhang H, Wang F, Xu B, Fiebig W (2022) Extending battery lifetime for electric wheel loaders with electric-hydraulic hybrid powertrain. Energy 261:125190. https://doi.org/10.1016/j.energy.2022.125190

[12] Costa GK, Sepehri N (2018) Understanding overall efficiency of hydrostatic pumps and motors. Int J Fluid Power 19:106–116. https://doi.org/10.1080/14399776.2018.1476306

[13] Grant M, Boyd S (2008) Graph implementations for nonsmooth convex programs. In: Blondel V, Boyd S, Kimura H (eds) Recent Advances in Learning and Control. Springer-Verlag Limited, pp 95–110

[14] CVX Research I (2012) {CVX}: Matlab Software for Disciplined Convex Programming, version 2.0

[15] Miretti F, Misul D, Spessa E (2021) DynaProg: Deterministic Dynamic Programming solver for finite horizon multi-stage decision problems. SoftwareX 14:100690. https://doi.org/10.1016/j.softx.2021.100690