

Sensors & Instrumentation and Aircraft/Aerospace Testing Techniques Vol. 8

Chad Walber
Matthew Stefanski



Proceedings of the 43rd IMAC, A Conference and
Exposition on Structural Dynamics 2025



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Preface

Sensors & Instrumentation and Aircraft/Aerospace Testing Techniques represent one of twelve volumes of technical papers presented at the 43rd IMAC, A Conference and Exposition on Structural Dynamics, organized by the Society for Experimental Mechanics, and held February 10-13, 2025. The full proceedings also include volumes on Nonlinear Structures & Systems; Dynamic Substructures; Model Validation and Uncertainty Quantification; Dynamic Substructuring & Transfer Path Analysis; Special Topics in Structural Dynamics & Experimental Techniques; Computer Vision & Laser Vibrometry; Dynamic Environments Testing; Topics in Modal Analysis & Parameter Identification I&II; Data Science in Engineering; and Structural Health Monitoring & Machine Learning.

Each collection presents early findings from experimental and computational investigations on an important area within Sensors and Instrumentation & other Structural Dynamics areas. Topics represent papers on calibration, smart sensors, shock calibration and shock environment synthesis and applications for aircraft/aerospace structures.

The organizers would like to thank the authors, presenters, session organizers, and session chairs for their participation in this track.

Editor:

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Chapter 1

X-57 Mod III Wing Ground Vibration Test

Truong Samson, Bhamidipati Keerti, Lung Shun-Fat, Spivey Natalie, Truax Roger, and Winkel James

Abstract The X-57 “Maxwell” all-electric experimental aircraft was an electric propulsion demonstrator developed by the National Aeronautics and Space Administration to inform airworthiness standards for electrified aircraft. The development of the X-57 aircraft was separated into configuration modifications culminating in a distributed electric propulsion flight demonstrator named Modification IV (a.k.a., Mod IV). One of these configurations, the X-57 Mod III aircraft, involved flight-testing of a long, slender, high-aspect-ratio wing with two electric motors located at the wingtips. The X-57 Mod III wing ground vibration test was performed in the Flight Loads Laboratory at the National Aeronautics and Space Administration Armstrong Flight Research Center to measure the structural dynamics properties of the newly developed wing, and to confirm airworthiness of the Mod III/IV aircraft. This paper describes the testing performed to acquire the modal data and the subsequent results from the test.

Keywords Electric · Ground Vibration Test · Modal · X-57

Introduction

The X-57 “Maxwell” all-electric experimental aircraft was based on a modified general aviation Tecnam P2006T (TECNAM S.p.A., Costruzioni Aeronautiche Tecnam, Capua, Italy) twin engine airplane [1, 2]. The plan was to have the X-57 aircraft undergo several iterative modifications, building up to a distributed-electric propulsion demonstrator aircraft. The originally planned modification sequence is shown in Fig. 1. Midway through the project lifecycle, the third and fourth modifications were merged as Modification (Mod III/IV). Modification (Mod I) is the baseline Tecnam P2006T airplane with two inboard gas-powered engines. Mod II expanded on Mod I by replacing the gas-powered engines with electric motors that were powered by traction batteries. The plan for Mod III/IV was to incorporate a new high-aspect-ratio wing design with the two electric cruise motors moved outboard to the wingtips as well as 12 high-lift electric motors mounted and distributed along the wing leading edge. The X-57 project, however, was cancelled before Mod III/IV took flight.

The National Aeronautics and Space Administration (NASA) Armstrong Flight Research Center (AFRC) Flight Loads Laboratory (FLL) [3–5] conducted a ground vibration test (GVT) on the X-57 Mod III wing, as shown in Fig. 2, in order to gather the wing modal data to correlate to the fundamental wing modes in the finite element model (FEM). The resulting tuned FEM helped with early updates to the classical and whirl flutter analyses for the Mod III and Mod IV aircraft. Also, having the GVT performed early in the wing design process helped reduced programmatic risk by identifying any problems before the wing was mounted to the aircraft fuselage. Two main configurations were tested, one was the Mod III wing with the cruise motor nacelle mass simulators at the wingtips, as shown in Fig. 2, and the other was the clean wing on its own, as shown in Fig. 3. The Mod III wing GVT was conducted prior to the fabrication of wingtip cruise motor nacelles; therefore, cruise motor nacelle mass simulators were used. The Mod III wing was mounted on the wing loads test fixture (WLTF) as the GVT was conducted using a test fixture shared with the wing proof loads test [6]. The test also incorporated various flap and aileron settings to measure the effect of deployment on the control surface modes. A total of 192 test runs were performed for the X-57 Mod III wing GVT from May to June 2019.

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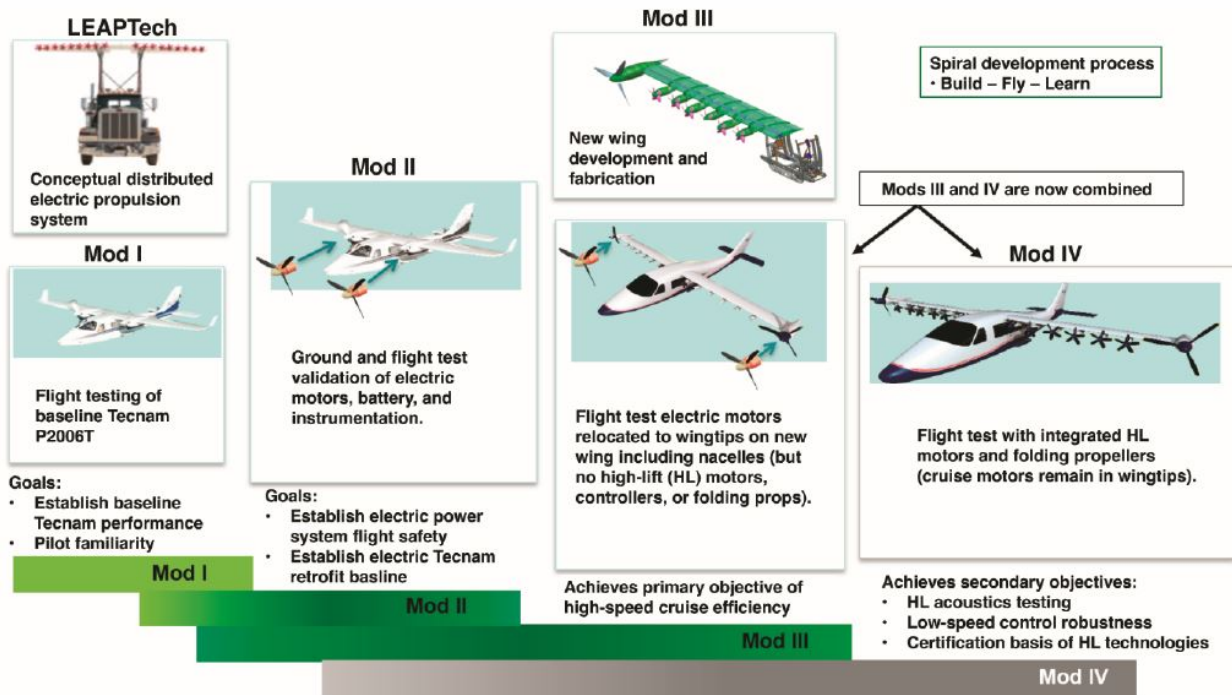


Fig. 1 X-57 Maxwell configuration modifications

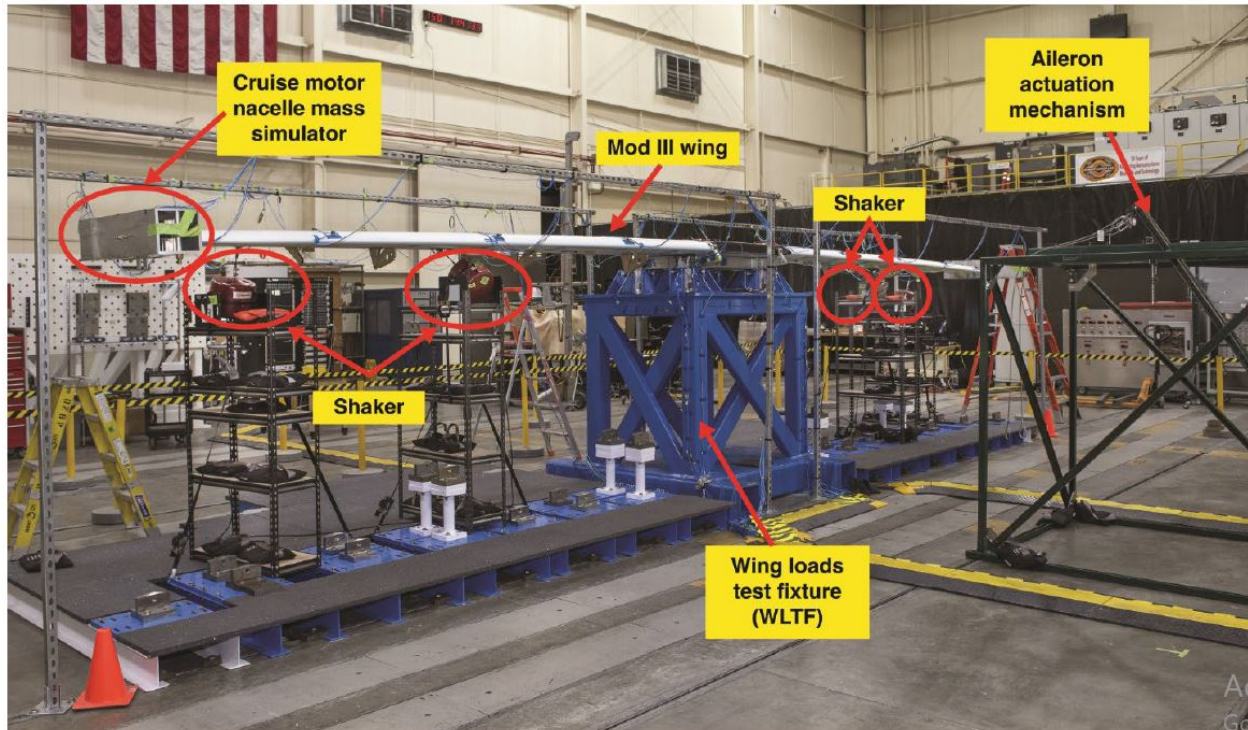


Fig. 2 X-57 Mod III wing ground vibration test setup in the Flight Loads Laboratory at Armstrong Flight Research Center. Setup shown in a four-shaker configuration

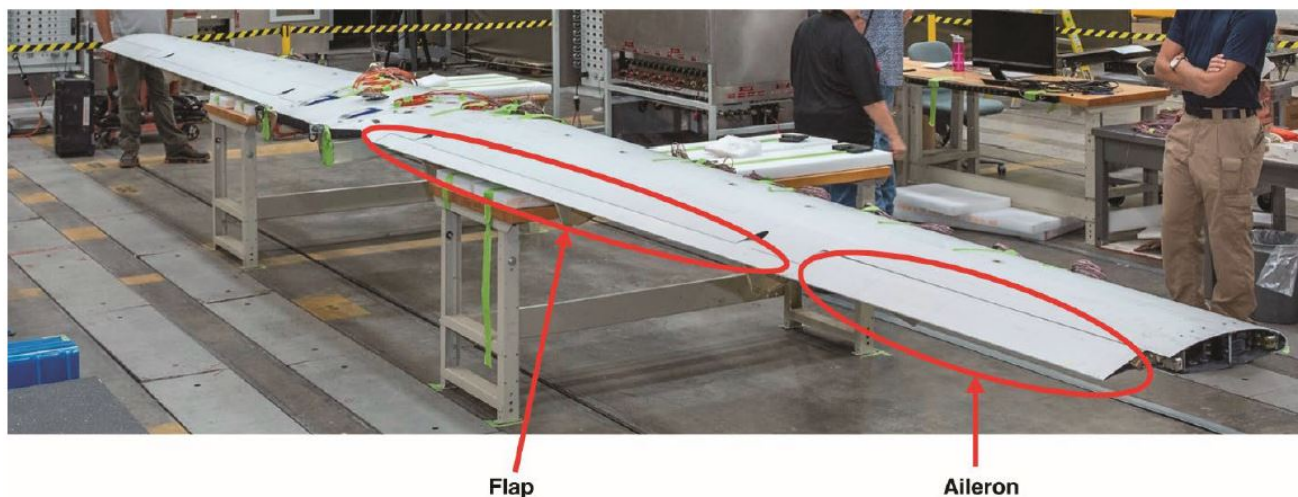


Fig. 3 X-57 Mod III clean wing at the National Aeronautics and Space Administration Armstrong Flight Research Center Flight Loads Laboratory

Test Objectives

The objective of the GVT was to capture modes that were predicted to be participants in the wing flutter mechanism [7]. For the primary objectives, frequencies, mode shapes, and damping were collected for wing 1st bending (W1B); wing 2nd bending (W2B); wing 1st torsion (W1T), wing fore/aft; and bending and rotation modes of the flaps and ailerons (early FEM predictions indicated significant responses from the control surfaces). Secondary objectives included measuring control surface modes for the clean wing configuration to understand the effect of wing deflection (resulting from the wingtip mounted motors) on the control surface modes. Data gathered during the GVT helped characterize the Mod III wing structure and facilitate updates to the wing FEM model. The tuned FEM was used to update both the classical and whirl flutter analyses after the GVT.

Test Description

The following sections go further into detail about the X-57 Mod III wing test article, the finite element model, and additional details of the GVT.

Test Article

The original Tecnam P2006T wing was to be replaced with a newly constructed wing for the X-57 Mod III/IV aircraft. The Mod III/IV wing was manufactured by Empirical Systems Aerospace, Inc. (ESAero) (San Luis Obispo, California). Unlike the original P2006T, the Mod III wing was designed to have a smaller chord and a higher-aspect-ratio wing when compared to the original P2006T wing, which would help increase cruise flight efficiency. Table 1 shows the various dimensions and weight specifications for the Mod III wing. Note that the weights in this table were provided in April 2019, which was prior to the X-57 Mod III wing GVT.

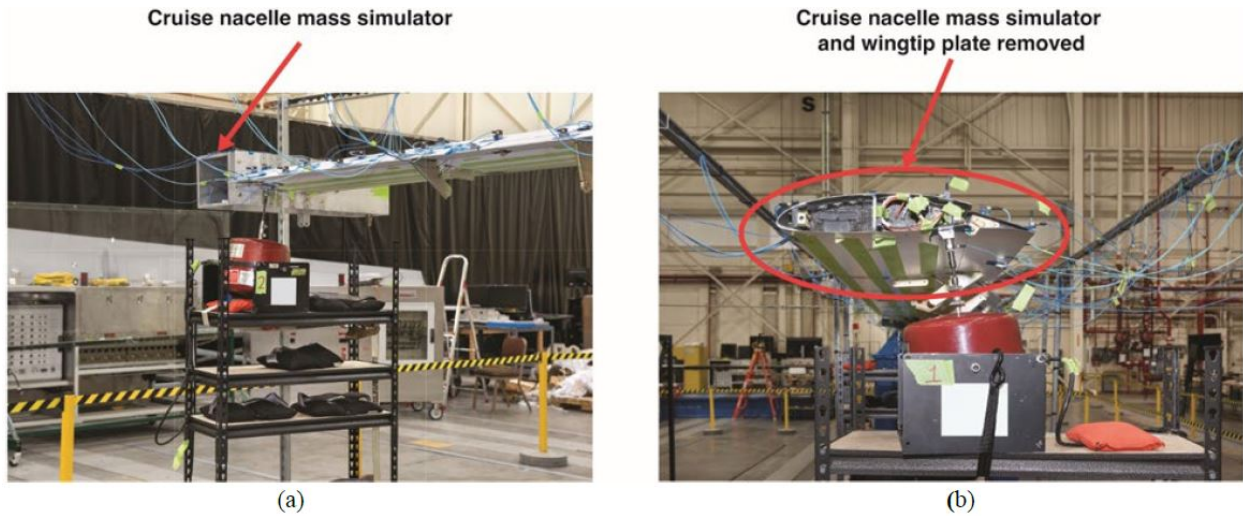
For this GVT, two different configurations were tested:

- Configuration A: wing + wingtip mounting plates + cruise nacelle mass simulators
- Configuration B: clean wing

Configuration A was tested first, and since the wingtip cruise nacelles and electric motors had not yet been fabricated at the time, mass simulators were added at the ends of each wingtip to simulate the actual mass and center-of-gravity locations of the real electric cruise motors. For these mass simulators to interface with the wing, wingtip plates had to be made and were installed at the end of each wing, which then allowed the mass simulators to be mounted onto the wing plates. After the GVT of configuration A was completed, the cruise nacelle mass simulators as well as the wingtip plates were removed

Table 1 X-57 Mod III wing specifications (as of April 2019) prior to the ground vibration test

Wing properties	Dimensions
Aspect ratio	15
Mean aerodynamic chord	2.13 ft
Wingspan	31.63 ft
Wing area	67 ft ²
Taper ratio	0.7
Tip chord	1.74 ft
Root chord	2.48 ft
Weight (clean wing including H-frame)	313.5 lb
Weight (wing + wingtip mounting plates)	352.5 lb
Weight (wing + wingtip mounting plates + cruise nacelle mass simulators ($\times 2$))	581.5 lb
Mod III estimate for weight (cruise nacelle ($\times 1$))	134 lb

**Fig. 4** X-57 Mod III wing ground vibration test configuration A (Fig. 4a); and configuration B (Fig. 4b)

for configuration B for the clean Mod III wing. Figure 4 shows the differences between the two GVT configurations at the wingtips (Fig. 4a and Fig. 4b).

Finite Element Model

The X-57 Mod III wing FEM was used for both modal and flutter analysis to generate pre-test predictions for the GVT. The FEM modal analysis was performed using MSC Nastran (MSC Software Corporation, Newport Beach, California) for the Mod III wing for boundary conditions of the wing fixed at the H-frame mounting points, as well as the wing mounted on the WLTF. Figure 5 shows the FEM models for configuration A (Fig. 5a) with the cruise nacelles, and configuration B (Fig. 5b) with only the clean wing.

For the fixed-wing boundary case, the four attachment points of the H-frame underneath the Mod III wing, which connect the wing to the fuselage, were rigidly fixed in all degrees of freedom (DOF). Figure 6 shows the location of the H-frame with respect to the Mod III wing.

In addition to the fixed-wing FEM predictions, modal analysis was performed with the Mod III wing FEM on the WLTF FEM for comparison between the two, as shown in Fig. 7. The WLTF was designed and fabricated by Pyramid Space, Inc. (Carlsbad, California) and allowed the Mod III wing to be mounted on the fixture in the FLL for both the GVT as well as the proof loads test.

The WLTF has eight bolt locations (two at each corner) so that it can be bolted into the floor tracks of the FLL. Initially, the WLTF FEM had all eight bolt locations rigidly fixed, resulting in a FEM prediction of 72.08 Hz for its 1st frequency.



Fig. 5 X-57 Mod III wing finite element model configuration A (Fig. 5a) with the cruise nacelles; and configuration B (Fig. 5b) with only the clean wing.

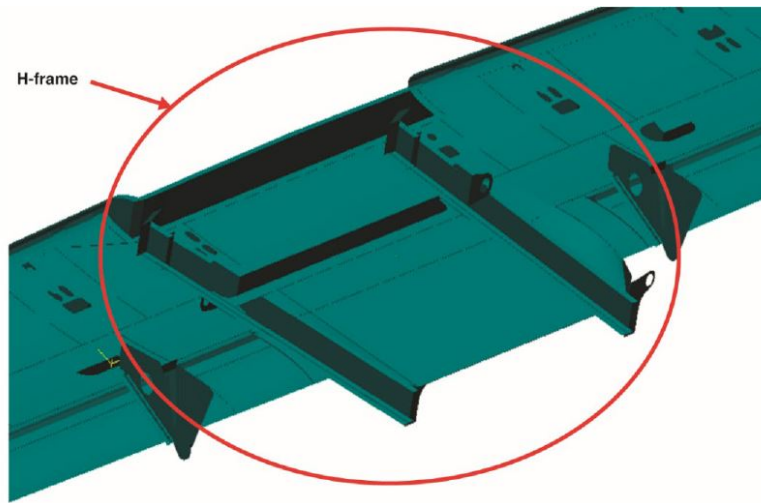


Fig. 6 H-frame location underneath the X-57 Mod III wing

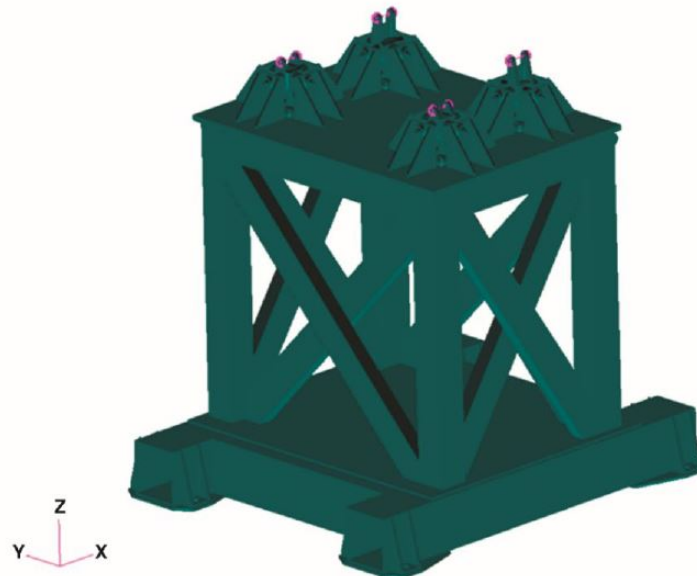


Fig. 7 Finite element model for the wing loads test fixture

For the FEM prediction verification, a small GVT was performed with the WLTF in the FLL prior to the X-57 Mod III Wing GVT. It was found that the 1st frequency of the WLTF was 58.38 Hz, about 23 percent lower than what was initially predicted. To model the boundary condition more closely to what was observed, the WLTF FEM was updated with each of the bolt locations having both X- and Y-axes rigidly fixed as a Rigid Body Element type 3 point, while the Z-axis translational vertical direction was fixed via a stiff CELAS2 spring, as shown in Fig. 8. The updated WLTF FEM boundary condition resulted in a 1st frequency of 55.68 Hz, which was close to what was seen in the WLTF GVT.

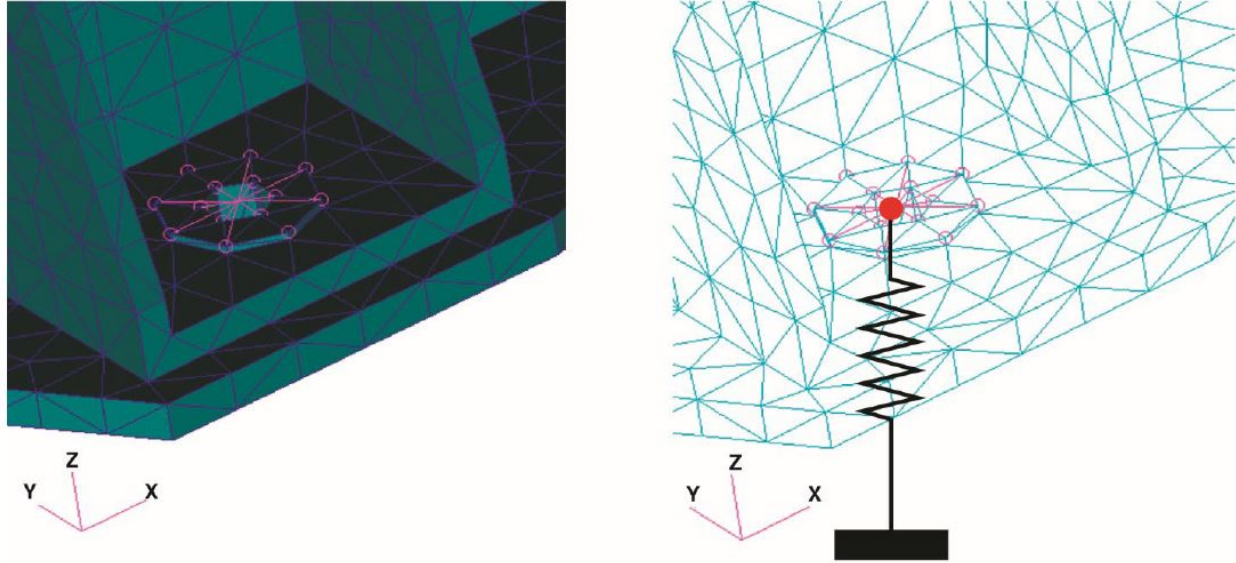


Fig. 8 Updated wing loads test fixture finite element model boundary condition, with the center of the Rigid Body Element type 3 point fixed in the Z-axis translational vertical direction via a stiff CELAS2 spring

The WLTF FEM with the updated boundary condition was then mated with both Mod III wing configurations, as shown in Fig. 9. The resulting FEM modal frequencies between the fixed-wing-only case and the wing with the WLTF were very similar for both configuration A (Fig. 9a) and configuration B (Fig. 9b). It should be noted that for both configurations, coupling was seen in the mode shapes between the Mod III wing and the WLTF at around 52 Hz.



Fig. 9 Finite element model for the X-57 Mod III wing on the wing loads test fixture for configuration A (Fig.9a); and configuration B (Fig.9b).

Test Configurations

Table 2 shows an example of the test configuration matrix used by the NASA AFRC structural dynamic test engineers to ensure that the test objectives were achieved. The table highlights the different test cases with varying aileron or flap position and excitation methods, locations, or directions. At the discretion of the test engineers, test cases B2 and B6 were

Table 2 Test configuration matrix for the X-57 Mod III wing ground vibration test

Configuration		Test case	Aileron position	Flap position	Shaker / impact	Shaker/impact locations	Shaker/impact direction	Shaker force level
Configuration A	Wing with cruise nacelle simulators	A1	Locked & neutral	Locked & stowed (0%)	Shaker (x2)	Wing TE spar	30° F/A, Vert, F/A, lat	Low, med, high
		A2			Shaker (x4)	Wing TE spar & flap TE center		
		A3			Unlocked & neutral	Shaker / impact	Control surfaces OB TE & TE center	Vert, F/A
		A4	Shaker (x2)			Wing TE spar	30° F/A, Vert, F/A, lat	Low, med, high
		A5	Shaker / impact			Control surfaces OB TE & TE center	Vert, F/A	Low
		A6		Locked & neutral	Locked & slightly deployed (0% - just off stops)		Vert, lat, F/A	
		A7			Locked & deployed (100% - relevel accels)			
Configuration B	Clean wing	B1	Locked & neutral	Locked & stowed (0%)	Shaker (x2)	Wing TE spar	30° F/A, vert, F/A, lat	Low, med, high
		B2			Shaker (x4)	Wing TE spar & flap TE center		
		B3			Unlocked & neutral	Shaker / impact	Control surfaces OB TE & TE center	Vert, F/A
		B4	Shaker (x2)			Wing TE spar	30° F/A, vert, F/A, lat	Low, med, high
		B5	Shaker / impact			Control surfaces OB TE & TE center	Vert, F/A	Low
		B6		Locked & neutral	Locked & slightly deployed (0% - just off stops)		Vert, lat, F/A	
		B7			Locked & deployed (100% - relevel accels)			

Accelerometer (accel); fore/aft (F/A); lateral (lat); outboard (OB); trailing edge (TE); vertical (vert)

not performed. For test case B2, it was found that a four-shaker setup in configuration A did not yield clean data as desired. Test case B6 was a secondary objective and was eliminated due to time constraints.

Ailerons were never deflected during the GVT and were either locked or unlocked at a neutral setting. Figure 10 shows an aileron actuation mechanism device, shown as the green Unistrut frame (Fig. 10a), and a close-up view that was constructed in front of the Mod III wing leading edge to lock the ailerons manually (Fig. 10b). Two cables were connected to the left and right ailerons, and each cable required about 44 lb of force to lock the ailerons in place. In-line load cells were placed on these cables to monitor the loads during the test. For any test cases that involved unlocked ailerons, the cables were disconnected and removed.



Fig. 10 Aileron actuation mechanism in front of the X-57 Mod III wing (Fig. 10a); and close-up view with the cables attached (Fig. 10b)

The Mod III wing flaps were driven by an actuator that locked the flap position when unpowered in all test cases. Flap position settings used during the GVT were stowed, slightly deployed (released just off their stops), and fully deployed at the maximum setting. It should be noted that the stowed and slightly deployed flap configurations were considered a 0-deg flap deflection setting, while the fully deployed flap case was considered a 30-deg flap deflection setting. For test cases that required the flaps to be slightly deployed or fully deployed, an external power supply was connected to electrically actuate the flaps to the desired position prior to test execution. Cables connecting the flap to the external power supply were disconnected before performing the test runs to avoid unwanted mass to the test article. Furthermore, for the fully deployed flap setting, GVT accelerometers on the flaps had to be releveled as their orientations had changed. Excitation was predominantly done with two shakers, one at each of the wingtips, oriented either vertically or fore/aft depending on the test run. There were several test runs that had a four-shaker configuration, as shown in Fig. 2, with an additional shaker located at each of the left and right flaps. An impact hammer excitation was also used, primarily for the control surface investigation.

Ground Vibration Test Setup

Prior to the GVT, both weight and balance measurements were performed on the X-57 Mod III wing. Due to the flap actuator mechanism located near the left outboard wingtip, about 5 lb of ballast had to be placed on the right-wing side to keep the wing level. This additional weight was considered for subsequent updated weight and center-of-gravity calculations. Before installing the Mod III wing onto the WLTF, the test fixture itself was torqued to 220lb (± 20 ft-lb) to the FLL floor tracks and shimmed to provide leveling. For configuration A, the wingtip plates and cruise nacelle mass simulators were installed onto the Mod III wingtips before the wing was mounted onto the WLTF. These wingtip plates and cruise nacelle mass simulators were later removed for configuration B for the clean wing case. At the time of the X-57 Mod III wing GVT, the flight accelerometers for the wing were not ready, so no data was collected during the GVT using flight instrumentation.

Ground Vibration Test Instrumentation

For the X-57 Mod III wing GVT, the data acquisition (DAQ) used was the LAN-XI DAQ hardware system (Brüel & Kjær, Nærum, Denmark), which was the first large-scale GVT by NASA AFRC using this DAQ system. Four mainframes were used, including two LAN-XI Type 3660-D11-slot mainframes and two LAN-XI Type 3660-C 5-slot mainframes, and were all connected via a network switch. Accelerometer outputs and force transducers for both the impact hammer and shakers were connected to the 12-channel accelerometer LAN-XI Type 3053 modules and shaker generator LAN-XI Type 3160 modules. The setup of the LAN-XI DAQ hardware is shown in Fig. 11.

Two different types of accelerometer sensors were used for this test, including the PCB T356A16 low-frequency triaxial and the PCB T33B32 low-frequency uniaxial (PCB Piezoelectronics, Inc., Depew, New York). The Mod III wing and its flaps and ailerons, as well as portions of the H-frame and the wingtip plates (used to interface the cruise nacelle mass simulator to the wing) were instrumented with these PCB T356A16 triaxial accelerometers. The WLTF, the cruise nacelle mass simulators, and the connection points between the WLTF and the H-frame were instrumented with PCB T33B32

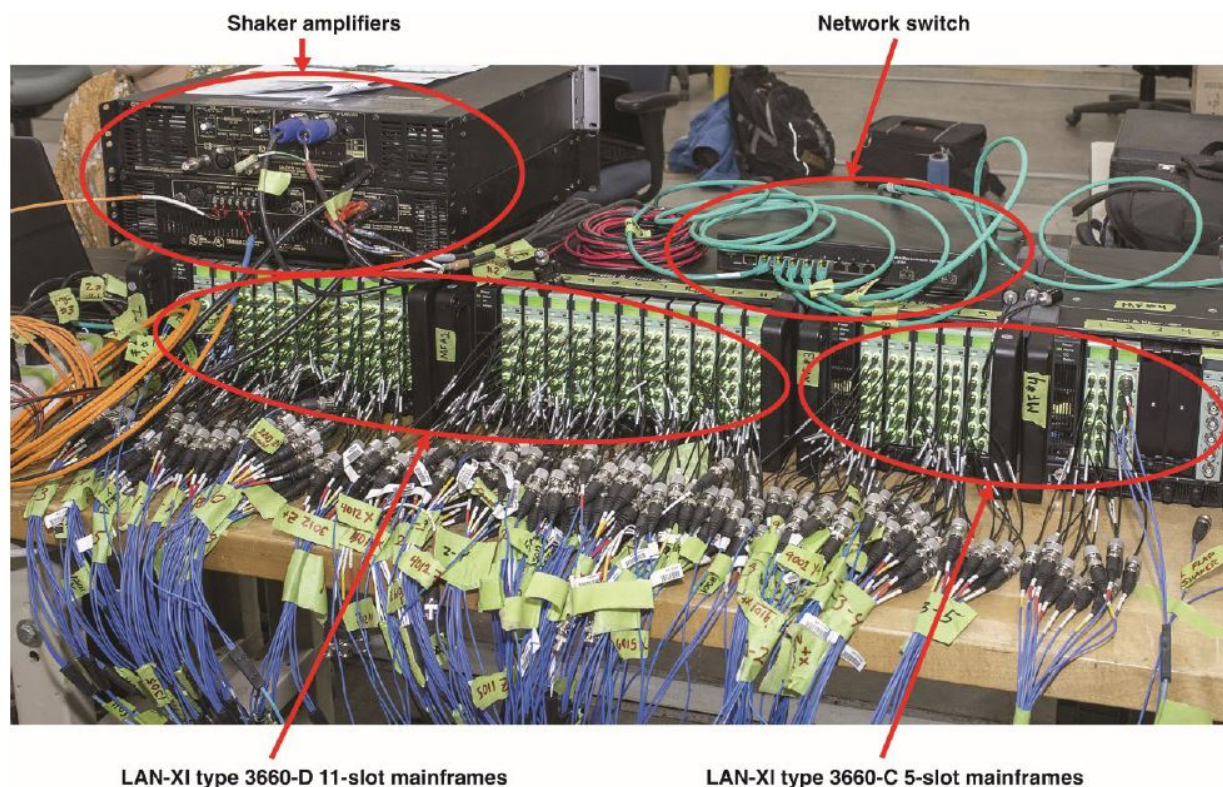


Fig. 11 LAN-XI data acquisition hardware system used for the X-57 Mod III wing ground vibration test

uniaxial accelerometers. It should be noted that for this GVT, data was gathered in all three degrees of freedom for all test node locations. At some test node locations, three uniaxial accelerometers were used to build up a triaxial measurement.

The excitation for this GVT was provided using both shakers and an impact hammer. The shakers were used to excite throughout the wing structure, while the impact hammer was used for local excitation of the individual control surfaces. Two different MB Dynamics (MB Dynamics, Inc., Cleveland, Ohio) electromagnetic shaker types were used, the Modal 110 and the Modal 50A. Depending on the test case for a particular run, either a two-shaker or a four-shaker configuration was u

Ground Vibration Test Accelerometer Layout

For configuration A of the X-57 Mod III wing GVT, a total of 104 accelerometer locations were installed measuring 312 DOF, whereas for configuration B with the clean wing (i.e., cruise nacelle mass simulators and wingtip plates removed), there were 96 total accelerometer locations measuring 288 DOF. Accelerometers were installed at various locations for the X-57 Mod III wing along the spars and ribs of the wing and along the hinge lines for the control surfaces. All sensors were installed using the global coordinate system of the airplane. It should be noted that several of the flap leading edge accelerometers had to be installed on the underside of the wing to avoid interference with the wing in the stowed-flaps configuration. Table 3 shows the number of accelerometers and DOF count for this GVT.

Table 3 Accelerometer and degrees of freedom count breakdown for the X-57 Mod III wing ground vibration test

Description	# of accelerometer locations	# of DOF
Wing	81	243
H-frame	7	21
Wing loads test fixture (WLTF)	8	24
Cruise nacelle mass simulators and wingtip plates	8	24
Total	104	312

The numbering convention for all the various accelerometer locations of this GVT is detailed in Table 4. The triaxial accelerometer locations for the wing are shown in Fig. 12.

Table 4 X-57 Mod III wing ground vibration test accelerometer node numbering convention

Description	Node Series #	Description	Node Series #
Left wing forward spar	1100	Right aileron	6000
Left wing mid spar	1200	Wing center	7000
Left wing aft spar	1300	Left cruise nacelle simulator	8100
Right wing forward spar	2100	Right cruise nacelle simulator	8200
Right wing mid spar	2200	Left mass simulator plate	8300
Right wing aft spar	2300	Right mass simulator plate	8400
Left flap	3000	Wing loads test fixture (WLTF)	9000
Right flap	4000	WLTF attachment points	9100
Left aileron	5000	H-frame	9200

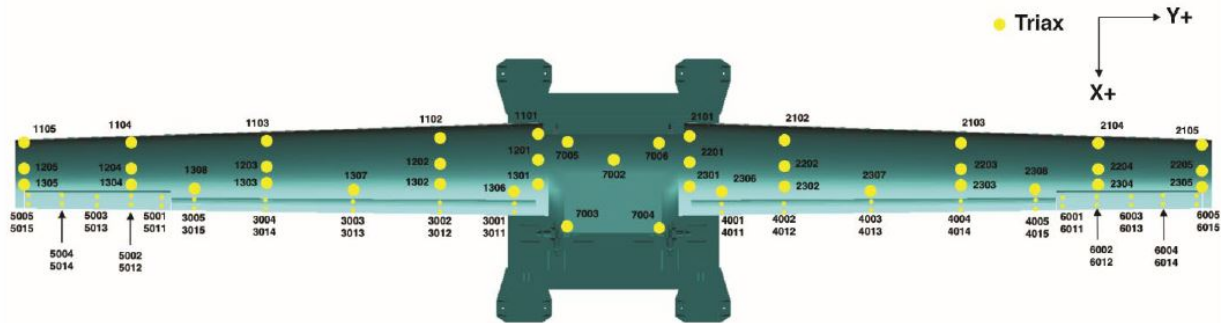


Fig. 12 Ground vibration test triaxial accelerometer locations for the X-57 Mod III wing

Figure 13 shows the triaxial accelerometer locations for the H-frame. Notice that the forward WLTF connection points to the H-frame do not line up linearly underneath, exactly where the forward wing points attach to the H-frame.

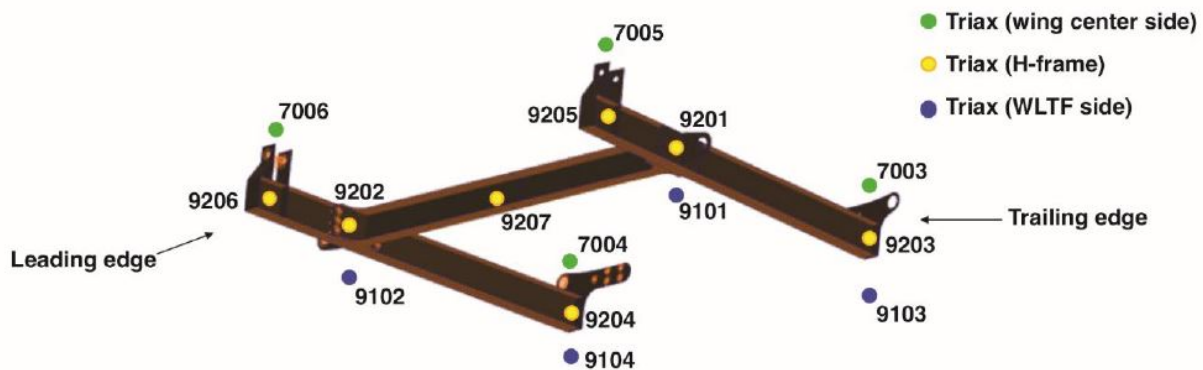


Fig. 13 Ground vibration test triaxial accelerometer locations for the H-frame, with other accelerometers on the wing and the wing loads test fixture side for reference

Figure 14 shows the triaxial accelerometer locations for the WLTF during the test. Since the GVT had been previously performed on the WLTF by itself and, at the time, confirmed that its 1st frequency was above most of the modal frequencies of interest for the Mod III wing, only the top portions of the WLTF and its connection points to the wing, via the H-frame adapter, were instrumented with accelerometers. For the cruise nacelle mass simulators and the wingtip plates, there were only eight total accelerometer locations, with one sensor each, located at both the forward and aft ends.

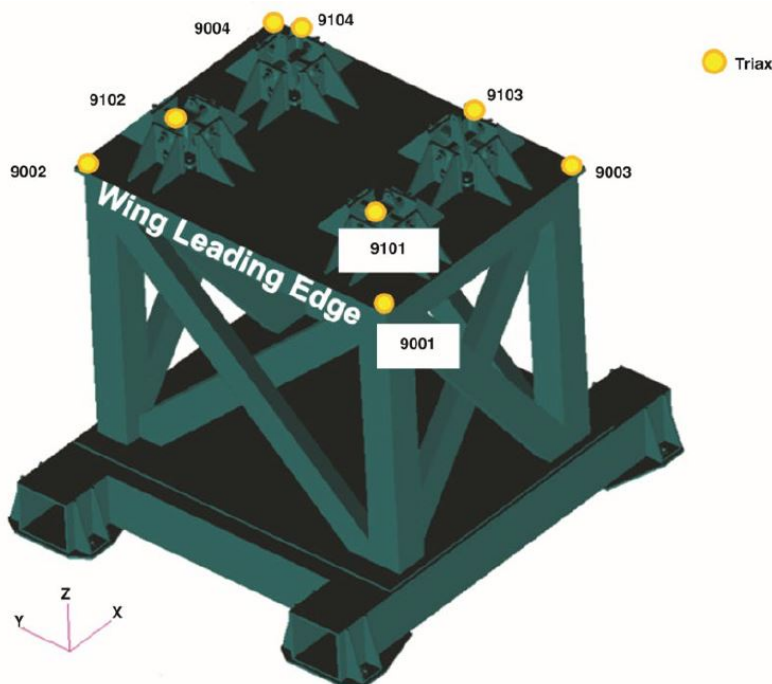


Fig. 14 Ground vibration test triaxial accelerometer locations for the wing loads test fixture

Test Display Model

The GVT test display models were created in the BK Connect[®] (Brüel & Kjær, Nærum, Denmark) software with triangular elements and assisted the test engineers with visualizing the test mode shapes throughout testing. Due to the large deflections between the stowed and the fully deployed flap cases, separate GVT test display models were created for configurations A and B. For configuration A, since there was little difference in the flap coordinate cases between the flap stowed and flaps slightly deployed, the same test display model was used for both cases. Figure 15 shows the test display models created for configuration A with the flaps stowed, slightly deployed, and fully deployed.

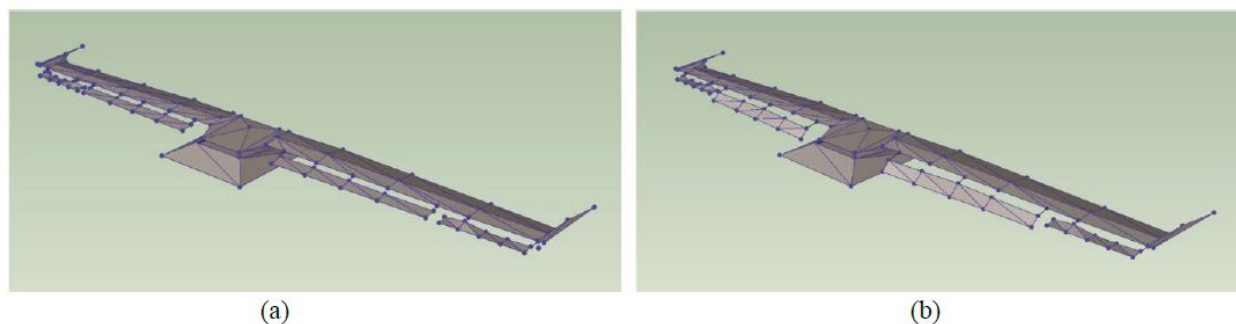


Fig. 15 Test display models or ground vibration test models for configuration A (Fig. 15a) with flaps stowed and flaps slightly deployed; and with flaps fully deployed configuration B (Fig. 15b)

Likewise, Fig. 16 shows the test display models created for configuration B, with the flaps stowed as well as the flaps fully deployed. The difference in configuration B versus configuration A is that the cruise nacelle mass simulator and wingtip plates were removed in the configuration B test display models.

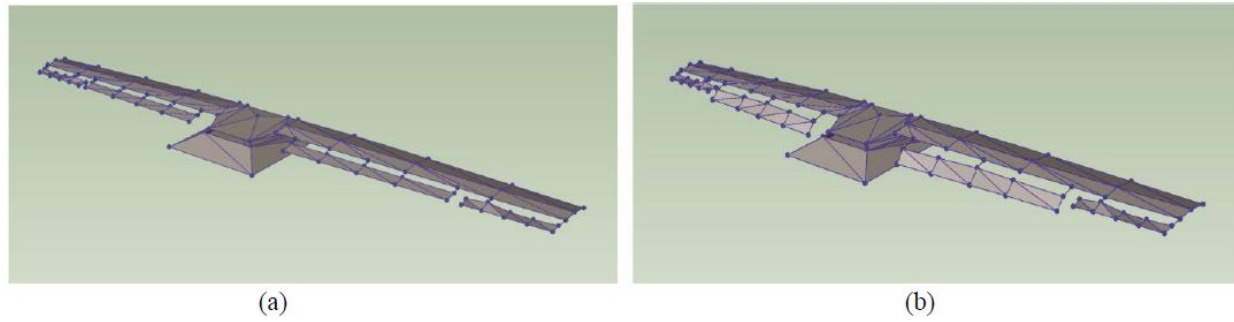


Fig. 16 Test display models or ground vibration test models for configuration B (Fig. 16a) with the flaps stowed; and the flaps fully deployed Configuration B (Fig. 16b)

Results

A total of 192 test runs were completed during the X-57 Mod III wing GVT. Data was gathered using BK Connect[®] software. The following sections discuss the modal test results for the two different wing configurations and at various flap settings, including configuration A with the flaps stowed, flaps fully deployed, and flaps slightly deployed, and configuration B with the flaps stowed and flaps fully deployed. With the flaps slightly deployed, configuration B was not performed as originally planned. Although GVT data was gathered for both ailerons in the locked, unlocked, and neutral position, only data from aileron locked cases is presented. Note that the FEM and GVT frequency comparisons, which are presented in the following sections, utilized the non-tuned FEM before correlation updates were performed.

Configuration A – Flaps Stowed

The Mod III GVT started with configuration A with the wingtip cruise nacelle mass simulators along with the flaps stowed and ailerons locked. Initial findings from this test configuration informed how the remainder of the GVT was to be conducted. There were several test runs that involved a four-shaker configuration with an additional two shakers (one at each of the left and right flaps); however, post-processing of those test runs indicated that the frequency response functions (FRFs) were not as clean as desired, and the four-shaker configuration was subsequently dropped for the remaining test runs. Normally, for shaker testing, burst random excitation is used. After the test conductors noticed that some of the accelerometers experienced a DC offset and that the time history decay in the data never settled out, the shaker excitation was switched to a continuous, random setting for the remainder of the test. In addition, impact hammer excitations were performed on the flaps and the ailerons to excite these control surfaces; however, post-processing of the impact hammer results during the GVT showed poor quality FRFs and mode shapes for the Mod III wing control surfaces. Data collected for the remainder of the test cases for configuration A exhibited similar results. As a result, the impact hammer excitation method was eliminated for the configuration B testing.

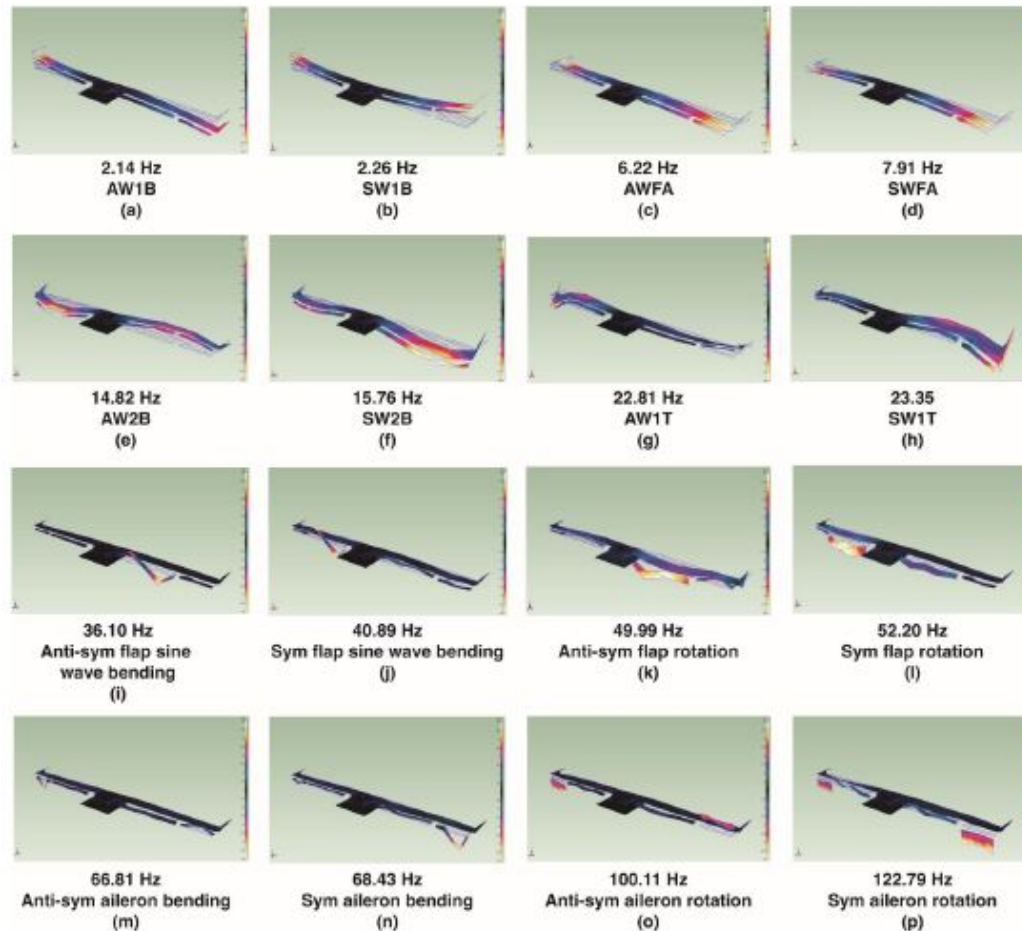
Table 5 lists the dominant wing and control surface modes with their GVT frequencies compared to FEM predictions for the Mod III wing in configuration A with the flaps stowed and ailerons locked. Mode names presented in Table 5 have been shortened to focus on the dominant motion. Percentage differences, highlighted in red, indicate that the error between GVT and the FEM frequency is greater than 5 percent. Frequency errors between the GVT and FEM predictions were consistently >5 percent starting at a GVT frequency of approximately 40Hz, which suggests coupling between WLTF and Mod III wing modes occurred at frequencies even lower than the 55 Hz (as predicted by the pre-test FEM of the Mod III wing mated to the WLTF). Visualization of mode shapes starting at approximately 40 Hz showed significant motion of the WLTF top plate (accelerometer node 9001-9004). This coupling between the WLTF and Mod III wing was observed in both configurations A and B.

Figure 17 shows the GVT mode shapes and frequencies for the dominant wing and the control surface modes that were listed in Table 5. What was noticed immediately was nearly all the modes exhibited asymmetry, with one side of either the wing or the control surface being more dominant than the other. Most of the wing modes were in line with the FEM predictions, with the exception of the anti-symmetric wing fore/aft (AWFA) and the symmetric wing fore/aft (SWFA) modes, as shown in Fig. 17c and Fig. 17d, respectively, which occurred at higher frequencies than expected. In addition, the AWFA and the SWFA modes were the only wing modes that showed no asymmetry between the left and right sides of the wing. The control surface modes with the flaps and ailerons also exhibited this asymmetry in their mode shapes, with the exception

Table 5 Configuration A – flaps stowed and ailerons locked: frequency comparison between finite element model and ground vibration test

Mode name	FEM frequency, Hz	GVT frequency, Hz	Percent difference from FEM	Notes on GVT mode shape
Anti-sym wing 1st bending (AW1B)	2.11	2.14	1.68%	Left wing dominant
Sym wing 1st bending (SW1B)	2.26	2.26	-0.05%	Left wing dominant
Anti-sym wing fore/aft (AWFA)	5.12	6.22	21.53%	Both wings dominant
Sym wing fore/aft (SWFA)	7.14	7.91	10.85%	Both wings dominant
Anti-sym wing 2nd bending (AW2B)	14.62	14.82	1.42%	Left wing dominant
Sym wing 2nd bending (SW2B)	15.62	15.76	0.89%	Right wing dominant
Anti-sym wing 1st torsion (AW1T)	21.89	22.81	4.21%	Left wing dominant
Sym wing 1st torsion (SW1T)	22.42	23.35	4.15%	Right wing dominant
Anti-sym flap sine wave bending	31.06	36.10	16.22%	Right-flap sine wave. GVT has no anti-sym shape. Only right flap responding.
Sym flap sine wave bending	31.12	40.89	31.42%	Left-flap sine wave. GVT has no sym shape. Only left flap responding.
Sym flap rotation	34.47	52.20	51.46%	Left-flap dominant
Anti-sym flap rotation	36.96	49.99	35.23%	Right-flap dominant with AWFA & right aileron bending
Anti-sym aileron rotation	46.21	100.11	116.66%	Left aileron dominant
Sym aileron rotation	46.93	122.79	161.67%	Both ailerons dominant
Anti-sym aileron bending	59.83	66.81	11.67%	Left aileron dominant
Sym aileron bending	61.41	68.43	11.43%	Right aileron dominant

symmetric (sym); anti-symmetric (anti-sym)

**Fig. 17** Ground vibration test mode shapes and frequencies for configuration A – flaps stowed and ailerons locked

of the symmetric aileron rotation mode in Fig. 17(p). It should be noted that all GVT flap and aileron mode frequencies were much higher than what the FEM had predicted. Particularly, the anti-symmetric and symmetric aileron rotation modes exhibited well over a 100-percent frequency difference between the GVT and FEM, indicating that the aileron wing hinge and/or the GVT aileron actuation mechanism connection stiffness was more rigid than expected.

Configuration A – Flaps Deployed

For the next flap setting of configuration A, the flaps were deployed at the maximum deflection setting of approximately 30 degrees. This configuration checked for any significant effects to the mode shapes or frequencies, particularly for the flaps, when compared to the stowed-flap case. The GVT accelerometers on the flaps were relevelled for orientation back into the aircraft coordinate system prior to the deployed flaps test runs since the flap accelerometers were initially installed when the flaps were stowed. Table 6 compares GVT frequencies between the wing and control surface modes of configuration A for the flaps stowed case in the left column, and the flaps deployed case in the right column. Percentage values in red indicate that the frequency shift between the flaps deployed case and the flaps-stowed case is greater than 5 percent magnitude. Note that pre-test FEM modal predictions were only done for both configuration A and B with the flaps stowed, so there were no direct GVT to FEM comparison for the cases where the flaps were either fully deployed or slightly deployed. Having the flaps fully deployed lowered frequencies for several flap modes compared to the flaps-stowed case. These modes included the right-flap sine wave bending, the left-flap sine wave bending, as well as the symmetric flap rotation mode. Although, the anti-symmetric flap rotation GVT mode frequency was lower than that of its counterpart in the flaps-stowed case, the difference was not as pronounced as the symmetric flap rotation mode, which was lower than the stowed GVT result by about 32 percent.

Table 6 Ground vibration test frequency comparison for configuration A between flaps stowed and ailerons locked versus flaps deployed and ailerons locked

Mode Name	Flaps stowed GVT frequency, Hz	Flaps deployed GVT frequency, Hz	% difference from flaps stowed
Anti-sym wing 1st bending (AW1B)	2.14	2.14	-0.15%
Sym wing 1st bending (SW1B)	2.26	2.26	0.03%
Anti-sym wing fore/aft (AWFA)	6.22	6.04	-2.95%
Sym wing fore/aft (SWFA)	7.91	7.54	-4.64%
Anti-sym wing 2nd bending (AW2B)	14.82	14.69	-0.89%
Sym wing 2nd bending (SW2B)	15.76	15.60	-0.96%
Anti-sym wing 1st torsion (AW1T)	22.81	22.84	0.14%
Sym wing 1st torsion (SW1T)	23.35	23.38	0.13%
Right-flap sine wave bending	36.10	30.59	-15.26%
Left-flap sine wave bending	40.89	32.58	-20.31%
Anti-sym flap rotation	49.99	47.50	-4.98%
Sym flap rotation	52.20	35.40	-32.18%
Anti-sym aileron bending	66.81	64.11	-4.05%
Sym aileron bending	68.43	68.90	0.69%
Anti-sym aileron rotation	100.11	96.96	-3.15%
Sym aileron rotation	122.79	115.27	-6.13%

Configuration A – Flaps Slightly Deployed

Configuration A with the flaps slightly deployed is similar to the flaps-stowed case with the following exception; when the flaps are released from their stops, a slight movement aft was created during flap positioning. In the case where flaps were slightly deployed, Table 7 shows a comparison of the GVT frequencies between the wing and control surface modes for configuration A with the flaps slightly deployed and the other two flap settings discussed earlier. The two right columns of the table show the percentage difference for the slightly deployed flap cases when compared to the stowed-flap and deployed-flap cases, respectively. Percentage values in red indicate frequency differences of greater than 5 percent. Wing modes did

Table 7 Configuration A – ground vibration test: frequency comparison between the flaps-stowed and flaps-deployed cases versus the flaps slightly deployed case

Mode name	Flaps stowed GVT frequency, Hz	Flaps deployed GVT frequency, Hz	Flaps slightly deployed GVT frequency, Hz	Percent difference from stowed	Percent difference from deployed
Anti-sym wing 1st bending (AW1B)	2.14	2.14	2.12	-1.26%	-1.11%
Sym wing 1st bending (SW1B)	2.26	2.26	2.23	-1.12%	-1.15%
Anti-sym wing fore/aft (AWFA)	6.22	6.04	6.11	-1.81%	1.17%
Sym wing fore/aft (SWFA)	7.91	7.54	7.54	-4.71%	-0.07%
Anti-sym wing 2nd bending (AW2B)	14.82	14.69	14.76	-0.45%	0.44%
Sym wing 2nd bending (SW2B)	15.76	15.60	15.76	0.04%	1.00%
Anti-sym wing 1st torsion (AW1T)	22.81	22.84	22.78	-0.12%	-0.25%
Sym wing 1st torsion (SW1T)	23.35	23.38	23.32	-0.11%	-0.23%
Right-flap sine wave vertical bending	36.10	30.59	30.56	-15.34%	-0.10%
Left-flap sine wave vertical bending	40.89	32.58	34.43	-15.80%	5.66%
Anti-sym flap rotation	49.99	47.50	N/A	N/A	N/A
Sym flap rotation	52.20	35.40	39.92	-23.53%	12.75%
Anti-sym aileron vertical bending	66.81	64.11	66.33	-0.72%	3.47%
Sym aileron vertical bending	68.43	68.90	67.23	-1.76%	-2.43%
Anti-sym aileron rotation	100.11	96.96	111.99	11.87%	15.51%
Sym aileron rotation	122.79	115.27	117.76	-4.10%	2.16%

not change much between the three different flap settings, less than 2 percent. Most of the significant change was seen in the control surface modes. Note that for this configuration (flaps slightly deployed) the anti-symmetric flap rotation mode was not found in the GVT. For the right- and left-flap sine wave vertical bending modes and flaps slightly deployed, the GVT frequencies were approximately 15 percent lower compared to the stowed-flap case. When compared to the deployed-flap case, the right-flap sine wave vertical bending was on par, whereas with the left-flap sine wave vertical bending, the slightly deployed flap case was slightly higher. The frequency difference between the slightly deployed flap case and the stowed case was also higher for the symmetric-flap rotation mode when compared to the deployed flap case; however, for the anti-symmetric aileron rotation mode (flaps slightly deployed), the GVT result was much higher for the deployed case as opposed to the stowed case.

Configuration B – Flaps Stowed

Configuration B represents the clean X-57 Mod III wing (i.e., the cruise nacelles mass simulator and its associated wingtip plates were removed prior to testing). Table 8 lists the dominant wing and control surface modes with their GVT frequencies, compared to the FEM for the Mod III wing in configuration B with the flaps stowed and ailerons locked. Mode names in Table 8 have been shortened to focus on the dominant motion. The FEM predictions presented are from the model using the fixed-wing boundary condition case without the WLTF incorporated. Percentage differences, highlighted in red, indicate that the error between GVT and FEM frequency is greater than 5 percent. Like the previous test configuration, configuration B saw coupling between the WLTF and the Mod III wing starting around a GVT frequency of approximately 40 Hz, which was lower than the 55 Hz predicted by the pre-test FEM.

Figure 18 shows the GVT mode shapes and frequencies for the wing and control surface modes listed in Table 8 for configuration B with the flaps stowed and ailerons locked. Similar to configuration A, many of the modes for both the wing and control surfaces exhibited asymmetry, with one side being more dominant than the other. As expected, removing the cruise nacelle mass simulators at the wingtips for the clean wing configuration increased the wing GVT frequencies. For the AWFA mode in Fig. 18(c), like in configuration A with the flaps stowed, the GVT frequency was higher when compared to the FEM; however, the GVT frequency for the SWFA mode in Fig. 18(d) was in line with what the FEM had predicted. Whereas both wings were equally dominant for the AWFA and SWFA modes for configuration A, with configuration B (with the flaps stowed), the asymmetry was seen again with the AWFA mode showing left-wing dominance and the SWFA mode showing right-wing dominance. In addition, both the anti-symmetric wing 1st bending (AW1B) and the symmetric wing 1st bending (SW1B) modes (shown in Fig. 18a and Fig. 18b, respectively), were about 8 percent lower than what the FEM had estimated. Unlike configuration A (flaps-stowed case), both the anti-symmetric wing 1st torsion (AW1T) and symmetric wing 1st torsion (SW1T) modes for this configuration were not exactly pure wing modes; the AW1T mode (shown

Table 8 Configuration B – flaps stowed and ailerons locked: Frequency comparison between finite element model and ground vibration test

Mode Name	FEM frequency, Hz	GVT frequency, Hz	Percent difference from FEM	Notes on GVT mode shape
Anti-sym wing 1st bending (AW1B)	6.32	5.76	-8.88%	Left wing dominant
Sym wing 1st bending (SW1B)	7.13	6.55	-8.21%	Right wing dominant
Anti-sym wing fore/aft (AWFA)	12.95	15.14	16.87%	Left wing dominant
Sym wing fore/aft (SWFA)	19.99	20.64	3.26%	Right wing dominant
Anti-sym wing 2nd bending (AW2B)	24.84	23.90	-3.78%	Left wing dominant
Sym wing 2nd bending (SW2B)	26.95	26.36	-2.20%	Right wing dominant
Anti-sym wing 1st torsion (AW1T)	N/A	84.43	N/A	NOT IN FEM: see Anti-sym aileron bending below.
Sym wing 1st torsion (SW1T)	N/A	51.15	N/A	NOT IN FEM: Left wing dominant with strong left-flap bending and left aileron bending
Anti-sym flap sine wave bending	31.09	41.61	33.86%	Left-flap sine wave. GVT has no anti-sym shape.
Sym flap sine wave bending	31.19	46.36	48.64%	Right-flap sine wave. GVT has no sym shape. Right-flap-outboard half dominant.
Sym flap rotation	35.65	N/A	N/A	DID NOT SHOW UP IN GVT
Anti-sym flap rotation	39.24	70.60	79.91%	Right-flap dominant with right aileron bending.
Anti-sym aileron rotation	46.42	89.28	92.32%	Both dominant with slight left wing torsion.
Sym aileron rotation	48.34	116.22	140.41%	Both dominant with slight left wing torsion.
Anti-sym aileron bending	58.66	84.43	43.93%	Left aileron dominant with anti-sym wing torsion (left dominant) & left-flap bending.
Sym aileron bending	58.82	78.71	33.81%	Left aileron dominant with sym wing torsion (left dominant) & left-flap bending.

symmetric (sym); anti-symmetric (anti-sym)

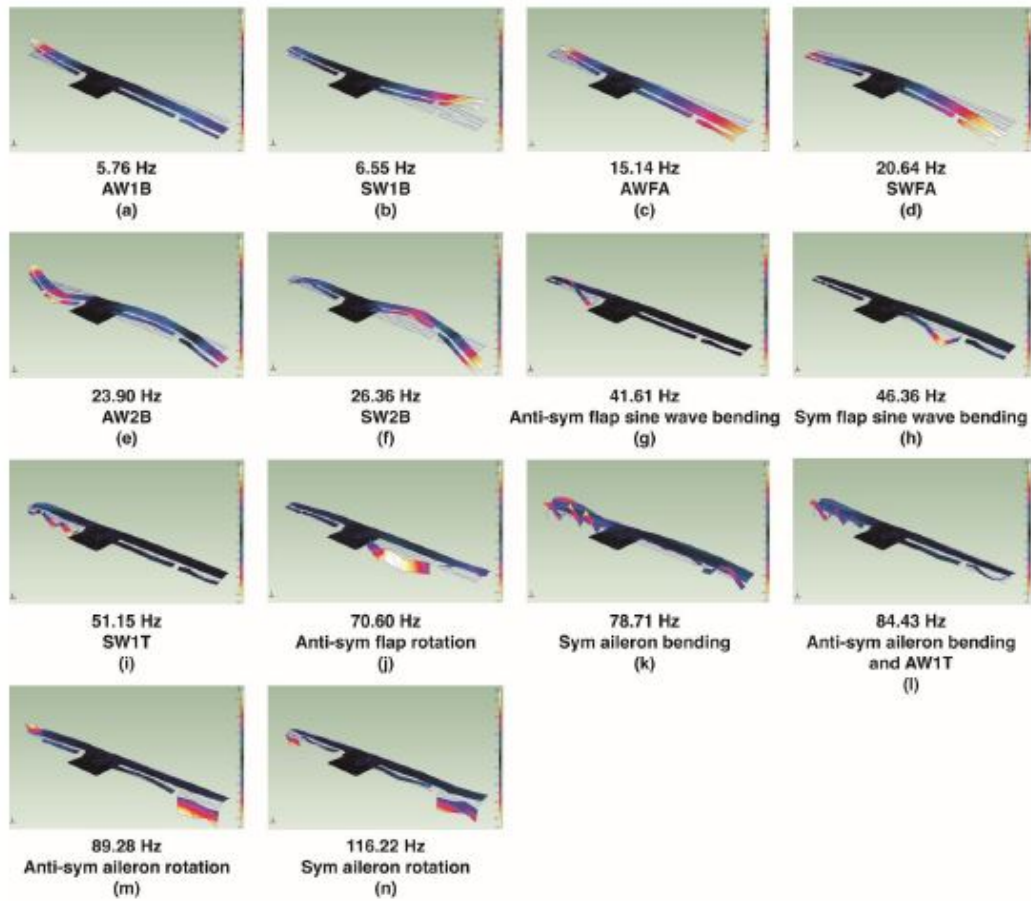
**Fig. 18** Ground vibration test mode shapes and frequencies for configuration B – flaps stowed and ailerons locked

Table 9 Ground vibration test frequency comparison for configuration B between flaps stowed versus flaps deployed

Mode Name	Flaps stowed GVT frequency, Hz	Flaps deployed GVT frequency, Hz	Percent difference from stowed
Anti-sym wing 1st bending (AW1B)	5.76	5.83	1.21%
Sym wing 1st bending (SW1B)	6.55	6.69	2.15%
Anti-sym wing fore/aft (AWFA)	15.14	14.77	-2.42%
Sym wing fore/aft (SWFA)	20.64	18.57	-10.01%
Anti-sym wing 2nd bending (AW2B)	23.90	23.91	0.02%
Sym wing 2nd bending (SW2B)	26.36	25.80	-2.13%
Right-flap sine wave vertical bending	46.36	30.87	-33.42%
Left-flap sine wave vertical bending	41.61	32.57	-21.73%
Sym wing 1st torsion (SW1T)	51.15	57.68	12.77%
Sym aileron vertical bending	61.22	71.07	16.08%
Sym flap rotation	N/A	35.93	N/A
Anti-sym flap rotation	70.60	42.87	-39.28%
Anti-sym wing 1st torsion (AW1T)	84.43	56.54	-33.03%
Anti-sym aileron vertical bending	84.43	64.34	-23.79%
Anti-sym aileron rotation	89.28	92.10	3.16%
Sym aileron rotation	116.22	124.10	6.78%

in Fig. 18(l)) had some dominant-left aileron bending as well as left-flap bending, and the SW1T mode (shown in Fig. 18(i)) also exhibited similar behavior with the left aileron and the left flap. These results make sense since the wing torsion modes shift to much higher frequencies with the removal of the wingtip motor masses and illustrate how the wing was designed to support large motors mounted at the wingtip. It should be noted that the clean wing FEM did not predict the presence of either the AW1T or SW1T modes. For the flap and aileron modes, the GVT frequencies were significantly higher than the FEM predictions, similar to the flaps-stowed case for configuration A with the cruise nacelle mass simulators. Also, the symmetric flap rotation mode did not manifest during the GVT for this case.

Configuration B – Flaps Deployed

Table 9 lists the frequencies for the wing and the control surface modes for the case with the clean wing, where the flaps are deployed compared to the previous case with the flaps stowed. Percentage values in red indicate that the frequency difference between the two GVT results are greater than 5 percent magnitude. The only wing mode that had a large difference from the flaps-stowed case was the SWFA mode, which was about 10 percent lower. The remainder of the control surface modes saw big differences between the flaps-stowed and flaps-deployed cases except for both the anti-symmetric and symmetric aileron rotation modes. Having the flaps deployed probably reduced the stiffness and frequencies for the flap modes. Although the symmetric flap rotation mode did not appear in the clean wing case with the flaps stowed, it did show up when the flaps were deployed. For the anti-symmetric flap rotation mode, the dominant flap changed; for the flaps-stowed case, the right flap was dominant, whereas in the flaps-deployed case, the left flap was dominant. This complex modal behavior of the flaps in both configurations A and B suggest that the control surface exhibits some nonlinear behavior, which will be further discussed in the next section. In the clean wing flaps-stowed case, the AW1T and anti-symmetric aileron vertical bending modes were coupled, which resulted in a shared frequency, whereas in the deployed flap case, the modes were more independent of one another but still slightly coupled.

Flap Nonlinearity and Finite Element Model Correlation

From this ground vibration test, it was observed that the different flap deployment settings affected flap modal response for both frequency and damping. From the frequency response plot, shown in Fig. 19, the right- and left-flap modes showed shifts in frequency when compared to one another between the various flap settings, despite having the same excitation location and force level input for the three different test runs. Also, the damping values for the flap modes were four to eight times larger than those of the wing modes, possibly due to nonlinear joints, such as the spherical bearings, associated with the control surface actuation system.

For the FEM correlation, a reduced-order model was used to compare the analytical results to the GVT data. Most of the model tuning effort was focused on updating the joint properties as well as making sure the FEM matched the as-built hardware of the test article. NASA Technical Standard (NASA-STD-5002), Load Analyses of Spacecraft and Payloads, defines that for significant modes, agreement between test and analytical frequencies should be within a 5-percent margin, whereas off-diagonal terms of the cross-orthogonality matrix should be within 10 percent [8]. Note that correlation of off-diagonal terms of the cross-orthogonality matrix is usually difficult at higher frequencies, particularly for complex aerospace structures. Figures 20, 21, and 22 present cross-orthogonality comparisons before and after FEM correlation to the GVT data for the stowed, deployed, and slightly deployed flap settings for configuration A. The cross-orthogonality plots show fundamental wing modes and primary flap bending modes up to approximately 40 Hz and were well correlated but also highlight some limitations that were encountered. The Mod III wing GVT was performed on a fixture designed for the wing loads proof test. This test fixture had flexible modes starting at approximately 40 Hz, which affected the GVT-measured mode shapes. A FEM of the test stand was included during the FEM to GVT correlation; however, more accelerometer instrumentation measuring test stand modes were required, yet not available, to improve wing mode correlation ≥ 40 Hz. Another challenge noted was with regards to the FEM versus GVT difference, where no significant response corresponding to FEM modes 7 and 8 (flap modes) was measured during the GVT when flaps were stowed and slightly deployed (Figs. 20 and 22, respectively); however, these flap modes only showed up in the deployed case, as shown in Fig. 21. The flap modes were heavily damped in the test data and difficult to extract, likely resulting from the flaps being restrained by contact with the wing flap-cove surface while in the stowed position.

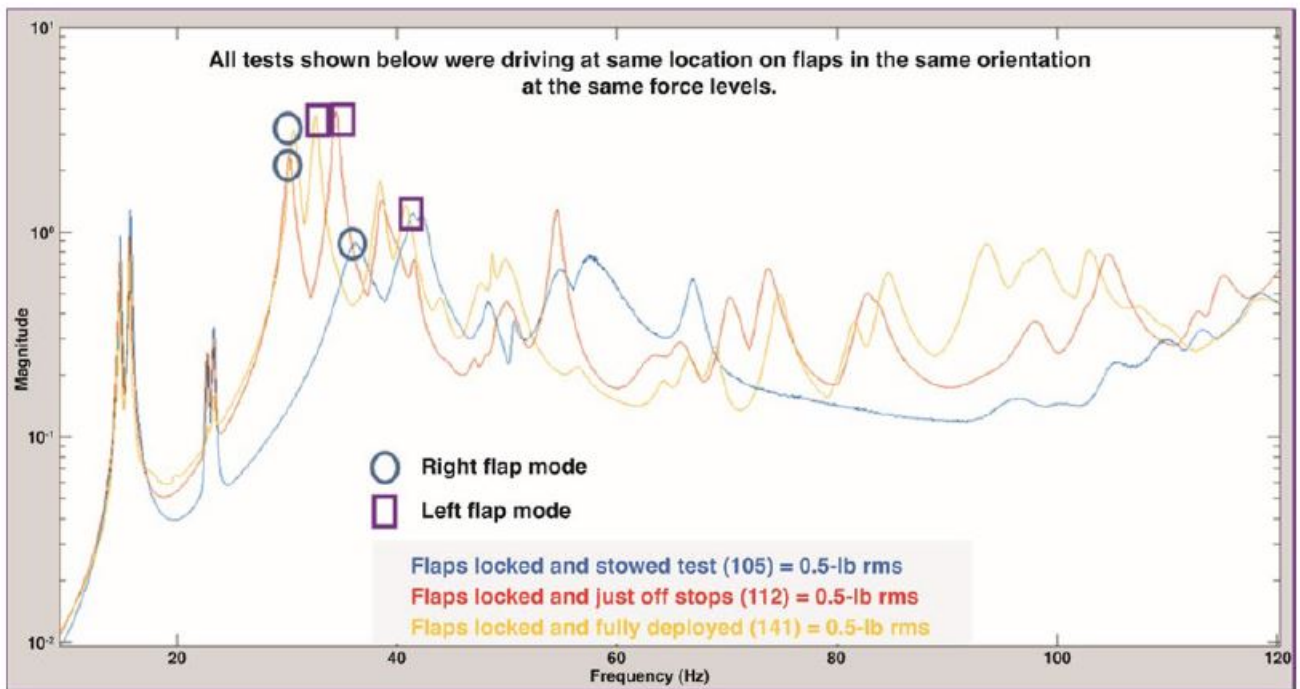


Fig. 19 Frequency response plot showing the nonlinearity behavior for the different flap settings for configuration A

		FEM/Test Cross Orthogonality Table																		
		FEM Shapes																		
Otg	Test Shapes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		2.17	2.35	4.62	7.37	15.11	16.32	19.07	19.27	22.74	23.54	31.87	33.10	33.17	38.02	41.60	42.24	45.99	47.87	48.86
100	1	2.13	100																5	
95	2	2.25		100																8
90	3	6.21			100															
85	4	7.88				100														
80	5	14.82	7				99			5									16	13
75	6	15.77		9				99			11									31
70	7	22.78				5	6			98	18								16	24
65	8	23.34				6	9			18	97								7	29
60	9	36.16											91	28			6		7	9
55	10	40.89	7						5			11	20	65		63	12	8	28	
50	11	41.35	9											18		75	52	9	42	6
45	12	42.30	7	9					5				10	19	9	55	65		6	47
40	13	44.26						23	7			10		18	89		21	7		11
35	14	48.30						7	6			60	5	7	5	9	6	67	42	19
30	15	52.32							8			40			26	6	5	68	42	12
25	16	54.77	5	7		6	6					8	6			7	10	5	40	17
20	17	57.21	5	6								5				22	16	10	22	
15	18	59.38	6	6			8	5				18				23	12	7	24	

Fig. 20a Cross-orthogonality matrix before correlation for configuration A – flaps stowed and ailerons locked

		FEM/Test Cross Orthogonality Table																		
		FEM Shapes																		
Otg	Test Shapes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		2.22	2.41	6.00	8.12	15.61	17.04	18.81	20.22	24.45	25.12	37.50	42.86	44.91	45.57	46.50	50.10	52.97	56.91	60.87
100	1	2.13	100	5																
95	2	2.25		100																
90	3	6.21			100															
85	4	7.88				100														
80	5	14.82	5				99	7												
75	6	15.77		7			6	99												
70	7	22.78							99	5										
65	8	23.34				6				99										
60	9	36.16										95			5	5			6	
55	10	40.89	6						5				36	87	20	13				
50	11	41.35	9								7		37	19	87	6				
45	12	42.30	6	8					5				25			91	11			6
40	13	44.26						17	13				80	10	27	40	23			
35	14	48.30							5				12		5	8	95	9		
30	15	52.32											7		8			94		27
25	16	54.77	5	6		6	5				6	7			7				91	
20	17	57.21		6														5	6	85
15	18	59.38	6	6			7				7									84

Fig. 20b Cross-orthogonality matrix after correlation for configuration A – flaps stowed and ailerons locked

FEM/Test Cross Orthogonality Table																				
		FEM Shapes																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Otg	Test Shapes	2.20	2.42	4.68	8.06	15.34	15.74	15.79	16.72	24.31	24.98	28.60	34.33	34.37	38.73	42.50	44.01	46.08	47.56	53.09
100	1	2.15	100																	
95	2	2.26	5	100																
90	3	6.00			100															
85	4	7.52				100														
80	5	14.74					99	9	6											
75	6	15.63		7					9	99										
70	7	17.45						78	61	7		11								
65	8	19.63				9	63	76	10											
60	9	22.81								99						5				
55	10	23.37			5						99	12								
50	11	30.57										24	27	90						
45	12	32.58					5	5					92	27						
40	13	34.11					6				10	87		17			38		5	
35	14	38.43	6							5		15		5		92	20	13	13	5
30	15	40.79						5				32		5	20	40	78		9	
25	16	43.99	5		5							5			94	7	15		22	10
20	17	46.73	5				6					13				10	15	5	29	60
15	18	47.59								5		9			18			9	91	17
10	19	48.59	6		5					5					6	17	13		10	85
5	20	49.70	7						5	6					15	20			25	87
0	21	50.40		9		5					5	5					27			15
	22	56.34							6		5				13	15	13		16	17

Fig. 21a Cross-orthogonality matrix before correlation for configuration A – flaps deployed and ailerons locked

FEM/Test Cross Orthogonality Table																				
		FEM Shapes																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Otg	Test Shapes	2.23	2.43	6.01	8.14	15.44	16.84	17.29	18.33	24.44	25.16	34.36	34.49	39.35	42.37	43.56	45.52	49.55	53.72	54.77
100	1	2.15	100																	
95	2	2.26	6	100																
90	3	6.00			100															
85	4	7.52				100														
80	5	14.74	5				100													
75	6	15.63		7				99	6											
70	7	17.45							99					9						
65	8	19.63								100										
60	9	22.81									99				6					
55	10	23.37			5							99								
50	11	30.57						6					96	12	5					
45	12	32.58							9					11	96					
40	13	34.11													96		10	13		7
35	14	38.43	6							5					12	65	68	18	6	5
30	15	40.79						5							8	41	10	86		
25	16	43.99	5													70	61	14	32	
20	17	46.73	5						5						15	11		12	25	73
15	18	47.59								5						13	23	5	91	9
10	19	48.59	6		6					5						13	9	12	21	84
5	20	49.70	7				5			6						8	16	6	16	91
0	21	50.40		9		5					5				5			23		93
	22	56.34					7				6					5	13	9	15	14

Fig. 21b Cross-orthogonality matrix after correlation for configuration A – flaps deployed and ailerons locked

		FEM/Test Cross Orthogonality Table																		
		FEM Shapes																		
Otg	Test Shapes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		2.17	2.35	4.62	7.37	15.11	16.32	19.07	19.27	22.74	23.54	31.87	33.10	33.17	38.02	41.60	42.24	45.99	47.87	48.86
100	1	2.08	100	6															5	
95	2	2.24		100																6
90	3	6.01			100															
85	4	7.51				100														
80	5	14.76	7				99			5									14	13
75	6	15.70		9				99			11									31
70	7	22.71				5	6			98	18								15	23
65	8	23.29			6		9			19	97								6	29
60	9	30.17						7				5	93	27						
55	10	34.43							9			13	26	85		25	14		12	14
50	11	38.54	5	5				5				36	6	5	5	45	74	20	52	27
45	12	39.93						9				82		7	29	31	25	15	7	10
40	13	41.54	9	6						6		11	8	26	8	76	41	10	23	37
35	14	44.51							7			24		10	94		10	7		
30	15	47.10		7				5	6		5	7			16	5	15	29	45	
25	16	48.30							15	7		5		12	15	5	17	91	63	27
20	17	49.14						5	15			26			21	25	8	78	57	12
15	18	49.89		7				5	10			14		5	7	7	17	46	6	14
10	19	50.32		5			5		12		5	8			24	17	17	70	58	10
5	20	54.50	7	7								5	7	24		15	8		17	15
0	21	63.11				5			13			14		11	8	6	7		5	
	22	65.65				13	7			15				6	6	7		10	9	17
	23	66.64		5			5	5			8	7								31
	24	70.14	7		6		18				5	6	6	5		10			23	23
	25	70.87		5				16		5			14							35

Fig. 22a Cross-orthogonality matrix before correlation for configuration A – flaps slightly deployed and ailerons locked

		FEM/Test Cross Orthogonality Table																		
		FEM Shapes																		
Otg	Test Shapes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		2.22	2.42	6.02	8.16	15.59	17.02	18.02	19.13	24.43	25.10	34.24	34.29	41.20	42.62	44.40	44.96	49.80	54.52	55.53
100	1	2.08	100	7																
95	2	2.24		100																
90	3	6.01			100															
85	4	7.51				100														
80	5	14.76	5				99	6												
75	6	15.70		7			6	99	6											
70	7	22.71								99	5				5					
65	8	23.29				5				5	99									
60	9	30.17						6				88	40	6						
55	10	34.43							9			37	82		8	22	14		9	5
50	11	38.54	5								5			36	38	38	70	11	6	6
45	12	39.93						6	5			7		86	12	33	31	14		
40	13	41.54	8	6						7		11	25	17	45	63	43		21	12
35	14	44.51							7				5	28	79	49		12		
30	15	47.10		6				6	5							10	23	30	56	68
25	16	48.30							16	7			11		7	14	20	89	12	
20	17	49.14							15					17	7	19	12	82	32	29
15	18	49.89		6						11					9		22	51	44	65
10	19	50.32					5		12		6					16	24	73	44	30
5	20	54.50	6	6								10	23			7			64	53

Fig. 22b Cross-orthogonality matrix after correlation for configuration A – flaps slightly deployed and ailerons locked

Summary

The X-57 Mod III wing ground vibration test was performed to gain a further understanding of the aeroelastic behavior of the Mod III wing structure. Two main configurations were investigated: the clean wing on its own, and a configuration that included the cruise nacelle mass simulators. For the most part, frequencies of the wing modes aligned well with finite element model predictions. Most of the error between the ground vibration test results and the finite element model predictions were for the flap and aileron modes and were significantly higher than anticipated, indicating the control surface modeling in the finite element model needed to be updated. Another problem in identifying and processing the ground vibration test results for these control surface modes was that these modes stay around or above the coupling frequency of the wing loads test fixture, which was lower than predicted. Using the fixed base correction method for this ground vibration test would have eliminated the coupling problem that was seen between the wing and the wing loads test fixture [9, 10]. Overall, the project gained some invaluable insight into the modal behavior of the Mod III wing, which contributed to the subsequent model updates to the Mod III wing finite element model as well as both the classical and whirl flutter analyses.

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References

1. National Aeronautics and Space Administration, "X-57 Maxwell Overview," X-57 Fact Sheet: <https://www.nasa.gov/aeronautics/x-57-max-well/> [accessed September 24, 2024].
2. Borer, Nicholas K., Geuther, Steven C., Litherland, Brandon L., and Kohlman, Lee, "Design and Performance of a Hybrid-Electric Fuel Cell Flight Demonstration Concept," AIAA Paper 2018-3357, June 2018. doi: [10.2514/6.2018-3357](https://doi.org/10.2514/6.2018-3357)
3. National Aeronautics and Space Administration, "Armstrong Flight Research Center," <https://www.nasa.gov/armstrong/> [accessed September 24, 2024].
4. National Aeronautics and Space Administration, "Flight Loads Laboratory Overview," <https://www.nasa.gov/flight-loads-laboratory-overview> [accessed September 24, 2024].
5. National Aeronautics and Space Administration, "Flight Loads Laboratory," Fact Sheet, https://www.nasa.gov/wp-content/uploads/2016/05/m187015fl-print_508.pdf [accessed September 24, 2024].
6. Miller, Eric J., Li, Wesley, Jordan, Ashante, and Lung, Shun-Fat., "X-57 Wing Structural Load Testing," AIAA Paper 2020-3090, June 2020. doi: [10.2514/6.2020-3090](https://doi.org/10.2514/6.2020-3090)
7. Bhamidipati, Keerti K., Pak, Chan-gi., Truax, Roger A., Winkel, James P., Rosario, Sev, Cutright, Stephen E., Massey, Steven J., and Wieseman, Carol D., "Classical Flutter Analysis of X-57 Aircraft Mod II, III, and IV Configurations," AIAA Paper 2024-4212. July 2024. doi: [10.2514/6.2024-4212](https://doi.org/10.2514/6.2024-4212)
8. NASA Technical Standard, "Load Analyses of Spacecraft and Payloads," NASA-STD-5002A, section 4.9.1, September 25, 2019.
9. Napolitano, Kevin L., and Yoder, Nathanael C., "Fixed Base FRF Using Boundary Measurements as References - Analytical Derivation," Topics in Modal Analysis I, Volume 5, Proceedings of the 30th IMAC, A Conference on Structural Dynamics, Jacksonville, Florida, pp. 299-308, 2012. doi: [10.1007/978-1-4614-2425-3_27](https://doi.org/10.1007/978-1-4614-2425-3_27)
10. Spivey, Natalie, Saltzman, Rachel, Wieseman, Carol, Napolitano, Kevin, and Smith, Benjamin., "Passive Aeroelastic Tailored Wing Modal Test Using the Fixed Base Correction Method," Topics in Modal Analysis & Testing, Volume 8, Proceedings of the 38th IMAC, A Conference and Exposition on Structural Dynamics 2020, pp. 61-83, Houston, Texas, 2020. doi: [10.1007/978-3-030-47717-2_7](https://doi.org/10.1007/978-3-030-47717-2_7)



Chapter 2

Exploring the Capabilities of Open-Source Hard- and Software for Experimental Dynamics in Rotating Systems

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Abstract This paper presents a first utilization of the Open Acquisition System for IEPE Sensors (OASIS) for experimental dynamic measurements of an academic test structure with a topology similar to a wind turbine designed to have periodically changing mass and stiffness parameters. After deriving the analytical equations of a minimal model, the vibrations of the corresponding test rig are measured using two OASIS boards, one attached to the rotating part and one remaining stationary. Using open-source Python packages, the modes of the system in different configurations are computed and compared to the analytical model. Lastly, the frequency response functions (FRF) for various rotational speeds are retrieved and analyzed. The results show that a complete measurement and analysis process of a mechanical system is possible using mainly open-source hardware- and software. For this reason, the further development of open-source tools, as well as the development of new ones, is desirable.

Keywords Open-Source · Acquisition Hardware · Experimental Dynamics · Experimental Modal Analysis · Operational Vibrations

Introduction

Open-source software brings great advantages; besides being free of charge, it allows everyone to look into the utilized algorithms and adapt the functionality based on individual requirements. For structural dynamics, many noteworthy Python packages exist today, e.g., pyFRF, pyFBS, pyUFF, SDynPy, pyIDI, and many more. The same benefits apply to open hardware, allowing to build on other people's work and customizing for the task at hand. Combining the software side with the steadily increasing amount of open hardware allows the entire measurement and analysis to be performed solely using open tools.

One reason for modifiable data acquisition hardware for the authors was the option to wirelessly synchronize multiple systems because this allows measurements on partly rotating structures, i.e., where one part is rotating while another remains stationary. While commercial systems exist for this purpose, they are too expensive for purely academic investigations. For this reason, the authors developed the Open Acquisition System for IEPE Sensors (OASIS) [1] with a wireless synchronization option.

This paper tests the feasibility of performing a mechanical system analysis using OASIS and open-source software. An academic test structure, with a topology similar to a wind turbine, is used as a test setup. After giving an overview of the test rig, a theoretical model is derived using analytical mechanic principles. The model's behavior is then studied using its eigenvalues and eigenvectors, which depend on the blade angle as well as the blade rotation speed. After detailing the instrumentation used for the test rig, two experimental campaigns are evaluated: A classical experimental modal analysis for different blade angles and an operational analysis for different rotation speeds.

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Test Rig Overview

The test rig analyzed in this paper, shown in fig. 1, was designed to mimic the topology of a wind turbine and is based on an Ampair 600 wind turbine. It consists of three main components: the blade assembly, the nacelle, and the tower with base:

Tower with base Separated from the ground by a spring-based suspension, the tower carries the wind turbine itself. Due to the four springs, the test rig can easily translate up and down as well as tilt around axes perpendicular to the ground. In the following, only the tilting degree of freedom (dof) in the direction of the rotation axis of the wind turbine is considered for this part.

Nacelle The nacelle holds the shaft of the wind turbine, supported by ball bearings and driven by an *Maxon* DC-motor through a planetary transmission and a belt drive. At the end of the shaft, the blade assembly is attached using a bolt and an interlocking socket.

Blade assembly Two blades, comprising of one standard flat-bar steel (cross-section 40×3 mm), are clamped between the two hub discs, denoted as the hub in the following. At the end of each blade, an additional mass m_C is mounted that is designed to increase centrifugal forces.

Compared to most other wind turbines, the academic test structure only has two blades and, additionally, there is an additional mass m_C attached at the ends of the blades. This design decision was made to amplify the change of system parameters, e.g., mass and stiffness, with a change of the blade angle as well as the rotational speed. For instance, the rotational inertia with respect to the forward tilting of the test rig changes with the blade angle. While this does not apply to wind turbines with three blades in theory, as they can be considered as an isotropic disc (assuming rigid blades), it still applies to imperfect or damaged systems.

Due to the parameters change over time, this system can not be considered Linear Time Invariant (LTI). However, when the rotation speed is kept constant, the system can be regarded as Linear Time Periodic (LTP). Considering this for the modeling process of real systems is relevant because LTP systems can exhibit parametric resonances that are not present in LTI-approximations of the system [2]. However, such effects are not analyzed using the proper LTP system theory in this paper, e.g., by applying Floquet theory as was done in [3].

Here, three selected effects will be modeled using analytical mechanics and the concept of a minimal model. Such a model aims to capture the fundamental physical effects using as few degrees of freedom as possible [4]. The effects specifically modeled are:

Periodic rotational inertia Due to the two blades, the rotational inertia changes proportional to twice the rotational speed. Because the rotational inertia J is identical after a complete revolution or period T_P , assuming constant speed, it is said to be periodic: $J(t + T_P) = J(T_P)$.

Centrifugal stiffening Another effect arises from the rotation speed $\dot{\alpha}$. A mass m traveling on a circular trajectory with constant angular speed $\dot{\alpha}$ and distance to the center r experiences a centrifugal force $F_C = mr\dot{\alpha}^2$, pointing away from the center of rotation. This introduces a prestress in the blades, which results in an increased stiffness for transverse vibrations when $\dot{\alpha} > 0$, which is proportional to the square of $\dot{\alpha}$, as will be shown in the following.

Periodic stiffening and softening through gravity Finally, how gravity affects the system depends on the blade angle α . When the blades are perpendicular to the ground ($\alpha = 0^\circ$), the upper blade experiences a compressive force, while the lower blade experiences a tensile force. This softens or stiffens the blades respectively, analogous to the former effect.

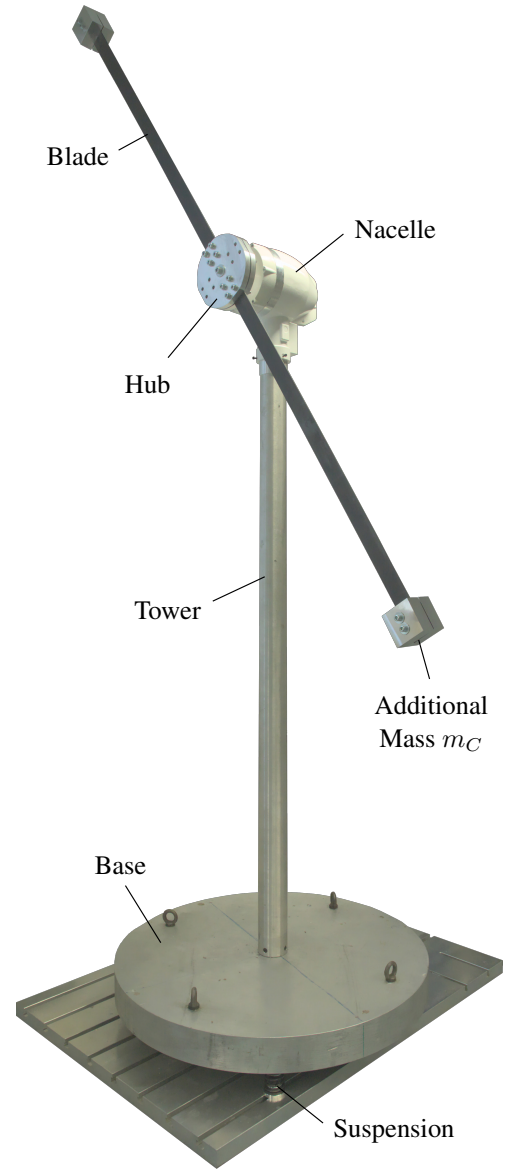
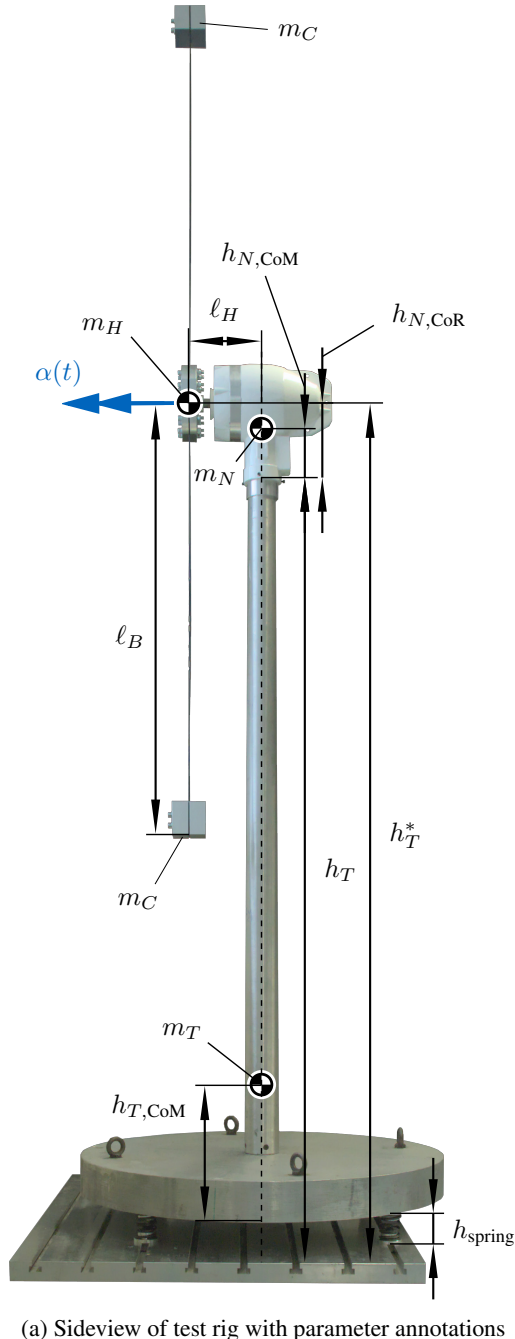


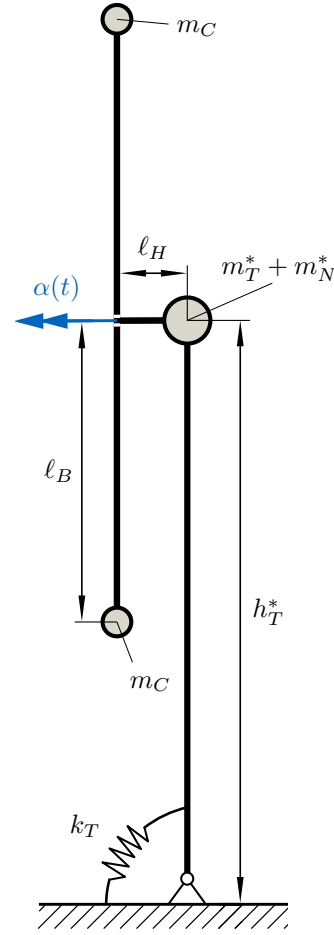
Fig. 1 Overview of wind turbine test rig

Since the minimal model is supposed to be as simple as possible, the geometry of the test rig is simplified, assuming the tower to be rigid, to an inverted pendulum with one rotational dof, as shown in fig. 2. For this, the mass of the nacelle m_N and the hub m_H are condensed to m_N^* , neglecting the hub length ℓ_H . The mass of the tower m_T is moved to the height of the wind turbine rotation axis $h_T + h_{N,CoR} = h_T^*$ and adjusted to be of equivalent rotational inertia J at the new position. All modified parameters are denoted by an asterisk superscript, e.g., m_T^* . The determined geometry parameters are summarized in the table in fig. 2c.

For simplicity, any damping in the system is neglected. The masses m_C , m_H , and m_N were determined using a scale; for the blades, generic steel parameters are assumed. Using an experimental modal analysis, the tilting stiffness k_T of the test rig without the hub assembly was identified. Those parameters are included in table 1.



(a) Sideview of test rig with parameter annotations



(b) Geometry of minimal model with parameters

Test rig parameter	Value
Height of suspension springs h_{spring}	0.07 m
Tower height (including springs) h_T	1.68 m
Nacelle height to center of rotation $h_{N,CoR}$	0.17 m
Height of wind turbine rotation axis h_T^*	1.85 m
Hub length ℓ_H	0.16 m
Blade length ℓ_B	1.00 m

(c) Wind turbine test rig geometry parameters

Fig. 2 Illustration of test rig geometry and simplification of test rig geometry for minimal model

Theoretical Model

To build a model of the test rig with the aforementioned effects, first, a model of an individual blade using an Euler-Bernoulli beam is derived that considers both the **centrifugal stiffening** as well as the **periodic stiffening and softening through gravity**, which is modeled by an additional, periodic pre-stress. Then, the tower is modeled as an inverted pendulum, to which two of the blade models are connected using primal assembly. Lastly, the **periodic rotational inertia** is added. For the model, the blade angle α and the rotational speed $\dot{\alpha}$ are considered a parameter and not a degree of freedom, i.e., all dynamics relating to a change of α are neglected.

Blade model The derivation starts with the differential equation for an Euler-Bernoulli beam, depicted in fig. 3a, including prestress through a normal force N , which can be found, for instance, in [5]:

$$\rho_B I_B \frac{\partial^2 \ddot{w}_B}{\partial x^2} - \frac{\partial^2}{\partial x^2} \left(E_B I_B \frac{\partial^2 w_B}{\partial x^2} \right) + \frac{\partial}{\partial x} \left(N \frac{\partial w_B}{\partial x} \right) = \bar{m}_B \ddot{w}_B \quad (1)$$

with the beam deflection w_B , density ρ_B , area moment of inertia I_B , Young's modulus E_B and mass per unit length $\bar{m}_B$. The normal force N can now be used to include centrifugal stiffening and periodic stiffening and softening through gravity.

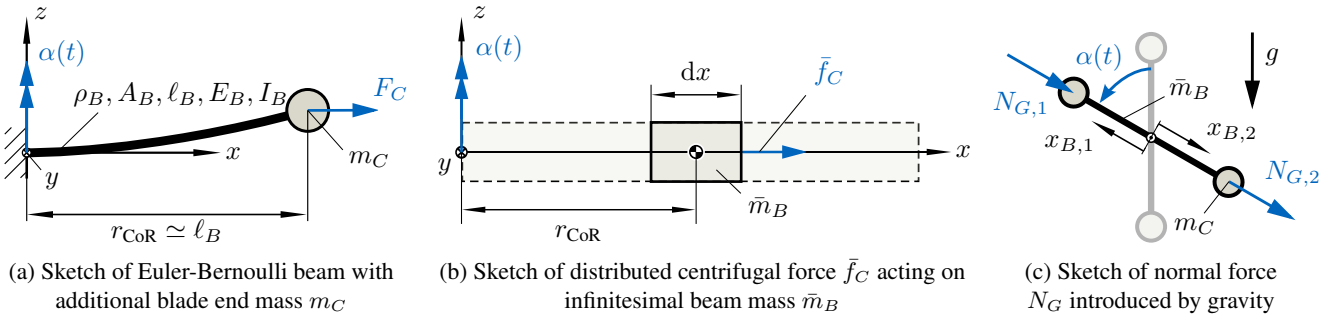


Fig. 3 Individual modeling steps used for the beam model representing the test rig blades

Centrifugal forces act, on the one hand, on the additional end mass m_C as shown in fig. 3a, and, on the other hand, on the distributed beam mass $\bar{m}_B$ as shown in fig. 3b. In general, the centrifugal force is given by $F_C = m_C r_{CoR} \dot{\alpha}^2$ with r_{CoR} being the distance to the axis of rotation and $\dot{\alpha}$ the angular velocity. For the end mass m_C , assuming only small deflections w_B and substituting $\dot{\alpha} = 2\pi f_\alpha$, the centrifugal force is then given by $F_C(f_\alpha) = 4\pi^2 \ell_B m_C f_\alpha^2$. For the latter, the distributed centrifugal force $\bar{f}_C$ per distributed mass $\bar{m}_B$ is given as $\bar{f}_C(x) = \bar{m}_B x \dot{\alpha}^2$, which can be integrated over x to the force $\bar{F}_C(x) = \frac{1}{2} \bar{m}_B x^2 \dot{\alpha}^2$, and with the substitution $\dot{\alpha} = 2\pi f_\alpha$ one finds $\bar{F}_C(x) = 2\pi^2 \bar{m}_B x^2 f_\alpha^2$.

The other force acting on the beams is induced by gravity, see fig. 3c. Since the shear force created by gravity acts in the transverse direction orthogonal to the beam deflection direction w_B , it is neglected. Projecting the forces created by gravity onto the beam normal forces yields $N_{G,1,2} = \mp \left(\bar{m}_B \ell_B g \left(1 - \frac{x}{\ell_B} \right) + m_C g \right) \cos \alpha$, with different signs for the two blades. For example, in the upward position with $\alpha = 0$, the first beam is compressed by gravity, and the second beam is elongated.

To summarize, the normal forces $N_{1,2}$ applied to the beams externally is given by:

$$N_{1,2}(x) = \underbrace{4\pi^2 \ell_B m_C f_\alpha^2}_{\text{Centrifugal force due to } m_C} + \underbrace{2\pi^2 \bar{m}_B x^2 f_\alpha^2}_{\text{Centrifugal force due to } \bar{m}_B} \mp \underbrace{\left(\bar{m}_B \ell_B g \left(1 - \frac{x}{\ell_B} \right) + m_C g \right) \cos \alpha}_{\text{Influence of gravity}} \quad (2)$$

In order to be able to get linearized masses and stiffnesses of the beams, a Rayleigh-Ritz ansatz $w_B(x, t) \simeq \mathbf{F}(x) \cdot \mathbf{q}(t)$, with $\mathbf{F}(x)$ being the matrix of shape functions, is made. Using only one shape function $\mathbf{F} = \left[\frac{x^2}{\ell_B^2} \right]$ yields the following:

$$K_B(\dot{\alpha}) = E_B \frac{b_B h_B^3}{3 \ell_B^3} + \left(\frac{16}{3} \pi^2 m_C + \frac{8}{5} \pi^2 \rho_B b_B h_B \ell_B \right) f_\alpha^2 \mp \left(\frac{4}{3 \ell_B} m_C g + \left(\frac{4}{3 \ell_B} - 1 \right) \bar{m}_B \ell_B g \right) \cos(\alpha) \quad (3)$$

$$M_B = \rho_B b_B h_B \frac{\ell_B}{5} + m_C \quad (4)$$

with the blade thickness b_B and blade height h_B .

Tower model The tower is modeled using an inverted pendulum as depicted in fig. 4a. Writing a rotational equilibrium around the center of rotation yields the equation of motion with respect to ψ , which can be transformed into the translational dof w_T when small angles are assumed.

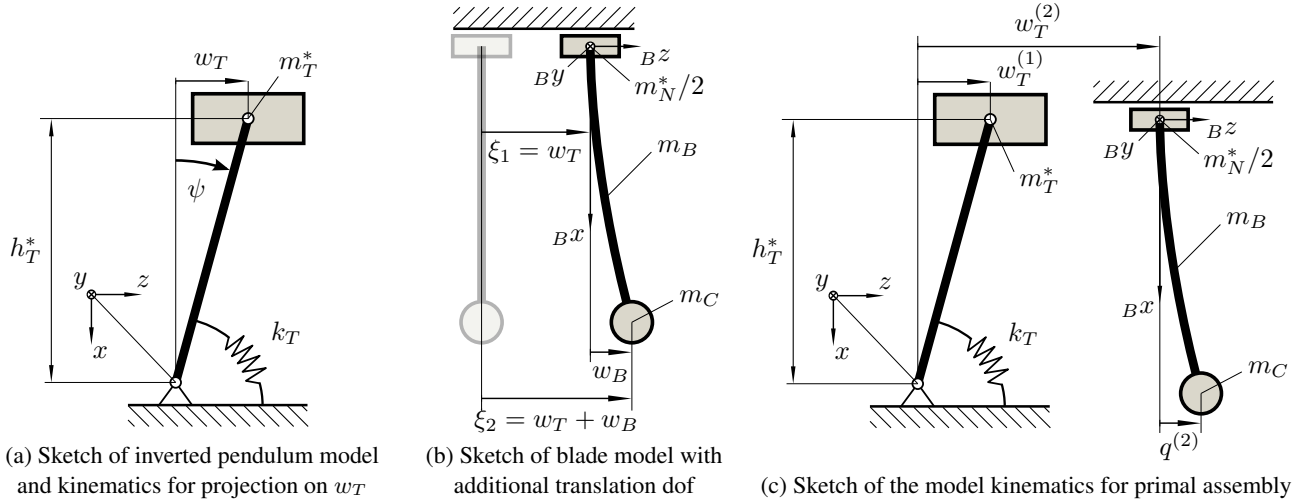


Fig. 4 Individual modeling steps for the tower and extended blade model as well as assembly kinematics

Assembly In order to be able to assemble the beam models to the tower model, the beam model has to be extended with a translational dof as depicted in fig. 4b. The mass of the added dof w_T of the beam model is half of the nacelle mass, which was not considered previously. Using Lagrange's equations, the mass matrix of this extended beam model can be derived:

$$\begin{bmatrix} \frac{m_N^*}{2} + m_C + m_B & \frac{1}{3}m_B \\ \frac{1}{3}m_B & M_B \end{bmatrix} \begin{bmatrix} \ddot{w}_T \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_B(\dot{\alpha}) \end{bmatrix} \begin{bmatrix} w_T \\ q \end{bmatrix} = \mathbf{0} \quad (5)$$

where $m_B = \rho_B b_B h_B \ell_B$ is the blade mass. No coupling exists between the blade deformation q and the horizontal tower displacement w_T via a stiffness. This is due to the chosen shape function enforcing the boundary condition $w_B(x=0) = 0$, i.e., a displacement q due to a deformation of the beam does not create a displacement of w_T .

Lastly, two extended blade models are assembled to the tower model using primal assembly, see fig. 4c. For this, the horizontal tower displacement $w_T^{(1)}$ has to match the blade displacement $w_T^{(2)}$ of the first blade, requiring $w_T^{(1)} - w_T^{(2)} = 0$. This condition is automatically satisfied by choosing unique global degrees of freedom, here $\mathbf{u}_{\text{global}} = [w_T \ q_1]^T$. Requiring the same for the second blade (not depicted) yields the assembled system matrices for the global dofs given by $\mathbf{u}_{\text{global}} = [w_T \ q_1 \ q_2]^T$:

$$\mathbf{M}_{\text{global}} = \begin{bmatrix} m_T^* + m_N^* + 2m_B + 2m_C & \frac{1}{3}m_B & \frac{1}{3}m_B \\ \frac{1}{3}m_B & M_B & 0 \\ \frac{1}{3}m_B & 0 & M_B \end{bmatrix} \quad \mathbf{K}_{\text{global}} = \begin{bmatrix} \frac{k_T}{h_T^{*2}} & 0 & 0 \\ 0 & K_B(\dot{\alpha}) & 0 \\ 0 & 0 & K_B(\dot{\alpha}) \end{bmatrix} \quad (6)$$

Periodic rotational inertia The effect of the changing rotational inertia can be added by first deriving the rotational inertia for tilting of the tower with the blades attached depending on the blade angle α . Only considering this change of inertia for the end masses m_C , the total rotational inertia is found as $J_{\text{total}}(\alpha) = (m_T^* + m_N^*) h_T^{*2} + 2m_C (\ell_H^2 + h_T^{*2} + \ell_B^2 \cos^2(\alpha))$.

Projecting this equation onto the translational dof w_T (by dividing with h_T^{*2}) and comparing the result to the previous diagonal entry of $\mathbf{M}_{\text{global}}$ shows that the mass m_C is now multiplied by a changing factor, yielding a periodic contribution:

$$J_{\text{periodic}}(\alpha) = 2m_C (\ell_H^2 + h_T^{*2} + \ell_B^2 \cos^2(\alpha)) \quad \leftrightarrow \quad m_C(\alpha) = m_C \cdot \frac{(\ell_H^2 + h_T^{*2} + \ell_B^2 \cos^2(\alpha))}{h_T^{*2}} \quad (7)$$

Final model Substituting this in the previously found global mass and stiffness matrix yields the final system matrices:

$$\mathbf{M}_{\text{global}} = \begin{bmatrix} m_T^* + m_N^* + 2m_B + 2m_C \cdot \frac{(\ell_H^2 + h_T^{*2} + \ell_B^2 \cos^2(\alpha))}{h_T^{*2}} & \frac{1}{3}m_B & \frac{1}{3}m_B \\ \frac{1}{3}m_B & M_B & 0 \\ \frac{1}{3}m_B & 0 & M_B \end{bmatrix} \quad \mathbf{K}_{\text{global}} = \begin{bmatrix} \frac{k_T}{h_T^{*2}} & 0 & 0 \\ 0 & K_B(\dot{\alpha}) & 0 \\ 0 & 0 & K_B(\dot{\alpha}) \end{bmatrix}$$

with $K_B(\dot{\alpha}) = E_B \frac{b_B h_B^3}{3\ell_B^3} + \left(\frac{16}{3}\pi^2 m_C + \frac{8}{5}\pi^2 m_B \right) f_\alpha^2 \mp \left(\frac{4}{3\ell_B} m_C g + \left(\frac{4}{3\ell_B} - 1 \right) \bar{m}_B \ell_B g \right) \cos(\alpha)$

$$M_B = \frac{1}{5}m_B + m_C \quad (8)$$

All parameters of this model are summarized in table 1. A sketch of the model described by eq. (8) is depicted in fig. 5. Note that this is not a quantitative model, i.e., highly accurate, but only a qualitative model that is supposed to show the dominating mechanical effects of the test rig while also giving insight as to what parameter influences which effect.

Model parameter	Value
Model tower height h_T^*	1.85 m
Hub length ℓ_H	0.16 m
Blade length ℓ_B	1.00 m
Blade Young's modulus E_B	$210 \times 10^9 \frac{\text{N}}{\text{m}^2}$
Blade density ρ_B	$7850 \frac{\text{kg}}{\text{m}^3}$
Blade height h_B	$3 \times 10^{-3} \text{ m}$
Blade thickness b_B	$40 \times 10^{-3} \text{ m}$
Blade area moment of inertia I_B	$9 \times 10^{-11} \text{ m}^4$
Model tower weight m_T^*	1.77 kg
Model nacelle weight m_N^*	8.15 kg
Blade mass per unit length $\bar{m}_B$	$0.942 \frac{\text{kg}}{\text{m}}$
Blade mass m_B	0.942 kg
Additional blade end mass m_C	1 kg
Torsional stiffness k_T	$8.4 \frac{\text{kN m}}{\text{rad}}$

Table 1 Summary of the minimal model parameters

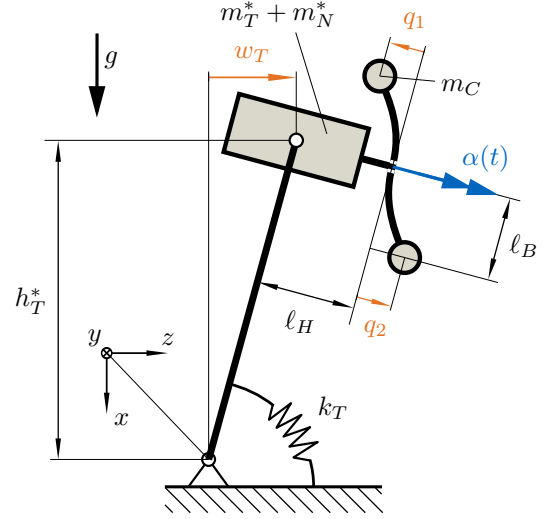


Fig. 5 Coupled, periodic wind turbine minimal model with three degrees of freedom, $\mathbf{u}_{\text{global}} = [w_T \ q_1 \ q_2]^T$

Theoretical Results

To analyze the system behavior, the eigenfrequencies and eigenvectors of the system are calculated using *SciPy*, and the mode shapes are visualized using *Manim* (Mathematical Animation Engine) [6]. The blade angle α and the rotational frequency f_α are substituted by a constant parameter for this. The eigenfrequencies and eigenvectors are then calculated for different values.

Starting with the stationary case, i.e., $\dot{\alpha} = 0$, the eigenfrequencies and the components of each eigenvector depending on the blade angle α are plotted in fig. 6. In the leftmost plot, the eigenfrequencies are depicted, where the first two eigenfrequencies between 1 and 1.5 Hz belong to the blades, and the third one is mainly a tilting of the tower. As can be seen, all three eigenfrequencies change with the blade angle. For the blades, this change is caused by gravity that compresses or stretches the blades depending on the angle as was derived using fig. 3c. For the tower, the change of eigenfrequency is caused by the change of rotational inertia. Note that this change is proportional to $\cos^2 \alpha$, while for the blades, it is proportional to $\cos \alpha$.

One interesting phenomenon is observable with the blade eigenfrequencies: While one would expect the eigenfrequencies to change continuously and proportional to $\cos \alpha$, there appears to be a kink at $\alpha = 90^\circ$. The first eigenfrequency f_1 is defined as the one with the lowest frequency, but this does not mean that it is the same physical eigenfrequency, i.e., that the corresponding mode shape is the same. This can be seen in the eigenvector components in fig. 6.

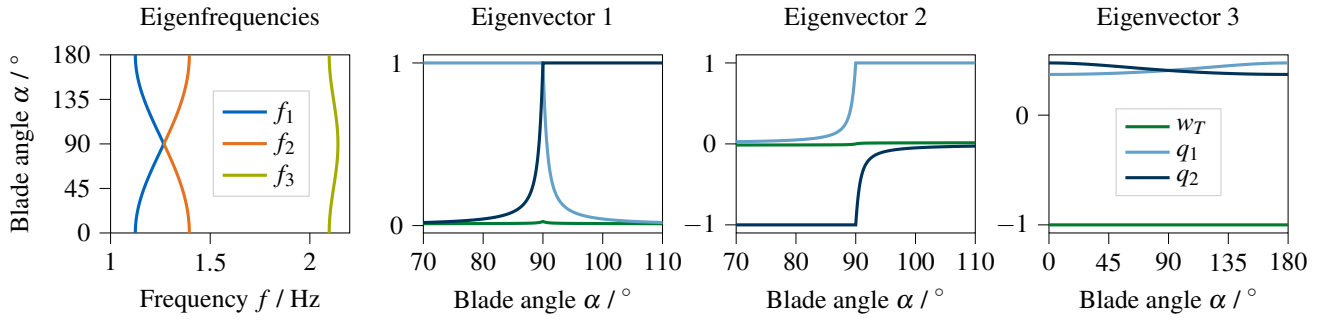


Fig. 6 Evolution of the individual, normalized eigenvectors component over the blade angle α

At $\alpha = 90^\circ$, where both blade eigenfrequencies are identical, it can be seen that the eigenvectors change, i.e., while eigenvector one is dominated by the q_1 component for $\alpha < 90^\circ$, it is dominated by the q_2 component for $\alpha > 90^\circ$. The same can be seen for eigenvector 2, where it changes from q_2 to q_1 . This effect is known as mode veering and can generally be observed with systems that have a varying parameter, here the beam stiffness $K_B(\dot{\alpha})$, and are coupled by arbitrarily small amount [7], here through the mass matrix. The rapid change of the eigenvectors in the area where the uncoupled eigenfrequencies cross, as seen here, is typical for mode veering [7].

Visualizing the mode shapes can also provide a physical interpretation of the system behavior. Figure 7 shows the first mode shape for different blade angles, fig. 8 the second mode shape. All mode shape visualizations are created by multiplying the respective normalized eigenvector with $\cos(2\pi t)$ for $t \in [0, 1 \text{ s}]$, scaling the amplitudes by a common factor and applying these values as displacements to the model geometry that is represented to scale. Every figure visualizing mode shapes contains two snapshots of the continuous animation, one with blue and one with orange dots that have a phase of 180° .

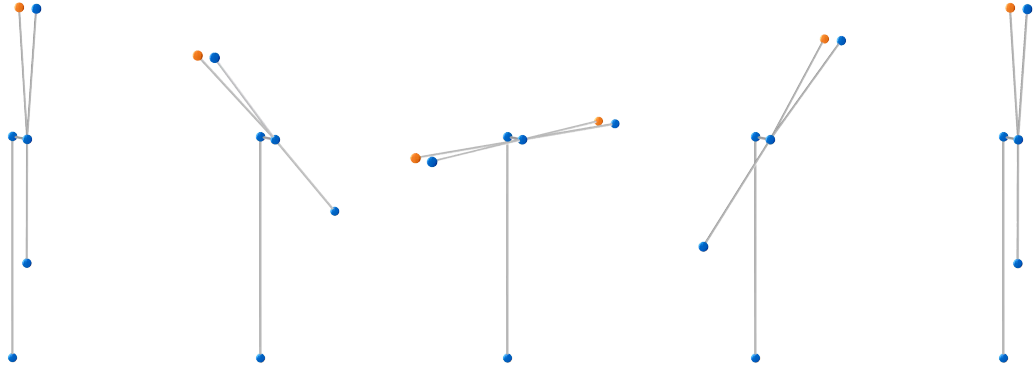


Fig. 7 Mode shape 1 of the minimal model for varying blade angle $\alpha = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ\}$ and $\dot{\alpha} = 0$

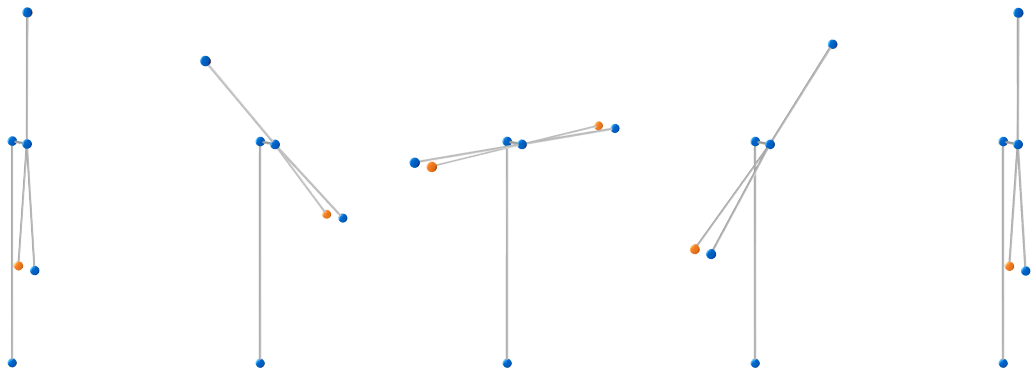


Fig. 8 Mode shape 2 of the minimal model for varying blade angle $\alpha = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ\}$ and $\dot{\alpha} = 0$

The first mode shape, which has the lowest eigenfrequency of the system, always represents the deflection of the upper blade. Since both beams are identical, there is no physical distinction between them. This means that the first mode describes the vibration of beam one for $-90^\circ < \alpha < 90^\circ$ and for $90^\circ < \alpha < 270^\circ$ it describes the vibration of beam two. Both beams vibrate in phase for $\alpha = 90^\circ$. For the second mode shape, the second beam is vibrating for $-90^\circ < \alpha < 90^\circ$ since it has the higher eigenfrequency. At $\alpha = 90^\circ$, both blades deflect in anti-phase, because the eigenvectors have to be linearly independent.

The third mode shape, see fig. 9, describes the tilting of the wind turbine tower coupled with the blade deformation in anti-phase. Because of the periodic stiffening and softening through gravity, the eigenfrequency of the lower beam is always higher, resulting in a higher amplitude in this mode shape since the corresponding eigenfrequency is closer to the third eigenfrequency.

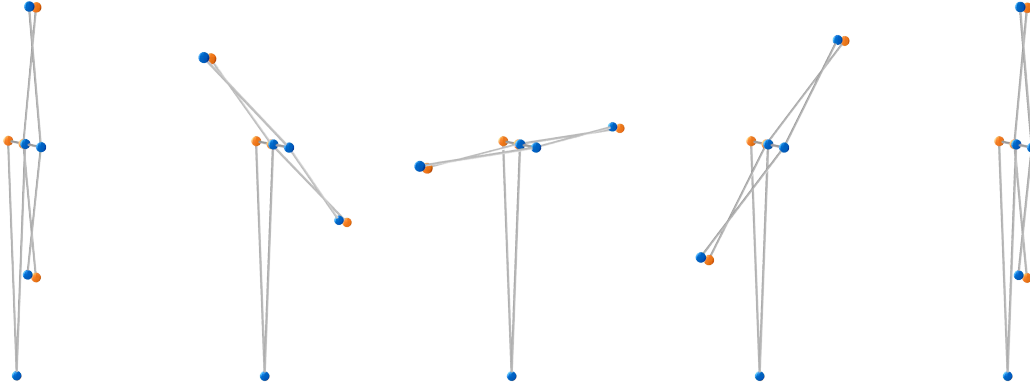


Fig. 9 Mode shape 3 of the minimal model for varying blade angle $\alpha = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ\}$ and $\dot{\alpha} = 0$

Lastly, the system's eigenfrequencies are calculated for different angular velocities $\dot{\alpha}$. The way the minimal model is set up allows to set the blade velocity $\dot{\alpha}$ independently from the blade angle α . An intensity plot is used to show the continuous change of the eigenfrequencies, similar to the phosphor display of analog oscilloscopes. For this, the values of the eigenfrequencies for a large number of angles within one full rotation are plotted over each other with partially transparent lines. This means the more saturated the color is at a specific point, the more often an eigenfrequency is located there, see fig. 10.

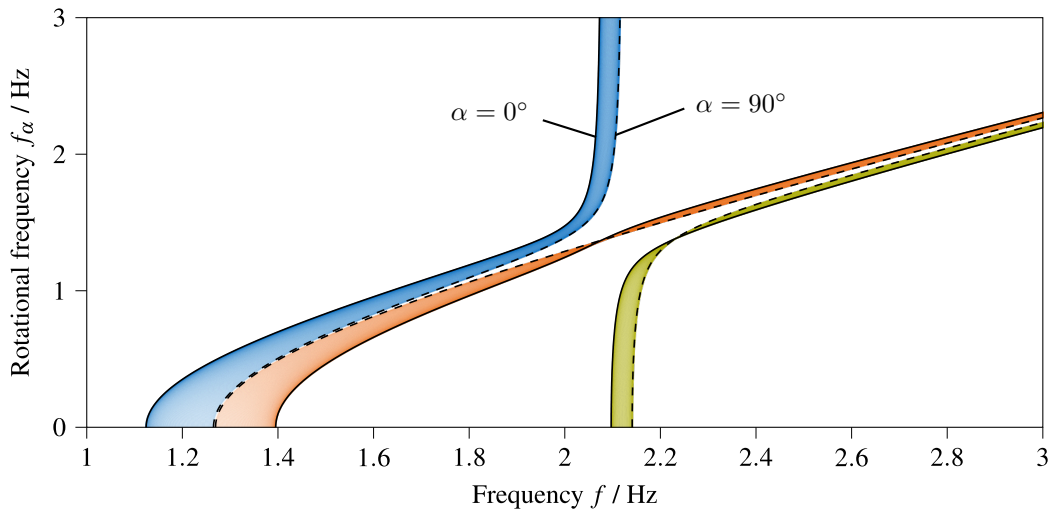


Fig. 10 Intensity plot of the fully assembled model's eigenfrequencies within one full rotation of the blades, for varying rotational blade frequency f_α ; the color saturation denotes the rate of the eigenfrequency appearing at the respective point

Also here, mode veering can be observed at the point where the tower eigenfrequency crosses the blade eigenfrequencies. The centrifugal stiffening can also be observed, where the centrifugal forces tension the blades, increasing the stiffness for the transverse vibrations. This leads to an increase of the blade eigenfrequencies when the rotational frequency f_α increases,

as can be seen in fig. 10. While this gives an idea of where the linear resonances of the system might be, this does not fully capture the nature of the periodic system that is modeled here. This would require proper modeling as an LTP system [2].

Testrig Instrumentation

Figure 11 shows the wind turbine test rig, including all used instrumentation. Different colors are used for system excitation, sensors, and data acquisition. Dashed lines show the signal flow from sensors to data acquisition. In the following the used components are described in detail. The raw measurement data can also be downloaded from [8].

Excitation The test rig is excited using a shaker attached to the tower with a stinger. Using a waveform generator, random noise with an amplitude of 12 V peak-to-peak and 30 MHz bandwidth is created. This signal is fed into a band-pass filter that removes the DC part and most of the higher frequencies using a cut-off frequency of 30 Hz. A gain of 40 dB is also applied to regain some signal level. This excitation signal is gradually turned on after the measurement begins by controlling the power amplifier's gain, kept at a constant level for approximately 200 s, and then gradually turned off.

The rotational speed of the test rig can be controlled using the DC-motor power supply. There is no control for the actual rotation speed, only a constant voltage can be set. Due to the relatively high rotational inertia, the rotational speed is fairly constant after some settling time.

Sensors Four uniaxial acceleration sensors are glued to the blades, symmetrically to the rotation axis of the wind turbine, such that the distance between the outer sensor and the sensor in the middle of the blade is identical to the distance between the latter and the rotation axis. For $\alpha = 0^\circ$ the test rig is in the configuration as shown in fig. 11, defined by the acquisition system *OASIS One* being oriented with the BNC connectors pointing downwards. The highest sensor in this orientation is denoted as *blade 1 tip*, followed by *blade 1 middle* and *blade 2 middle*, the lowest sensor is denoted *blade 2 tip*.

An impedance sensor between the shaker stinger and the tower measures the force applied by the shaker. A little bit higher up, a triaxial accelerometer measures the acceleration of the tower in the excitation direction. Additionally, the acceleration lateral to the excitation is measured but not further used.

Lastly, the blades' rotation speed, receptively the blades' passing frequency, is sensed using a light barrier. Its circuitry is adapted to be used with a single BNC connection. The supply voltage is provided by the IEPE power supply of the *OASIS* board. When a blade passes, the LED within the light barrier illuminates, drawing more current than allowed by the limiting circuit (adjusted to 2 mA from the regular 5 mA). This results in the supply voltage dropping in order to not exceed the set current limit. While the acquisition system can not measure the constant voltage due to the high-pass filter, the change of voltage can be measured. This manifests in voltage spikes with an amplitude of roughly 3 V that decay moderately fast. These spikes are negative as the blade enters the sensor area and positive as it leaves. Due to the system configuration there are two impulses (four spikes) per complete revolution. A counter, denoted as the blade passing counter, is used to evaluate the rotational frequency during the measurements.

Data acquisition The sensor and excitation signals are digitized using *OASIS*; specifically, two boards are used. *OASIS One* is attached to the wind turbine hub using a 3D-printed adapter and rotates with the hub. The second system, *OASIS Two*, is stationary. Measurements on this partly rotating system are possible due to the self-contained, Wireless Sample Synchronization (WSS) feature of *OASIS*, details can be found in [1]. In short, one board, the WSS source, sends a synchronization pulse when the measurement is started. All other boards configured as a WSS sink will start sampling when they receive this pulse. As was shown in [1], the delay between the source and sink measurements is well below 100 μ s. The samples are transferred to a computer during the measurement using WiFi and UDP, processed, and then saved as .mat files.

For all measurements a sample rate of 10 kHz and a measurement duration of 250 s was used. In each test rig configuration, five measurements were taken. The channel assignment is summarized in table 2.

Table 2 Summary of channel assignment for the two utilized boards, *OASIS One* and *OASIS Two*

	OASIS One	OASIS Two
Channel 1	Acceleration sensor blade 1 tip	Impedance sensor (force)
Channel 2	Acceleration sensor blade 1 middle	Light barrier signal
Channel 3	Acceleration sensor blade 2 middle	Acceleration sensor in excitation direction
Channel 4	Acceleration sensor blade 2 tip	Acceleration sensor in direction lateral to excitation

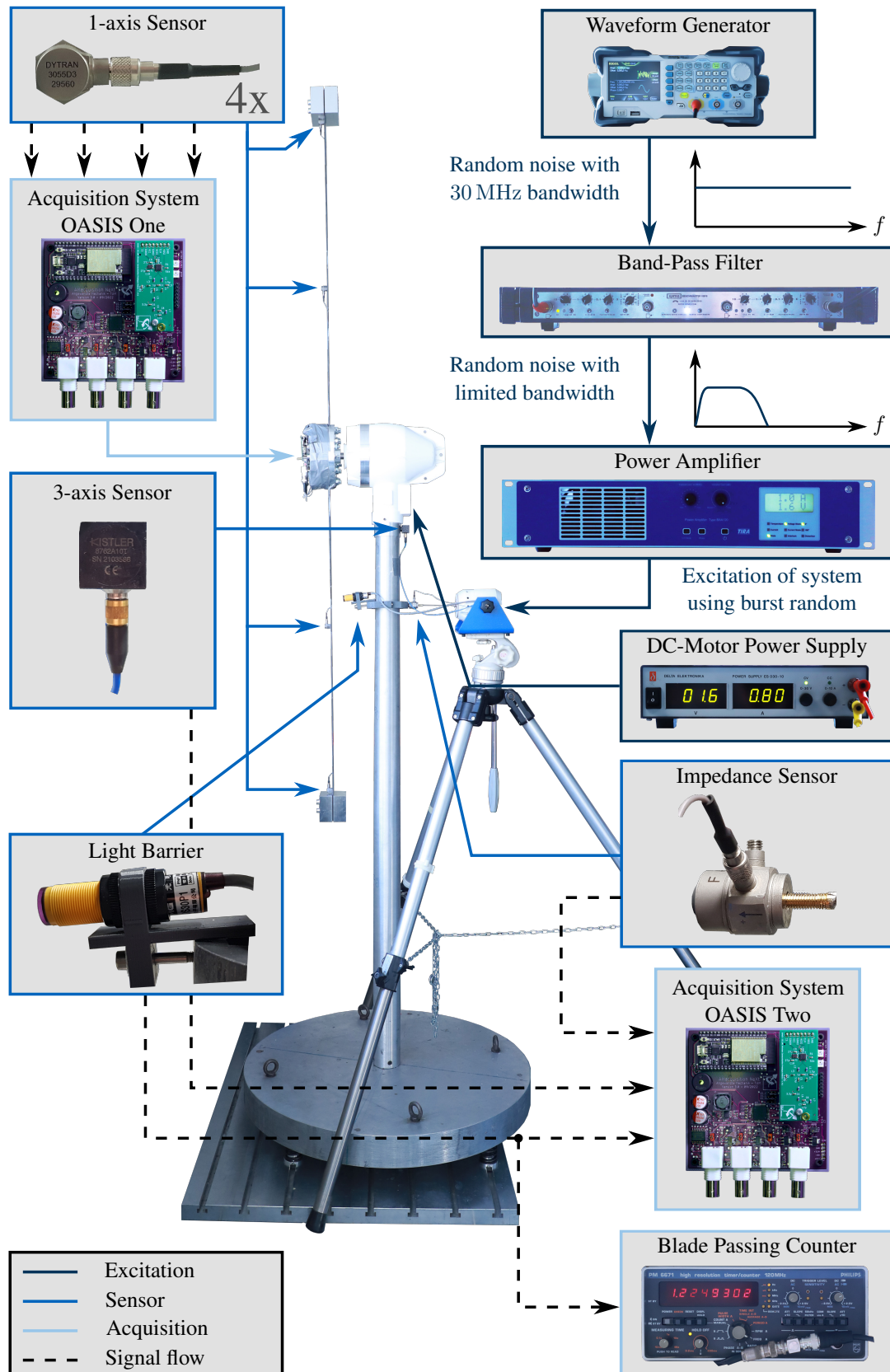


Fig. 11 Illustration of the used test rig instrumentation and signal flow from sensors to acquisition systems

Experimental Modal Analysis

For the stationary measurements used here, the blade angle α was varied between 0 and 180° with 30° increments. From the raw time signals, available at [8], the frequency response functions (FRF) from the shaker excitation to the tower degree of freedom as well as the four blade dofs were calculated with *pyFRF* [9] using five averages and the H_1 estimator. The FRFs of the tower dof for various blade angles α are shown in fig. 12.

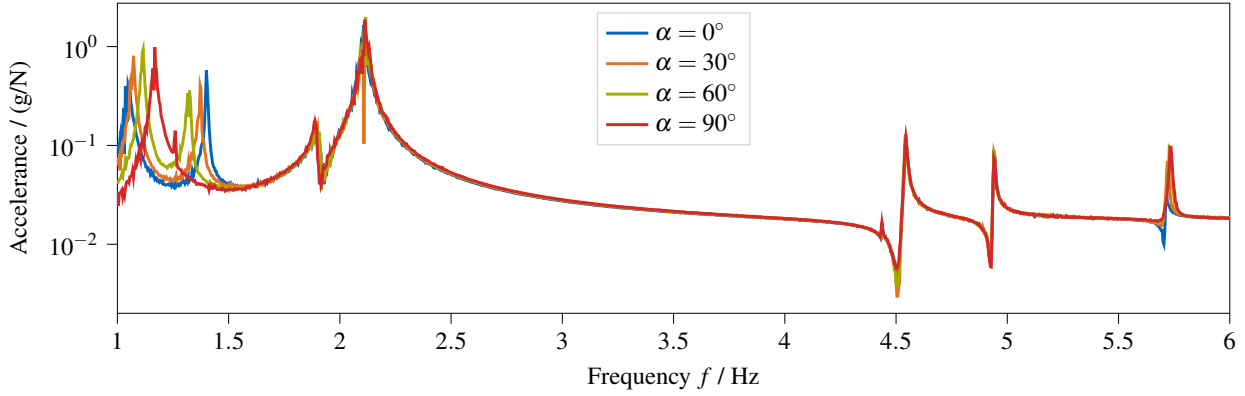


Fig. 12 Plot of the FRF corresponding to the tower degree of freedom for different blade angles α

The greatest variation with the blade angle α can be seen for the two eigenfrequencies around 1 Hz, which belong to the two blades. Slight changes can be seen as well with the two eigenfrequencies around 2 Hz and the eigenfrequency around 5.7 Hz.

For further analysis, the eigenfrequencies f_i , damping ratios ϑ_i and mode shapes are identified using the *pyFBS* [10] implementation of the least-squares complex frequency-domain estimator [11]. Table 3 summarizes the estimated parameters for the different blade angles. To visualize the change of eigenfrequencies and damping ratios, the cells in table 3 are colored green if the parameter increased compared to the measurement at $\alpha = 0$ and are colored red for decreasing values.

The most apparent change of eigenfrequency can be seen for modes one and two that behave as predicted by the minimal model, i.e., f_1 increasing as the compression of blade 1 due to gravity is reduced and f_2 decreasing. For the tilting tower mode shape, presumably mode 3 or 4, no significant trend is visible. The higher frequencies are not covered by the minimal model. Mode 5 and 6 appear to be invariant with respect to α , while mode 7 shows slight variations with the blade angle α .

Mode		$\alpha = 0^\circ$	$\alpha = 30^\circ$	$\alpha = 60^\circ$	$\alpha = 90^\circ$	$\alpha = 120^\circ$	$\alpha = 150^\circ$	$\alpha = 180^\circ$
Mode 1	f_1	1.049 Hz	1.070 Hz	1.112 Hz	1.165 Hz	1.139 Hz	1.092 Hz	1.059 Hz
	ϑ_1	0.51 %	0.37 %	0.34 %	0.25 %	0.27 %	0.26 %	0.43 %
Mode 2	f_2	1.402 Hz	1.371 Hz	1.321 Hz	1.259 Hz	1.290 Hz	1.339 Hz	1.390 Hz
	ϑ_2	0.23 %	0.22 %	0.25 %	0.24 %	0.29 %	0.31 %	0.27 %
Mode 3	f_3	1.904 Hz	1.902 Hz	1.904 Hz	1.891 Hz	1.894 Hz	1.888 Hz	1.895 Hz
	ϑ_3	0.75 %	0.65 %	0.56 %	0.59 %	0.52 %	0.75 %	0.71 %
Mode 4	f_4	2.108 Hz	2.111 Hz	2.112 Hz	2.112 Hz	2.112 Hz	2.105 Hz	2.106 Hz
	ϑ_4	0.40 %	0.49 %	0.45 %	0.77 %	0.41 %	0.33 %	0.44 %
Mode 5	f_5	4.544 Hz	4.543 Hz	4.541 Hz	4.543 Hz	4.543 Hz	4.544 Hz	4.542 Hz
	ϑ_5	0.11 %	0.10 %	0.11 %	0.12 %	0.11 %	0.11 %	0.12 %
Mode 6	f_6	4.941 Hz	4.940 Hz	4.940 Hz	4.939 Hz	4.940 Hz	4.941 Hz	4.941 Hz
	ϑ_6	0.06 %	0.06 %	0.06 %	0.07 %	0.05 %	0.06 %	0.06 %
Mode 7	f_7	5.712 Hz	5.720 Hz	5.729 Hz	5.734 Hz	5.731 Hz	5.721 Hz	5.712 Hz
	ϑ_7	0.06 %	0.07 %	0.08 %	0.08 %	0.08 %	0.09 %	0.10 %

Table 3 Summary of identified eigenfrequencies f_i and damping ratios ϑ_i for the wind turbine test rig at standstill; coloring shows trend of parameters compared to measurement at $\alpha = 0^\circ$ (green = increasing, red = decreasing)

For the experimentally identified mode shapes, *Manim* [6] was also used to create visualizations. The first two mode shapes are displayed in fig. 13 and fig. 14. As with the minimal model, mode shape 1 describes the deflection of the upper blade, and mode shape 2 the deflection of the lower blade. At $\alpha = 90^\circ$, both blades vibrate in phase for mode 1 and in anti-phase for mode 2. Except for some slight deflections of the opposing blade, the experimental mode shapes match the model.

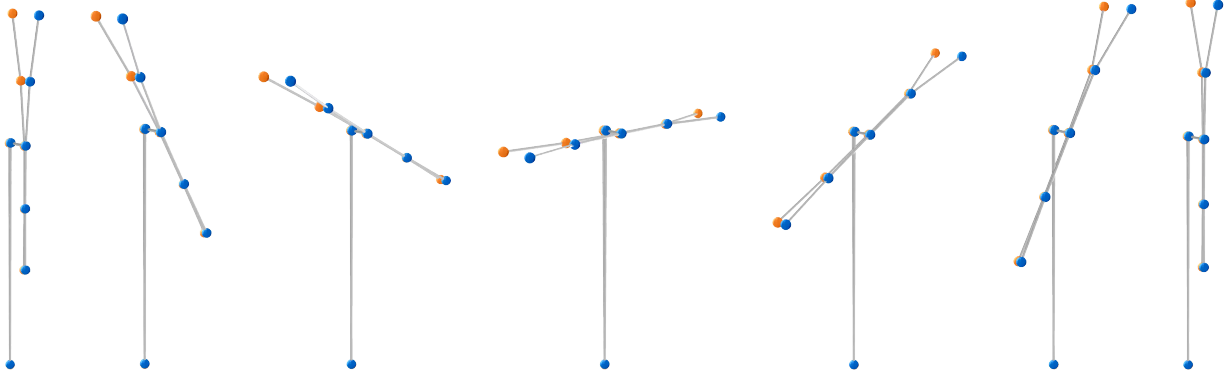


Fig. 13 Test rig mode shape 1 - Upper blade deformation - for $\alpha = \{0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ\}$

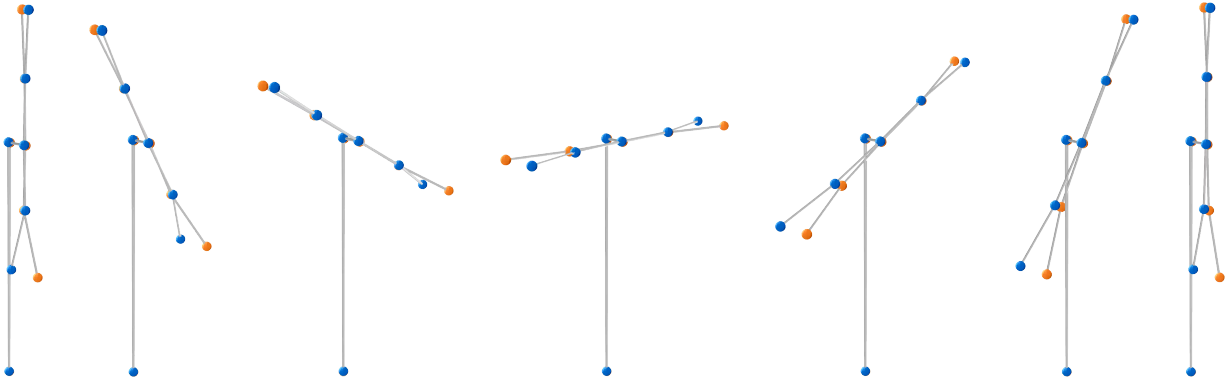


Fig. 14 Test rig mode shape 2 - Lower blade deformation - for $\alpha = \{0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ\}$

For the third mode shape of the model, the tilting of the tower, there exist two candidates from the experiments, mode shape 3 (fig. 15) and mode shape 4 (fig. 16). With the utilized dofs, there is no distinction possible. Calculating the values of the Auto Modal Assurance Criterion (Auto-MAC) also reveals a very high dependence ($\approx 96\%$) of the modes 3 and 4. Besides that, both mode shapes also show the same vibration behavior as the third mode shape of the model, i.e., the tower dof exhibits displacements in one direction, while all blade dofs move in the opposite direction.

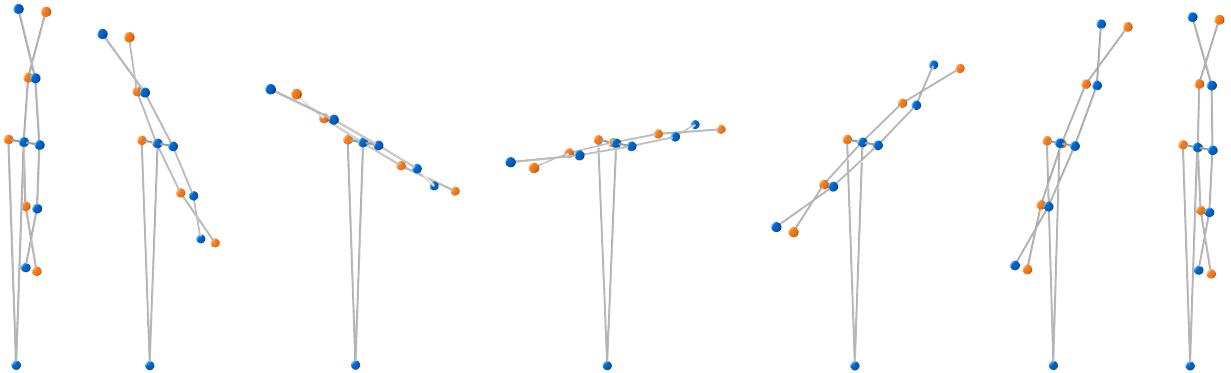


Fig. 15 Test rig mode shape 3 - Tilting tower multiplicity 1 - for $\alpha = \{0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ\}$

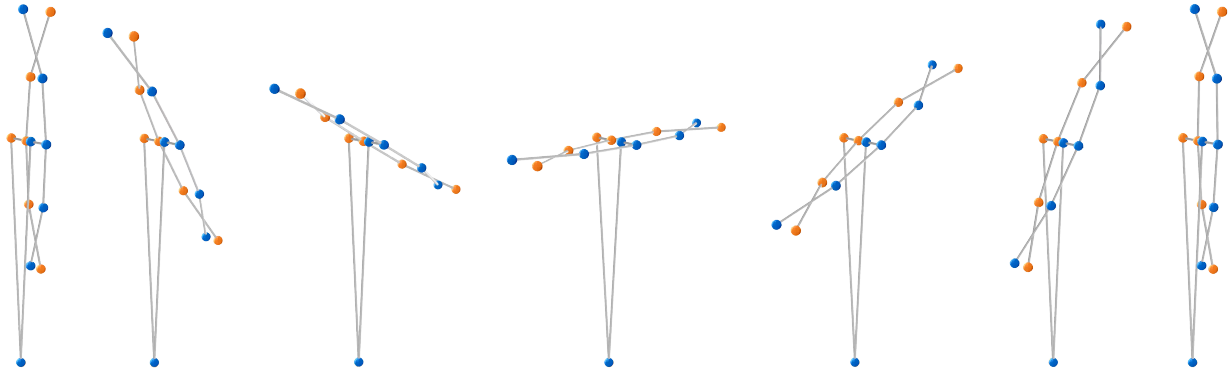


Fig. 16 Test rig mode shape 4 - Tilting tower multiplicity 2 - for $\alpha = \{0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ\}$

The fifth and sixth experimental mode shapes behave very similar to the previously described ones and also show a moderately high Auto-MAC value ($\approx 55\%$) and are therefore not shown here. Lastly, the seventh mode shows a second-order bending mode of the blades, see fig. 17. Since this mode shape is focused on the blades, the frequency dependence can probably also be explained by the stiffening/softening effect due to gravity.

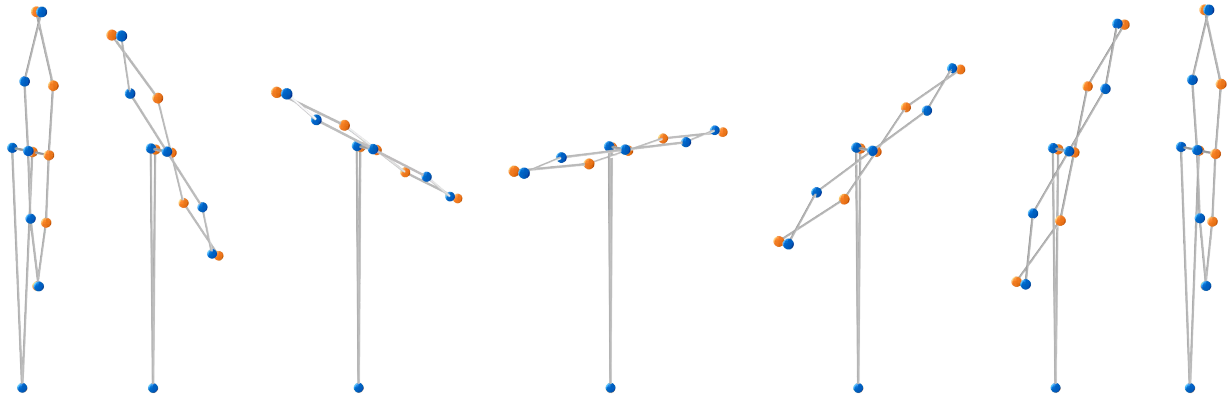


Fig. 17 Test rig mode shape 7 - Higher Order Blade Deformation - for $\alpha = \{0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ\}$

Finally, the values of the modeled and experimentally identified eigenfrequencies are compared using fig. 18. The eigenfrequencies of the blades (blue and orange) correspond pretty well and show the same qualitative change with the blade angle α . At the point with $\alpha = 90^\circ$, where the eigenfrequencies should be identical, there is a difference of about 0.1 Hz. Since mode veering occurs in a region around the critical points, that is in the order of the off-diagonal terms that couple the individual degrees of freedom [7], this implies that the model might be missing some coupling terms. Also, neither of the two experimentally identified tower tilting eigenfrequencies show a dependence on the blade angle α as suggested by the theoretical model. The second identified tower tilting eigenfrequency is most likely a sideways motion of the tower, not covered by the model.

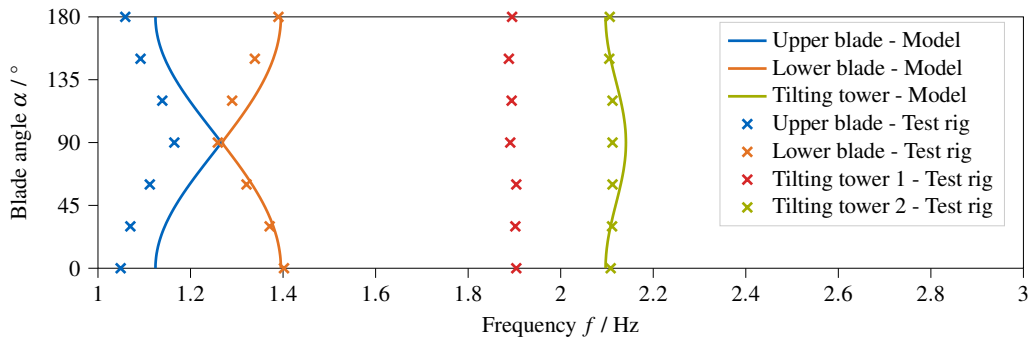


Fig. 18 Plot of the modeled and the experimentally identified eigenfrequencies for varying blade angle α with $\dot{\alpha} = 0$

Operational Measurements

Next, the dynamic measurements with $\dot{\alpha} \neq 0$ shall be analyzed. Measurements were taken for rotational frequencies f_α in 0.1 Hz steps, from 0.1 Hz to 1.2 Hz, whereby no measurements were taken for 1.0 Hz since the test rig exhibited a resonance at this rotational frequency. During the dynamic measurements the system was also excited by the shaker with burst random. For the evaluation, FRFs are calculated using *pyFRF* [9]. However, no modal parameters are extracted due to the high amount of noise and because the system can not be considered time-invariant. The rotational frequency f_α was set during the measurements using the DC-motor power supply and checked using the blade passing counter as depicted in fig. 11. Using peak detection on the light barrier signal, the average rotational frequency can be calculated.

For the plots of the dynamic results, the desired frequency f_α is used. However, note that due to some deviations of the average frequencies, especially for 0.3 and 0.4 Hz, there might be some distortion. The spectrograms are created using *Matplotlib's imshow* command, which interpolates the plots. This can especially be noticed around $f_\alpha = 1$ Hz, where no measurement data exists. To start the analysis of the system's behavior, the FRF trace, i.e., the sum of all diagonal entries of the FRF matrix, for each measured rotational frequency is plotted in fig. 19. The x -axis represents the frequency axis of the FRFs for each rotational frequency f_α , starting with the FRF for $f_\alpha = 0.1$ Hz at the bottom to the one on the top for $f_\alpha = 1.2$ Hz.

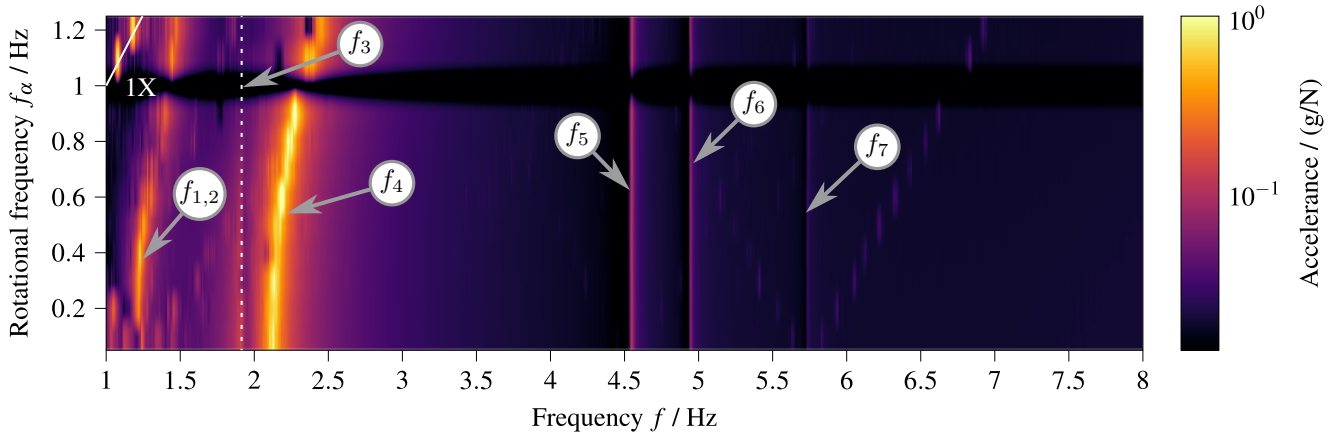


Fig. 19 Spectrogram showing the **FRF Traces** for different rotational frequencies f_α with eigenfrequencies f_i annotated

Visible most clearly are the eigenfrequencies of the two blades $f_{1,2}$ that merged to one line and one of the eigenfrequencies belonging to the tower, namely f_4 , while f_3 is only faintly visible. All three of those eigenfrequencies, not including f_3 , show a clear dependence on the rotational frequency and increase with f_α . Also clearly visible is the first excitation order, marked as 1X, due to some unbalance. The higher eigenfrequencies are less pronounced and are constant. One interesting effect can be seen for f_7 , the second-order beam deformation, which is only visible in the FRFs corresponding to the blade

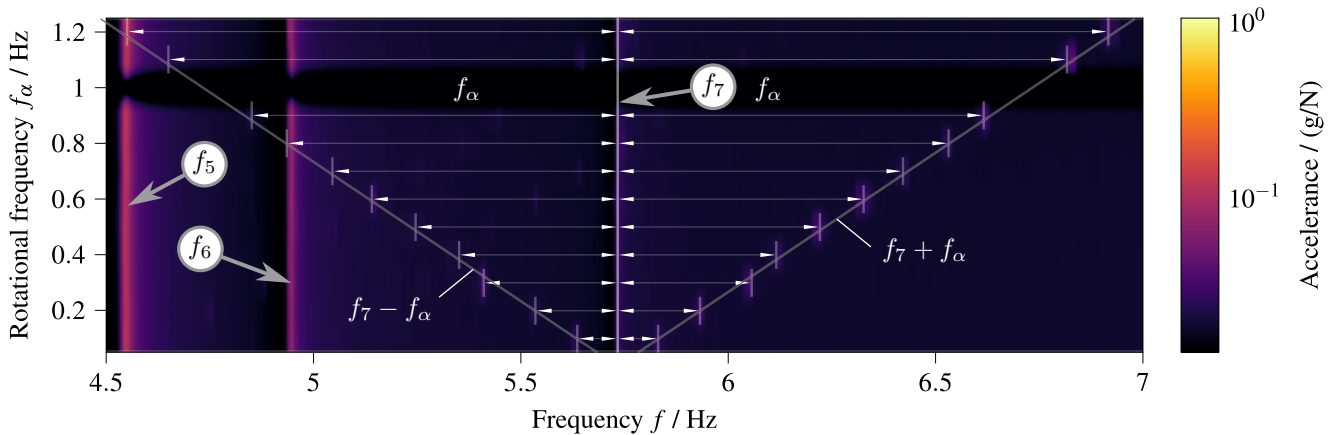


Fig. 20 Magnified spectrogram showing the **FRF Traces** for different f_α within 4.5-7 Hz

acceleration sensors. While the eigenfrequency is constant, there are sidebands that have an increasing distance to f_7 for higher rotational frequencies. This part of the plot is enlarged in fig. 20 and analyzed in more detail.

In fig. 20, there are two lines denoting the frequencies $f_7 - f_\alpha$ and $f_7 + f_\alpha$ as well as vertical lines marking the aforementioned quantities for the actual rotational frequency f_α during the measurements. As can be seen, the sidebands are exactly spaced by f_α from the eigenfrequency f_7 . The presence of these sidebands at a distance equal to an integer multiple of the fundamental frequency, here the rotational frequency f_α , is a consequence of the periodic system behavior and typical for LTP-systems [12]. Here, the periodically changing parameter is most likely the stiffness of the blades due to gravity.

A closer look shall also be taken at the lower eigenfrequencies the minimal model covers. The appropriate segment from 1 to 3 Hz is shown in fig. 21. Similar sidebands can also be found for the blade mode shapes $f_{1,2}$, although the distance from the dominant peak does not equate to an integer multiple of f_α . Tracing the respective average rotational speed from the visible sideband peaks toward the location of the beam eigenfrequencies $f_{1,2}$ leads to what is shown in fig. 21.

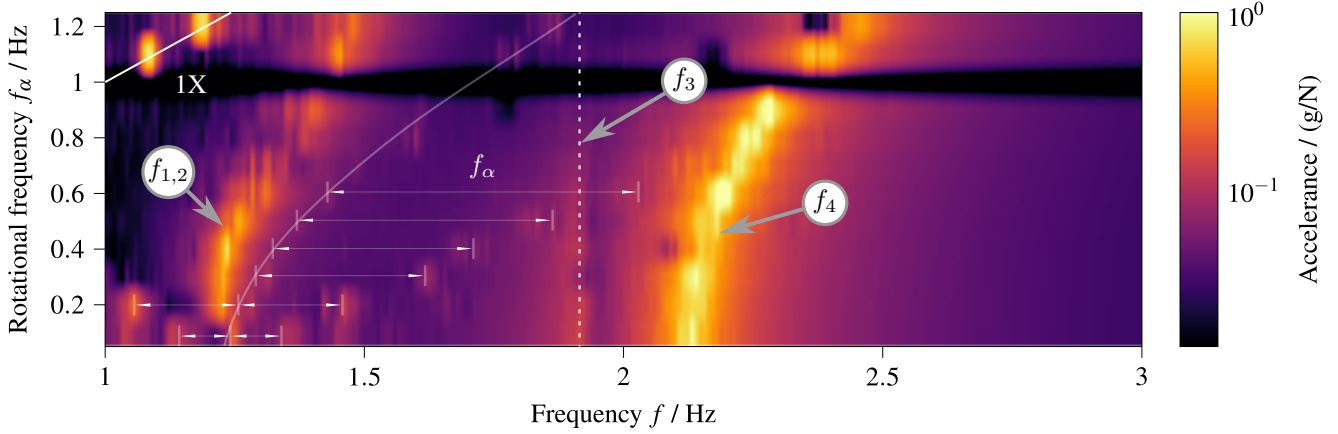


Fig. 21 Magnified spectrogram showing the **FRF Traces** for different f_α within 1-3 Hz

The sidebands have a distance equal to an integer multiple of the average rotational frequency f_α , but not to the visible blade eigenfrequencies $f_{1,2}$ and instead to other sporadic peaks, traced with a bent line. This line also corresponds to an antiresonance visible in the FRFs of the tower, see fig. 22. Since the system is excited at the tower, fig. 22 shows a driving point FRF. At the antiresonance frequency, by definition, the tower does not exhibit any movement and only the attached system, the blade assembly, vibrates at its eigenfrequency. The sidebands are visible at a distance equal to an integer multiple of the average rotational frequency when measured from the uncoupled eigenfrequencies of the periodically varying system.

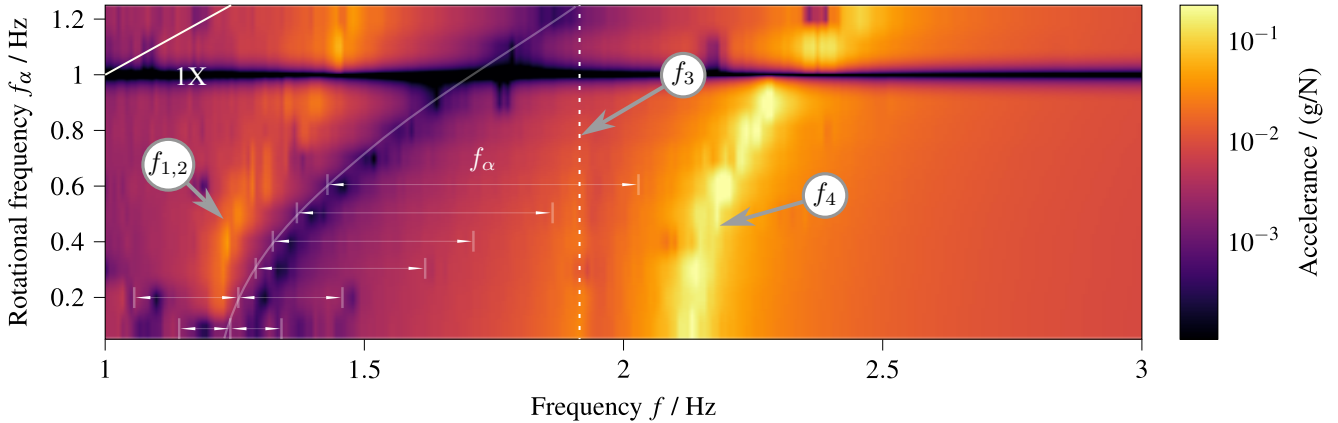


Fig. 22 Spectrogram showing the FRFs of the **Tower** for different f_α within 1-3 Hz

Lastly, the agreement of the minimal model and the experimental results shall be assessed. Since the minimal model's behavior with varying rotational frequency f_α was determined by calculating eigenfrequencies at fixed values for the blade angle α and rotational speed $\dot{\alpha}$, essentially linearizing the system around this configuration, no behavior typical for LTP-systems can be analyzed with the model. Therefore, the only possible comparison is between the values of the modeled eigenfrequencies for all angles α and the basically averaged eigenfrequencies observed experimentally and how they develop

with the rotational frequency f_α . For this, the intensity plot of the minimal model is plotted over the spectrogram of the FRF traces, see fig. 23.

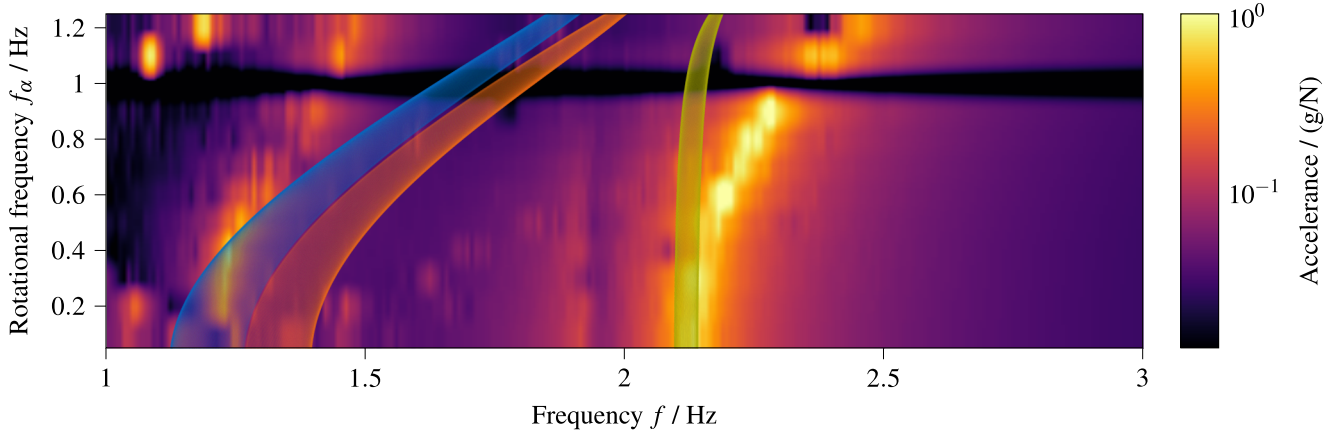


Fig. 23 Magnified spectrogram showing the **FRF Traces** and model intensity plot for different f_α

The agreement with the minimal model is not very good for higher rotational frequencies f_α . Since the blade assembly has an angular momentum for rotational speeds $\dot{\alpha} \neq 0$, a displacement of the tower requires a tilting of the rotation axis. Due to gyroscopic effects, with an increase of the rotational frequency f_α , there is an increasing resistance against the tower tilting that increases the eigenfrequency. This was not considered in the minimal model. Also, the eigenfrequencies of the blades are off and become too stiff for higher rotational frequencies f_α . Looking at the tower FRFs in fig. 24, the agreement of the model's blade eigenfrequencies with the antiresonance frequency/frequencies, i.e., when the tower is fixed, is pretty good, implying that the coupling between the blades and the tower is not modeled precisely enough.

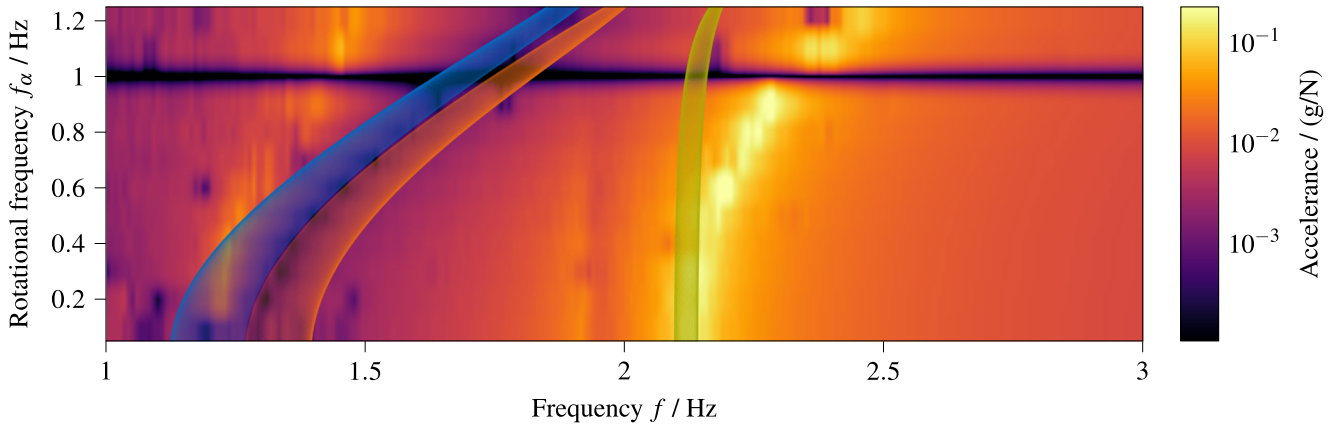


Fig. 24 Magnified spectrogram showing the FRFs of the **Tower** and model intensity plot for different f_α

Conclusion and Outlook

While the utilized minimal model could not fully capture the dynamics observed for the test rig, mainly because some effects were missed in the modeling processes, it was still able to predict most of the quantitative behavior. If the minimal model were to be updated, a sideways degree of freedom should be added to the tower to allow representing gyroscopic effects on the global system dynamics. The true experimental dynamics could be analyzed here using the measured vibrations on both the tower and the blades during rotation. While no LTP-theory was applied here, an evaluation of the existing data [8] might be interesting.

Building on preexisting open-source tools enabled the analysis of this academic test rig, both quickly and cost-effectively. While open-source tools cannot replace commercially available tools in all circumstances, here, no noticeable limitations

were found. Another big advantage is the openness and adaptability, which are vital for enabling the quick development of new measurement procedures or evaluation techniques within academia. For this reason, the further development of available open-source tools, as well as the development of new ones, is desirable. Therefore, the authors decided to further develop the *Open Acquisition System for IEPE Sensors* and build an improved version, called *OASIS-UIROS: Open Acquisition System for IEPE Sensors - Upgraded, Refined, and Overhauled Software*, which is further detailed in [13].

References

- [1] Zobel, O. M., Maierhofer, J., and Rixen, D. J. "OASIS: Open Acquisition System for IEPE Sensors: For Academic Research and Teaching Purposes". In: *Conference Proceedings of the Society for Experimental Mechanics Series* (2024), pp. 79–86. DOI: [10.1007/978-3-031-34938-6_9](https://doi.org/10.1007/978-3-031-34938-6_9).
- [2] Allen, M. S. "Frequency-Domain Identification of Linear Time-Periodic Systems Using LTI Techniques". In: *Journal of Computational and Nonlinear Dynamics* 4.4 (2009). ISSN: 1555-1415. DOI: [10.1115/1.3187151](https://doi.org/10.1115/1.3187151).
- [3] Allen, M. "Floquet Experimental Modal Analysis for System Identification of Linear Time-Periodic Systems". In: *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference Volume 5: 6th International Conference on Multibody Systems, Nonlinear Dynamics, and Control, Parts A, B, and C* (2007). DOI: [10.1115/DETC2007-34790](https://doi.org/10.1115/DETC2007-34790).
- [4] Dresig, H. and Holzweißig, F. *Dynamics of Machinery: Theory and Applications*. Springer Berlin Heidelberg, 2010. ISBN: 9783540899402.
- [5] G  radin, M. and Rixen, D. J. *Mechanical vibrations: theory and application to structural dynamics*. 3rd. John Wiley & Sons, Ltd, 2014. ISBN: 978-1-118-90020-8.
- [6] The Manim Community Developers. *Manim – Mathematical Animation Framework*. Version v0.17.2. 2022. URL: <https://www.manim.community/>.
- [7] Mace, B. and Manconi, E. "Mode veering in weakly coupled systems". In: *Proceedings of ISMA2012-USD2012* (2012).
- [8] Zobel, O. M., Maierhofer, J., and Rixen, D. J. *Rotating System Measurements using the Open Acquisition System for IEPE Sensors (OASIS)*. Dataset. 2024. DOI: [10.14459/2024mp1752772](https://doi.org/10.14459/2024mp1752772).
- [9] Slavi  , J.,   esnik, M., Zaletelj, K., and Novak, L. *pyFRF: Frequency response function as used in structural dynamics (v0.40)*. 2019. URL: <https://github.com/ladisk/pyFRF>.
- [10] Bregar, T., El Mahmoudi, A., Kodri  , M., Ocepek, D., Trainotti, F., Poga  ar, M., and G  ldeli, M. *pyFBS (v0.2.9)*. 2022. URL: <https://pyfbs.readthedocs.io/en/latest/index.html>.
- [11] Guillaume, P., Verboven, P., Vanlanduit, S., Van der Auweraer, H., and Peeters, B. "A poly-reference implementation of the least-squares complex frequency-domain estimator". In: *Proceedings of IMAC XXI* (2003).
- [12] Tcherniak, D. and Allen, M. "Experimental characterization of an operating Vestas V27 wind turbine using harmonic power spectra and OMA SSI". In: May 2015.
- [13] Zobel, O. M., Maierhofer, J., K  stler, A., and Rixen, D. J. "OASIS-UIROS: Open Acquisition System for IEPE Sensors - Upgraded, Refined, and Overhauled Software". In: *HardwareX* (2025). ISSN: 2468-0672. DOI: [10.1016/j.ohx.2025.e00650](https://doi.org/10.1016/j.ohx.2025.e00650).



Chapter 3

Investigation of Output Metrics for Dynamic Topology Optimization of PCB Reinforcement

Ryan Semler, John Bell, and Laura Redmond

Abstract Rigid-flex Printed Circuit Board (PCB) robotics have been considered as a future architecture for the exploration of high-risk extraterrestrial environments such as the moon. One major challenge with this architecture is the lack of modeling methodologies and design strategies for the use of PCB as the primary structure of the system. Previous research has established the need for external reinforcement on PCB panels to survive the expected dynamic loading for an extraterrestrial mission. Because the primary advantages of the system are low mass and cost-effectiveness, topology optimization is a prime candidate for the design of lightweight exoskeletons to reinforce the PCB structure. This paper begins to formulate a code to interact with the commercial software Abaqus to formulate custom Equivalent Static Loads (ESLs) and output metrics that control the optimization of a reinforcing exoskeleton for a PCB panel subjected to dynamic loads. First, the paper outlines several methods for selection of ESLs in the literature. Second, common failure mechanisms of PCB are presented to inform the parameters that are investigated in the study. Third, the algorithm for generating the ESL and to pass customized parameters to the Abaqus TOSCA solver is presented. Fourth, preliminary results are presented to demonstrate the optimization routine when optimizing for the output metric of strain energy. The paper concludes with future various output metrics to be used to inform the optimization, and the presentation will include recommendations for the dynamic topology optimization of PCB structures based on the findings in this case study.

Keywords topology optimization · dynamic analysis · PCB

Introduction

NASA JPL's rigid-flex PCB Pop-Up Folding-Flat Explorer Robot (PUFFER) was originally designed to be a lightweight, cost-efficient explorer robot for Mars, which included Nomex fabric joints to improve the fatigue resistance of the rover [1]. Clemson University has collaborated with NASA JPL to develop modeling methodologies for rigid-flex PCB robotics validated with dynamic testing to be used for investigations of potential loading scenarios to PUFFER if it were used for a Lunar scouting mission. This research determined that the current PUFFER structure is inadequate for an 80-meter drop into a lunar pit [2–5] and that external reinforcing would be needed to preserve large panel sizes. Research has also been conducted to optimize a reinforcing skeleton on a single-degree of freedom (SDOF) model of a PCB panel with a frequency-based approach [6]. The frequency-based optimization prioritized minimizing displacement while constraining the mass to be less than or equal to 100% and utilized a looping process to conduct the optimization multiple times as the system's natural frequency shifted during optimization.

This work furthers the research on the structural reinforcement by leveraging commercial software to conduct the opti-

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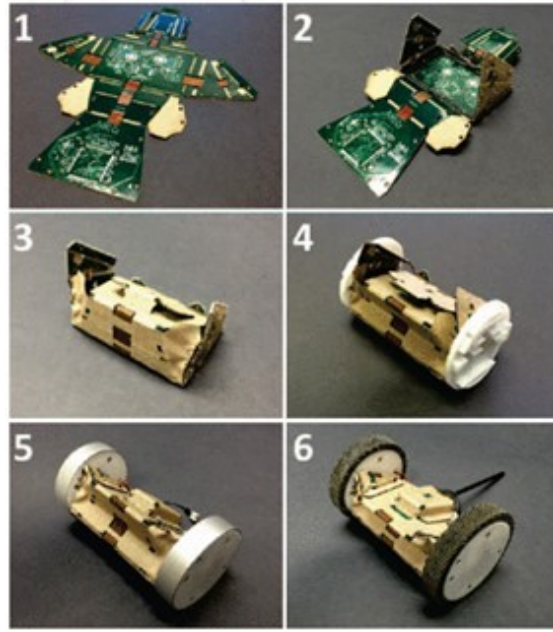


Fig. 1 Rigid-flex NASA JPL rover, PUFFER [1]

mization process with various output metrics on a multi-degree of freedom (MDOF) system utilizing the Equivalent Static Load (ESL) approach to determine loads at each iteration of the optimization. This preliminary case study presents the first step in this investigation with a baseline sizing optimization of the reinforcement after applying a random base excitation to the system. The overall goals of this paper are to 1) present the ESL method and integration with Abaqus TOSCA to conduct the optimization process, 2) present the various parameters to be considered for the objective function in PCB exoskeleton design and possible formulations of the ESL, and 3) show preliminary results of the optimization routine for a baseline set of optimization parameters.

ESL Generation in Literature

There exist several methods of creating static-equivalent loads (ESLs) from dynamic loading. The most common method is a time-domain approach in which the static-equivalent loads are calculated using the stiffness matrix and dynamic displacements at a specific time point. Choi & Park examine an analytical method that utilizes this approach with a transient analysis for displacements and generates ESLs at all locations within a model [7]. Park then applies this approach to a structural optimization problem by generating ESLs from a displacement field at each time step of the dynamic analysis [8]. In later years, Jang et al. have used this approach for dynamic response topology optimization in the time domain [9, 10].

Preliminary research shows that a frequency-based approach can be utilized with commercial software to derive approximate static-equivalent loads for dynamic topology optimization of a single-degree of freedom system subjected to sinusoidal loads [6]. The approach generates static-equivalent loads as a load which generates mode 1 deformations modified by a dynamic response amplification factor, R_d . This factor is calculated from the applied excitation frequency and the natural frequency of the system. The approach was verified at each intermediate step of the optimization by comparing the results from the ESL(s) to the maximum steady-state response of the model run in time domain with the sinusoidal base excitation. For the single degree-of-freedom system approach, a mode participation of 80-90% is sufficient for determining the dynamic response of the structure [11]. However, this approach is limited to sinusoidal loading.

Static-equivalent loads are typically approximated for optimization problems with displacement or stress. Displacement-based approximations use inequalities to calculate ESLs that are then optimized to find the smallest load at a critical time [7]. Stress-based approximations utilize stress as a constraint condition when calculating ESLs. This is because displacement and stress are not proportional when a rigid body is dominant in the displacement component due to the internal strains of the body being negligibly small [7].

A time step for equivalent static loads is selected based on the type and objective of the analysis or optimization problem. For a minimization problem, the time step is usually chosen at the point at which the objective function, usually displacement

or stress, is at its maximum. ESLs are calculated from the dynamic displacement field at this specific time step [7]. This method of time step selection prioritizes a single time point for a single area of a model. In this work, the analytical method for obtaining the ESL will be utilized, and we will later investigate whether selecting the time point based on localized maxima around critical mounted components or selecting it based on global maxima results in a more robust design for the PCB exoskeleton.

PCB Failure Mechanisms

A plethora of research studies have been conducted to identify, investigate, and mitigate several failure mechanisms present in PCB panels. Such failure mechanisms include Conductive Anodic Filament (CAF), cracking, and solder joint failure. CAF is an iterative failure induced by PCB depenalization, primarily due to moisture exposure caused by small cracks and the formation of dendrite, which leads to burn-out of the panel [12]. Cracking is a corrosion-induced failure due to the presence of dendrite growth [13]. Dendritic growth is an electrochemical phenomenon resulting in conductive metal filaments forming along the PCB panel; it is most notably caused by high relative humidity and contamination [14]. In a dynamic environment, PCB panels can fail when the yield stress of a ply is exceeded from board deformation. These deformations can be caused by shock waves or stress distributions generated from impact, vibration, or panel bending. Differential flexing and elongation of the board may damage sensitive interconnections [15–17].

Solder joint failure itself has several potential causes. They are critical components of PCB panels as they anchor physical components to the panel and can provide essential electrical connection to the PCB itself. Manufacturing errors and unsatisfactory environmental conditions can result in failure. However, the most prominent cause of solder joint failure is cracking due to dynamic flexing as the stress in the solder joint exceeds the joint strength. Drop impacts, for example, may produce panel differential bending and/or large inertial forces from the heavier mounted components that induce severe stresses in the solder joints [18]. As such, the use of stress and curvature as objective functions within the optimization routine will be examined.

Summary of Optimization Parameters to be Investigated

The optimization parameters investigated are outlined in Table 1. ESL time point selections for global (full model) and local (partial model) areas will be examined for maximum principal stress (abs.), maximum curvature, and both stress and curvature simultaneously. The preliminary case study examines a global determination of the ESL time point for maximum strain energy as a whole model quantity for the initial baseline analysis to demonstrate the optimization routine.

Table 1 Matrix of output metrics

	ESL time point selection method	
	Single time point, single area	Multiple time points and areas
Output metric measured	Whole model strain energy(abs.)	
	Maximum element strain energy over whole model	Maximum element strain energy near each component
	Maximum curvature	Maximum curvature
	Max. principal stress (abs.)	Max. principal stress (abs.) near each component

**Blue represents the subset shown in this abstract, green is the additional data that will be presented at IMAC, and grey are suggested metrics for future investigations*

Table 2 System Material Properties

	Density [tonne/mm ³]	Young's modulus [MPa]	Poisson's ratio [-]
FR4 [19]	1.85E-09	18,500	0.136
Stainless Steel 316L	8.00E-09 [20]	193,000 [20]	0.28 [21]
Electronic Component ^a	1.97E-07	N/A	N/A

^aDensity is a reasonable approximation for instrumentation on a planetary robot. Stiffness is unaccounted for

Model Setup

A finite element model (FEM) was created based on an instrumented PCB panel for modal testing and model correlation shown in Figure 2 [19]. The model was modified to add three representative electronic components by increasing the density of certain areas of the panel where they could be located. A system can be idealized as an SDOF system if the natural mode has a modal participation of at least 80%, meaning it captures the majority of the system's dynamic response with one mode. The components are arranged in such a way that the system is a multiple degree-of-freedom (MDOF) system that cannot be idealized as an SDOF system due to the necessity of multiple modes to increase the modal participation.

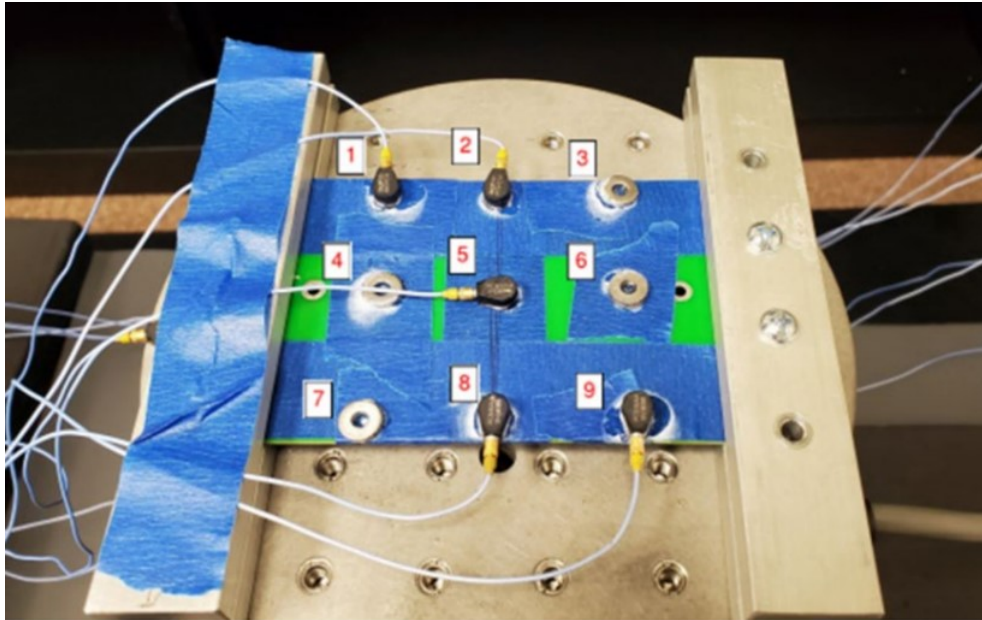
**Fig. 2** Instrumented PCB panel for fixed—fixed frequency test [18]

Figure 3 shows the layout of the 150 mm x 75 mm PCB panel. The PCB panel has a thickness of 1.5 mm, and the reinforcement (not shown) of the same area has a thickness of 6.35 mm. The portion of the PCB & reinforcement shown in orange is clamped to account for physical testing situations. Tie constraints between the PCB panel and reinforcement are shown in dark blue. The area of the representative electronic components, shown in maroon, is varied to induce an MDOF system and to emulate the varying size of physical instrumentation. A reinforcement layer consisting of an isotropic, printable material (Stainless Steel 316L) was added and constrained to the PCB panel with several ties representing physical fasteners. The shape of this reinforcing layer will be optimized to minimize the objective function in each scenario. The reinforcement is shown on the PCB in Figure 4. The material properties of the model are denoted in Table 2.

Time History Excitation

A random base motion excitation is applied to the clamped regions of the system through an acceleration time history generated from a power spectral density curve. Table 3 provides the input power spectral density, which uses values converted into the model units from the General Environmental Verification Standard (GEVS) for GSFC Flight Programs and Projects [22].

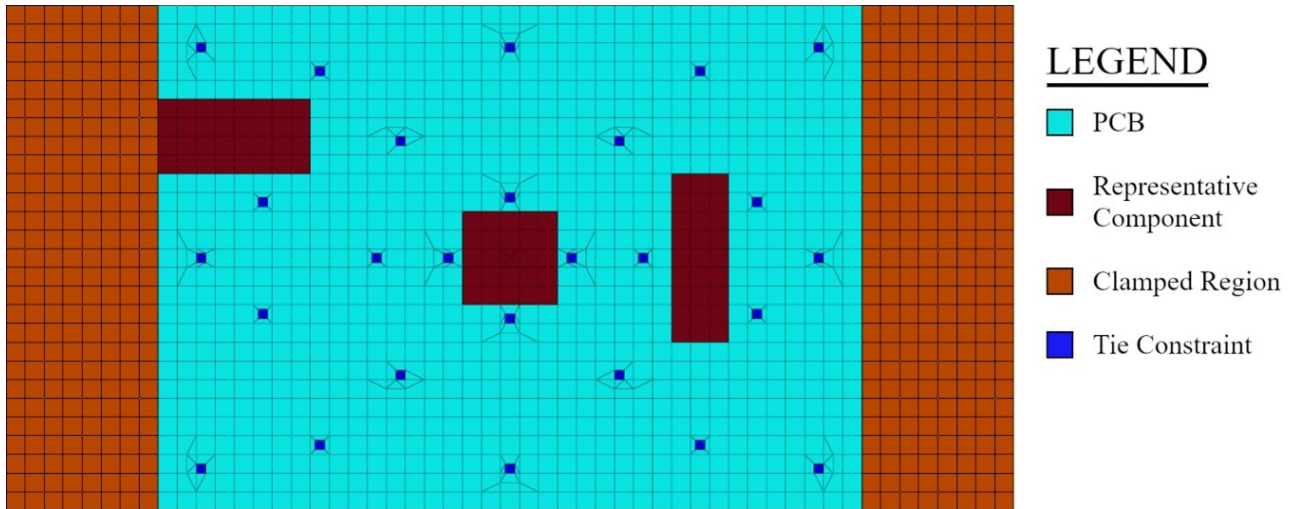


Fig. 3 PCB panel layout

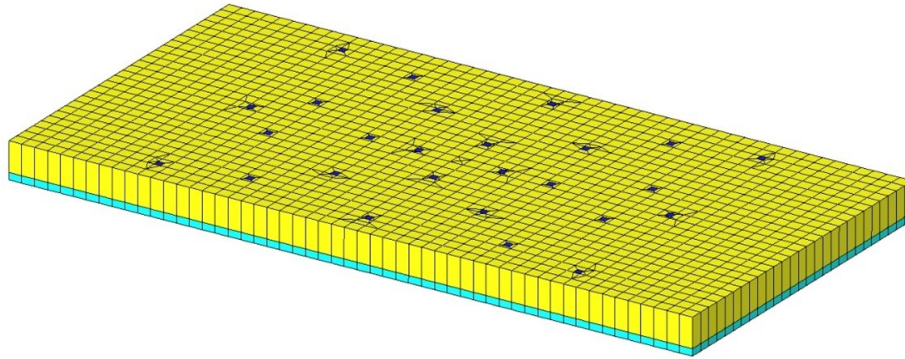


Fig. 4 PCB model with reinforcement (yellow)

Table 3 Power spectral density inputs

Frequency (Hz)	Amplitude (mm^2/s^3)
20	2500430.0
50	15387261.5
800	15387261.5
2000	2500430.0

The PSD was used to generate a time history for the optimization process such that a time step in the simulation could be selected for the calculation of static-equivalent loads. The time history was synthesized as a series of sinusoids resembling a random signal and satisfying the specified PSD [23]. A custom MATLAB script was created to calculate the amplitudes of the specified PSD and sum the spectral components at each time point. The time history was verified to fit the PSD visually in Figure 5 using Welch's method in MATLAB with a window division length of 6 as well as 15 samples of overlap between segments. The GRMS values between the time history and input power spectral density were found to be 14.163 g's and 14.136 g's, respectively, with a reasonably small difference of 0.19%. Then, a normalized PDF, shown in Figure 6, was plotted to verify the generated time history followed a Gaussian distribution. The bin width for the histogram was 0.038. The kurtosis was found to be 2.9932; since this is very close to 3, it can be said that the generated time history follows a normal or Gaussian distribution similarly to the power spectral density input.

The modal damping ratio was approximated based testing conducted on solid PCBs by Bell et. al [4] and were calculated using the half power bandwidth method [24]. The estimated damping ratio from these experiments ranged from 1.26% to 1.82% for the first three modes in a clamped condition. Thus, for the first 50 modes of our model, a damping ratio of 1.5%

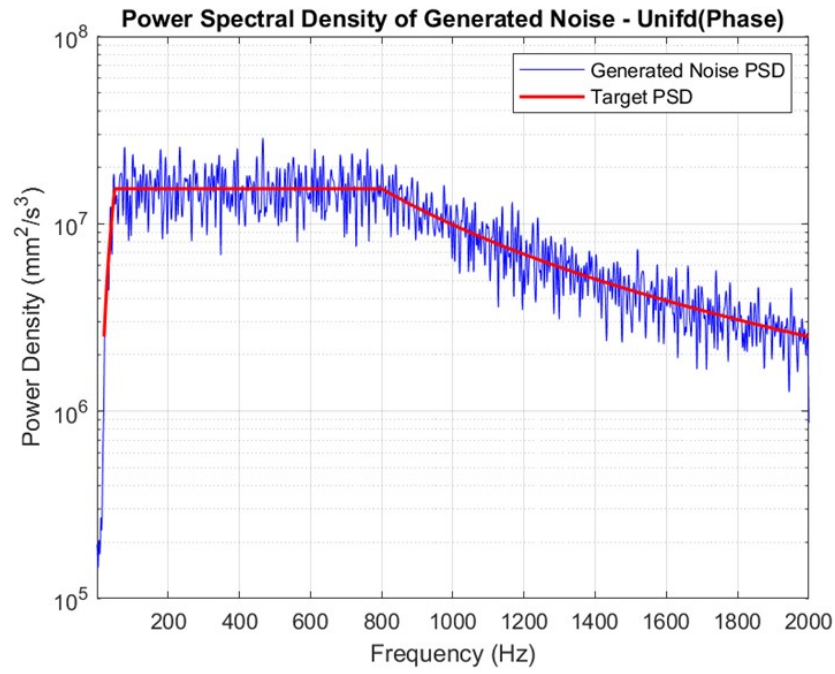


Fig. 5 Input power spectral density (red) vs time history (blue)

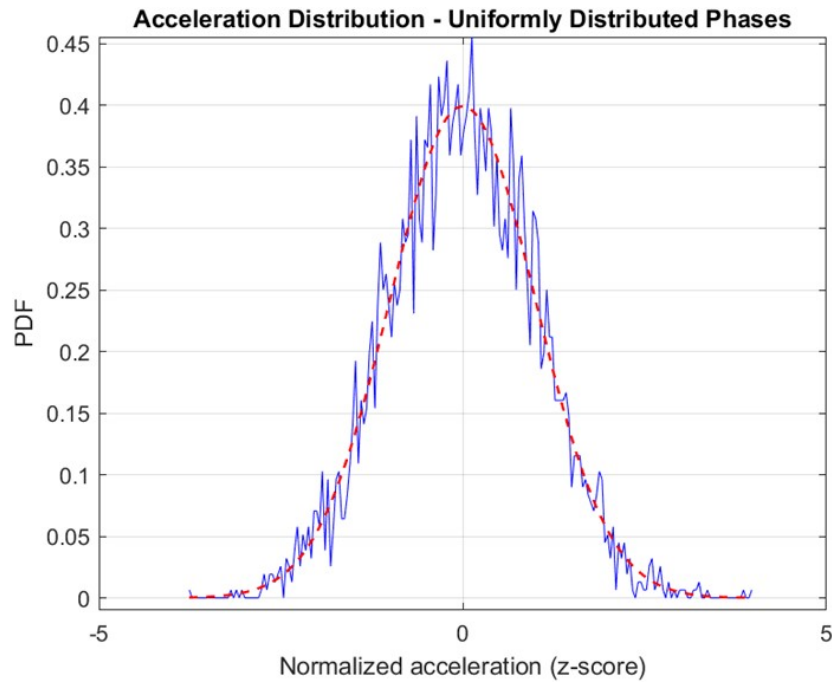


Fig. 6 True Gaussian distribution (red) vs normalized Gaussian PDF of time history (blue)

was applied for each mode as a reasonable estimate of damping. For reference, the fundamental frequency of the baseline model with uniform reinforcement thickness was 1193.1 Hz.

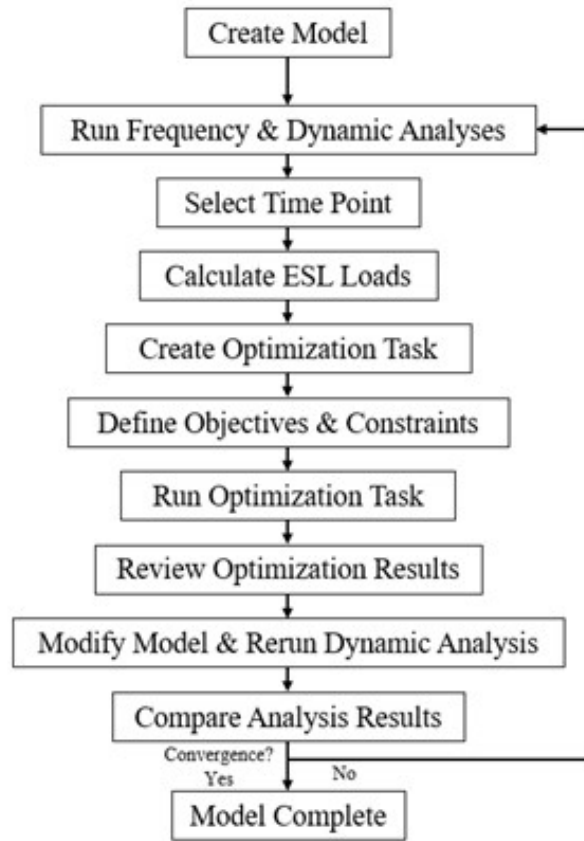


Fig. 7 Optimization procedure

Optimization Routine and Code

The optimization method followed is an iterative process in which the reinforcement panel is repeatedly optimized until the results are satisfactory. The full optimization process is outlined in the following flow chart.

Several scripts were written and utilized throughout the process. Most prominently, a script was written to extract the displacement history data from the dynamic analyses results for use in calculating the static-equivalent loads for the system at the specified time step. The calculation of the equivalent-static loads itself was carried out with a separate MATLAB script. The optimization task was a general (sensitivity-based) optimization of the reinforcement exoskeleton with default settings modified to freeze the boundary regions, unfreeze the load regions, and allow for 0.5 (50%) maximum change per cycle. Sizing optimization was used in lieu of topology optimization for the preliminary case study to ensure applied ESLs were unaffected by the deletion of elements throughout the process. For the sizing optimization, a thickness control was defined for the reinforcement to allow the thickness of its elements to change from 0.635 mm (10% thickness) to 12.7 mm (200% thickness). The objective function is defined as the selected output metric to be measured, and the constraint limits the mass to less than or equal to 100% of its initial mass. For consistency within the optimization loop, the mass was kept consistent between iterations by increasing the thickness of the remaining reinforcement after remodeling. The proposed optimization process output metric comparison matrix was outlined in Table 1. The preliminary case study presents the results of the first round of optimization with whole element strain energy as the output metric and a single time point, single area ESL generation method. The time point chosen was the time at which strain energy in the model was maximum.

Sample Results

In this preliminary case study, the optimization task was used to optimize the reinforcement exoskeleton to minimize the strain energy globally. The mass of the system remained unchanged throughout the process for consistency when comparing

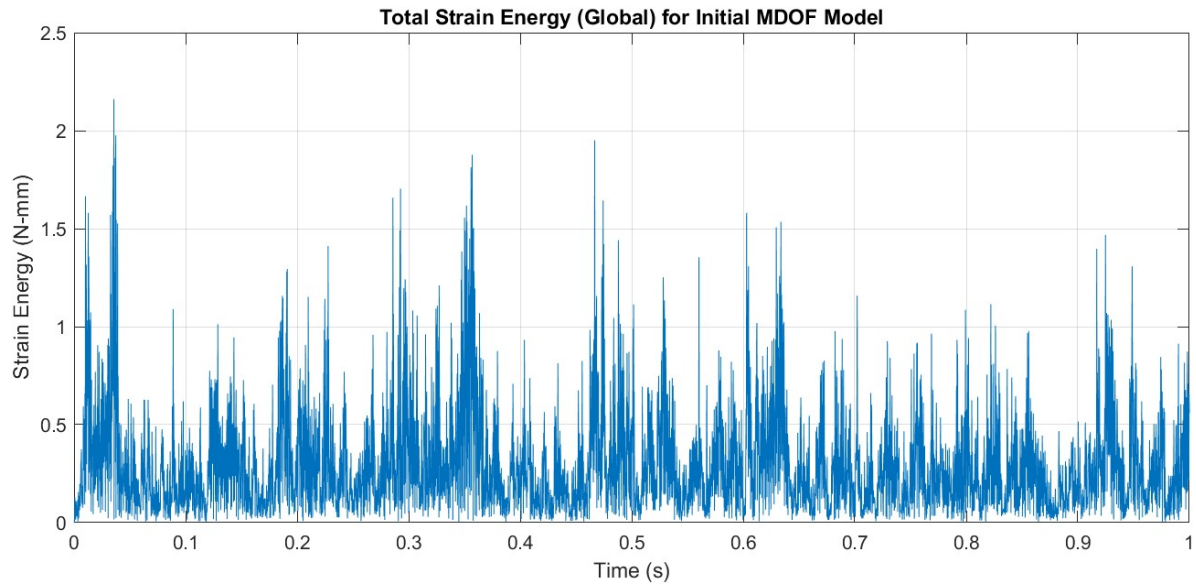


Fig. 8 Total strain energy over time (global) for initial MDOF model

results. Equivalent-static loads used for the optimization were calculated from the stiffness and displacements due to the time history random base excitation at the critical time point which strain energy was maximum. These loads were applied to the model at every node. Figure 8 shows the strain energy in the baseline model (uniform reinforcement) at all time points due to the random base excitation. The critical time point was found to be 0.0356 s with a strain energy of 2.161 N-mm.

The result of the first round of optimization is shown in Figure 9. Clamped reinforcement is hidden for clarity. As anticipated, reinforcement was removed at areas where overall density was lower. The reinforcement primarily remained where strain energy was higher in the model, which is where density was higher in the PCB panel due to the components. Note that even though mass was permitted to decrease during the optimization routine in TOSCA, the optimization preserved about 99% of the original system mass. This optimization result was remodeled to adjust the thickness of the indicated reinforcement elements, which were modified rather than deleted for consistency between runs and to allow for the strain energy of the remade model to be compared to the previous model with the same ESLs.

The remade model, shown in Figure 10, was found to have a reduced strain energy of 2.126 N-mm for the ESL generated with the baseline model. In the model, the red region has a thickness of 12.7 mm, the blue region has a thickness of 0.635 mm, and the orange region to be optimized has a thickness of 5.816 mm. The thickness of the orange region to be optimized was adjusted so the total mass of the system remained unchanged. The yellow region represents the reinforcement that is clamped and unoptimized. The same random base excitation was applied to the first-round optimized model to determine the critical time point and maximum strain energy for the next step in the optimization procedure. Figure 11 shows the maximum strain energy in the model at all time points of the analysis. The critical time point was found to be 0.0356 s with a maximum strain energy of 2.262 N-mm. Note the total strain energy slightly increased from the initial model as dynamic properties of the system have shifted.

The result of the second round of optimization is shown in Figure 12. Clamped reinforcement is hidden for clarity. Like the first optimization run, the thickness of reinforcement within the region to be optimized was increased in areas where the overall density was higher and decreased where it was lower. This optimization result was remodeled in a manner similar to that of the remodel of the previous optimization result, though the regions of maximum thickness at isolated regions from the previous result were modified to reflect the thickness of nearby regions instead due to the second optimization result representing a final run.

The model remade from the second and final round of optimization is shown in Figure 13. Unlike the previous model, the orange region now has a reduced thickness of 5.320 mm. This model was found to have a reduced strain energy of 2.236 N-mm when analyzing it with the ESLs generated from the previous model run. The model was also found to have the same strain energy as the previous model of 2.383 N-mm when analyzing it dynamically. Thus, overall, the whole model strain energy was relatively flat throughout the optimization process and slightly increasing as the dynamics of the system changed. It is likely not an ideal optimization parameter. A summary of the progression of the whole model strain energy values throughout the optimization process is shown in Table 4.

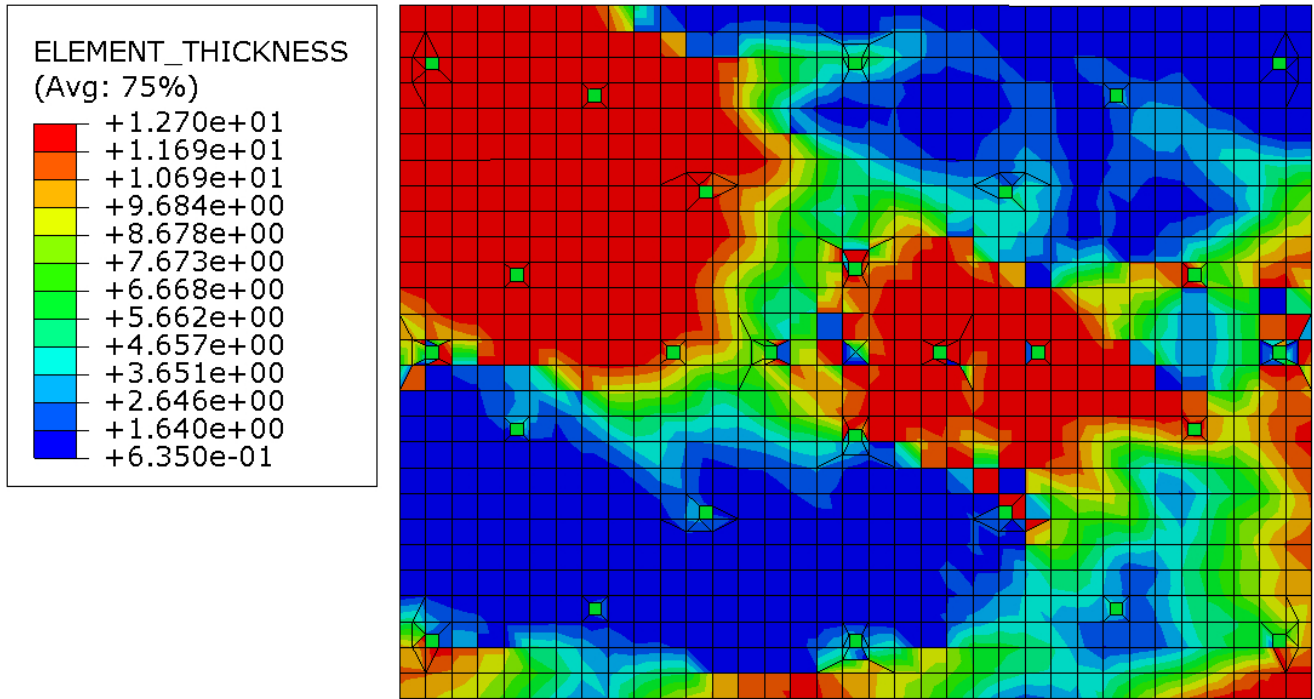


Fig. 9 First-round optimization result (element thickness of the reinforcement layer)

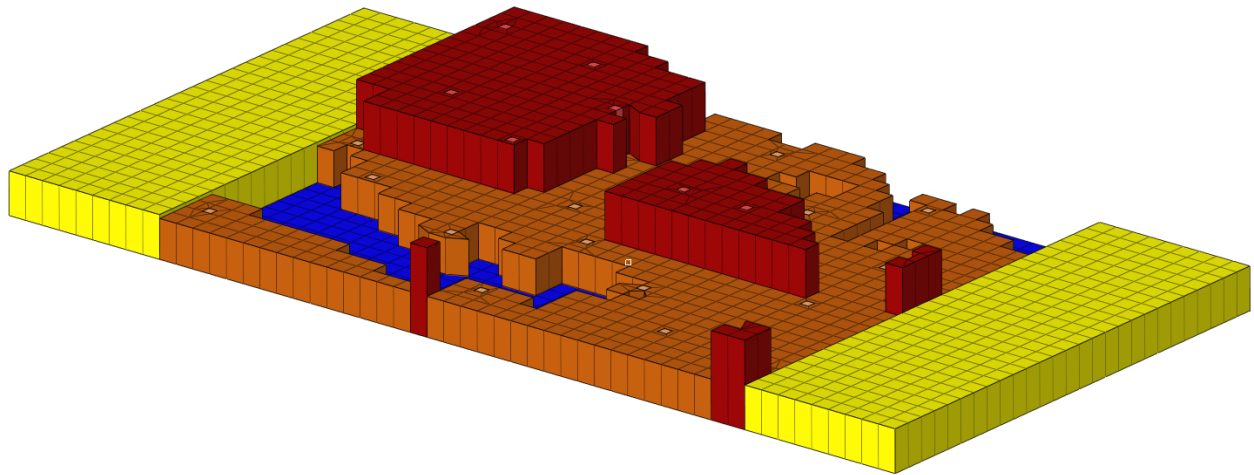


Fig. 10 Remodel of first-round optimization result (reinforcement only)

A final comparison was made between the strain energy in the PCB without reinforcement and the second-round optimization result to verify that reinforcement was at least beneficial in reducing the whole model strain energy. Without reinforcement, the maximum strain energy in the PCB alone is 106.421 N-mm when analyzing it with the same time history random base excitation, which is monumentally higher than that of the reinforced PCB. The strain energy over time for the unreinforced PCB is shown in Figure 15.

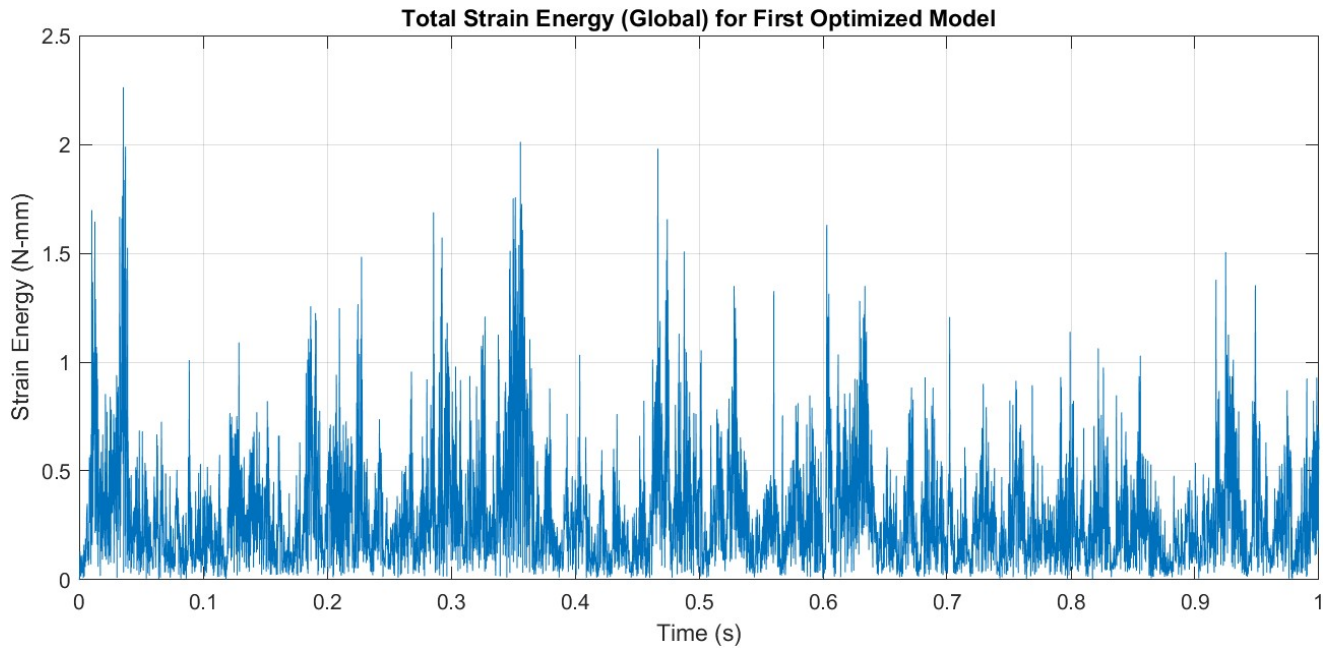


Fig. 11 Total strain energy over time (global) for first optimized model

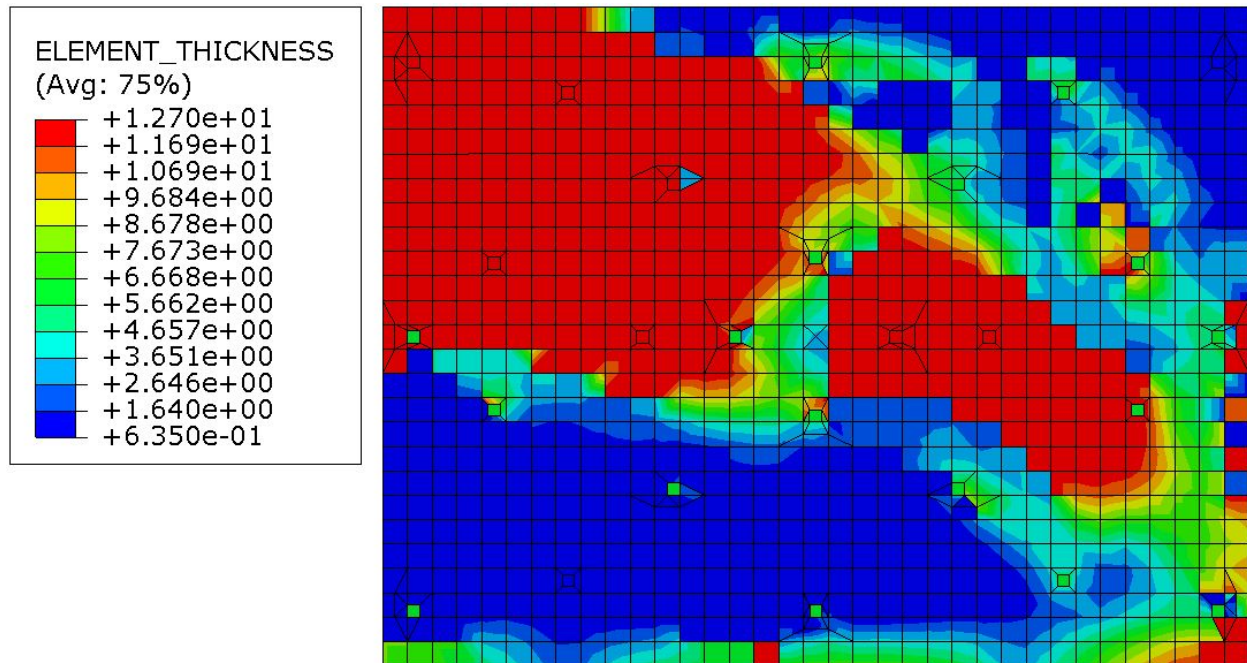


Fig. 12 Second-round optimization result: reinforcement (green) and component locations (red)

Conclusions and Future Work

The preliminary case study work has shown that the dynamic topology optimization process presented results in a non-uniform optimized structural reinforcement with a significant reduction in the objective function. The conference presentation will include and compare results for the outputs that can be included by default in the TOSCA solver and compare the use of global vs. localized regions for identifying the critical time point(s) for ESL(s). The presentation will additionally introduce a method of extracting custom output metrics from Abaqus. Future work will include utilizing custom optimization

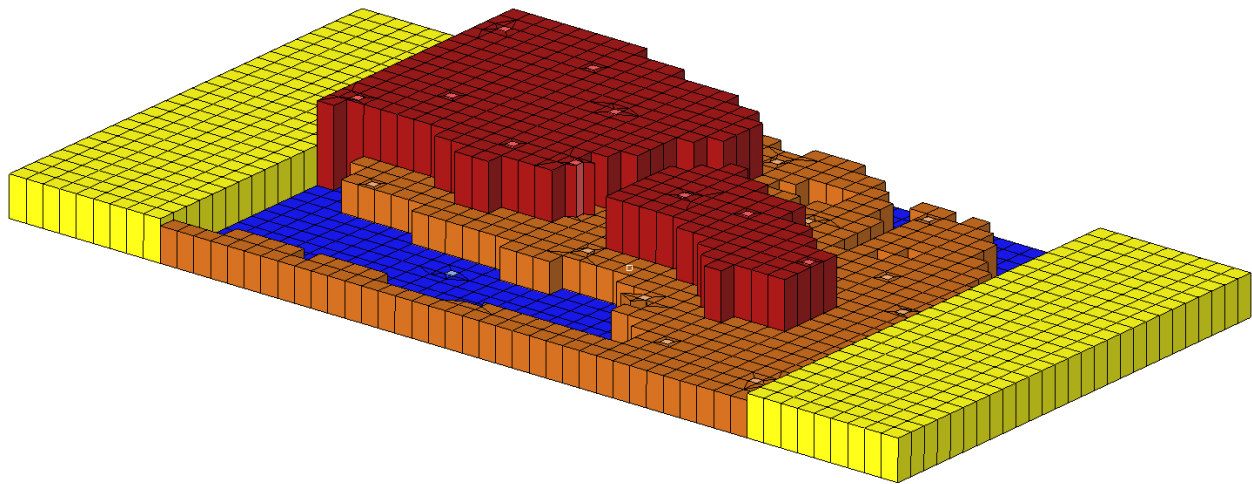


Fig. 13 Remodel of second-round optimization result (reinforcement only)

routines to leverage the custom output metrics as objective functions and experimental testing to validate that the optimized reinforcement performs as expected.

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References

1. Karras, J.T., Fuller, C.L., Carpenter, K.C., Buscicchio, A., McKeeby, D., Norman, C.J., Parcheta, C.E., Davy-dychev, I., Fearing, R.S.: Pop-up Mars rover with textile-enhanced rigid-flex PCB body. In: 2017 IEEE International Conference on Robotics and Automation (ICRA) (2017). <https://doi.org/10.1109/icra.2017.7989642>
2. Bell, J., Redmond, L., Carpenter, K., and de la Croix, J.P., "Finite Element Modeling of Rigid-Flex Printed-Circuit-Board Origami Robotics Subjected to Large-Scale Deformations," *AIAA Journal*, 62(4), 2024. <https://doi.org/10.2514/1.J062738>
3. Bell, J., Redmond, L., Carpenter, K., de la Croix, J.P.: Experimental dynamic characterization of rigid-flex PCB systems. *Exp. Tech.* (2022). <https://doi.org/10.1007/s40799-022-00559-4>
4. Bell, J., Redmond, L., Carpenter, K., de la Croix, J.P.: Finite element modeling of rigid-flex PCB robotic structures for dynamic environments. *J. Microelectron. Electron. Packag.* **19**(1) (2022)
5. Bell, J., Redmond, L., Carpenter, K., de la Croix, J.P.: Design of rigid-flex PCB robotics leveraging validated finite element simulations. In: *IEEE Aerospace, Big Sky, MT*, 5–12 Mar 2022
6. Semler, R., Carlson, D., Crowder, J., & Redmond, L., "Dynamic Topology Optimization of Rigid-Flex PCB Robotic Systems," *IMAC XLII*, Orlando, FL, January 29-February 1, 2024.
7. Choi, W.-S., Park, K. B., & Park, G. J. (2005). Calculation of equivalent static loads and its application. *Nuclear Engineering and Design*, 235(22), 2337–2348. <https://doi.org/10.1016/j.nucengdes.2005.05.030>
8. Park, G.-J. (2005). *Structural Optimization Using the Equivalent Static Load Concept*. <https://doi.org/10.21236/ada451871>
9. Jang, H. H., Lee, H. A., and Park, G. J., "Dynamic Response Topology Optimization Using Equivalent Static Loads," 8th World Congress on Structural and Multidisciplinary Optimization, Lisbon, Portugal, June 1–5 2009.
10. Jang, H. H., Lee, H. A., Lee, J. Y., & Park, G. J. (2012). Dynamic Response Topology Optimization in the Time Domain Using Equivalent Static Loads. *AIAA Journal*, 501, 226–234. <https://doi.org/10.2514/1.j051256>
11. Priestley, M. J. N., Seible, F., & Calvi, G. M. (1996). *Seismic Design and Retrofit of Bridges*. John Wiley & Sons, Inc., p. 242.
12. Tan, C.-M., Chen, H.-H., Wu, J.-P., Sangwan, V., Tsai, K.-Y., & Huang, W.-C. (2022). Root Cause Analysis of a Printed Circuit Board (PCB) Failure in a Public Transport Communication System. *Applied Sciences*, 12(2), 640. <https://doi.org/10.3390/app12020640>
13. Lu, L. N., Huang, H. Z., Su, X. X., Wu, B. Y., & Cai, M. (2009). Investigation on PCB related failures in high-density electronic assemblies. *2009 International Conference on Electronic Packaging Technology & High Density Packaging*, 128–132. <https://doi.org/10.1109/icept.2009.5270780>
14. Singh, P., Sunder, M., Campbell, E., & Palmer, L. (2020). A Case Study of Nickel Dendritic Growth on Printed-Circuit Boards. *2020 Pan Pacific Microelectronics Symposium (Pan Pacific)*, 1–8. <https://doi.org/10.23919/panpacific48324.2020.9059398>
15. Chong, D. Y. R., Che, F. X., Pang, J. H. L., Xu, L., Xiong, B. S., Toh, H. J., & Lim, B. K. (2008). Evaluation on influencing factors of Board-Level drop reliability for chip scale packages (Fine-Pitch Ball Grid array). *IEEE Transactions on Advanced Packaging*, 31(1), 66–75. <https://doi.org/10.1109/tadvp.2007.908024>
16. E. H. Wong, K. M. Lim, N. Lee, S. Seah, C. Hoe and J. Wang, "Drop impact test - mechanics & physics of failure," 4th Electronics Packaging Technology Conference, 2002., Singapore, 2002, pp. 327-333, doi: [10.1109/EPTC.2002.1185692](https://doi.org/10.1109/EPTC.2002.1185692).

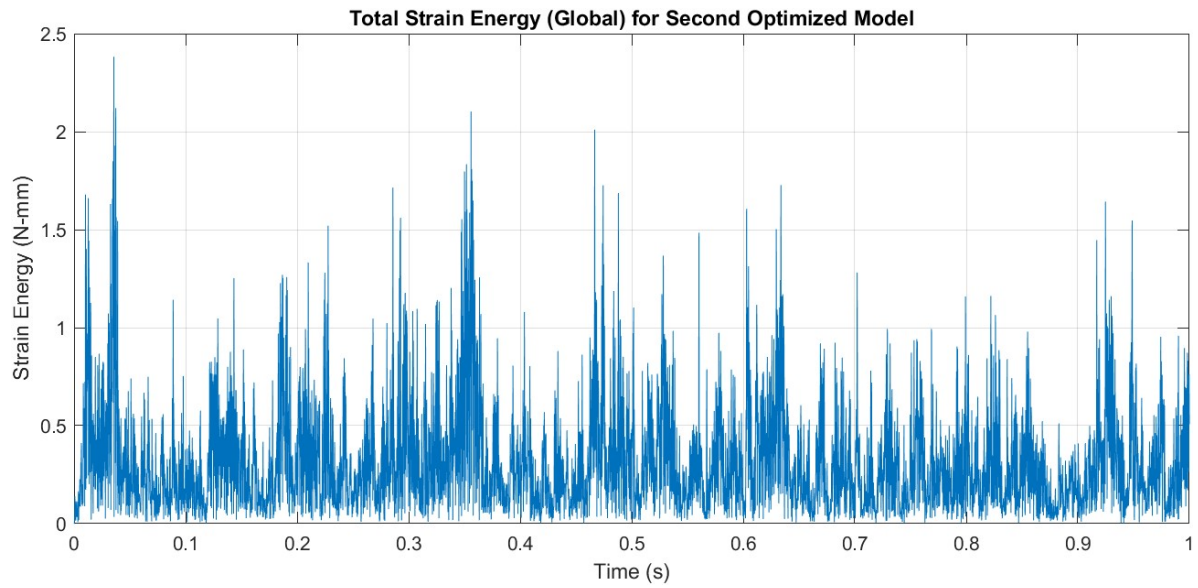


Fig. 14 Total strain energy over time (global) for second optimized model

Table 4 Summary of whole model strain energy progression throughout optimization process

Model iteration	Load description	Maximum whole model strain energy (N-mm)
Baseline model (solid uniform reinforcement)	ESL 1 (from initial time history analysis with Baseline Model)	2.161
First-round optimization model	ESL 1	2.126
First-round optimization model	ESL 2 (from time history analysis with first-round optimization model)	2.262
Second-round optimization model	ESL 2	2.236
Second-round optimization model	ESL 3 (from time history analysis with second-round optimization model)	2.383

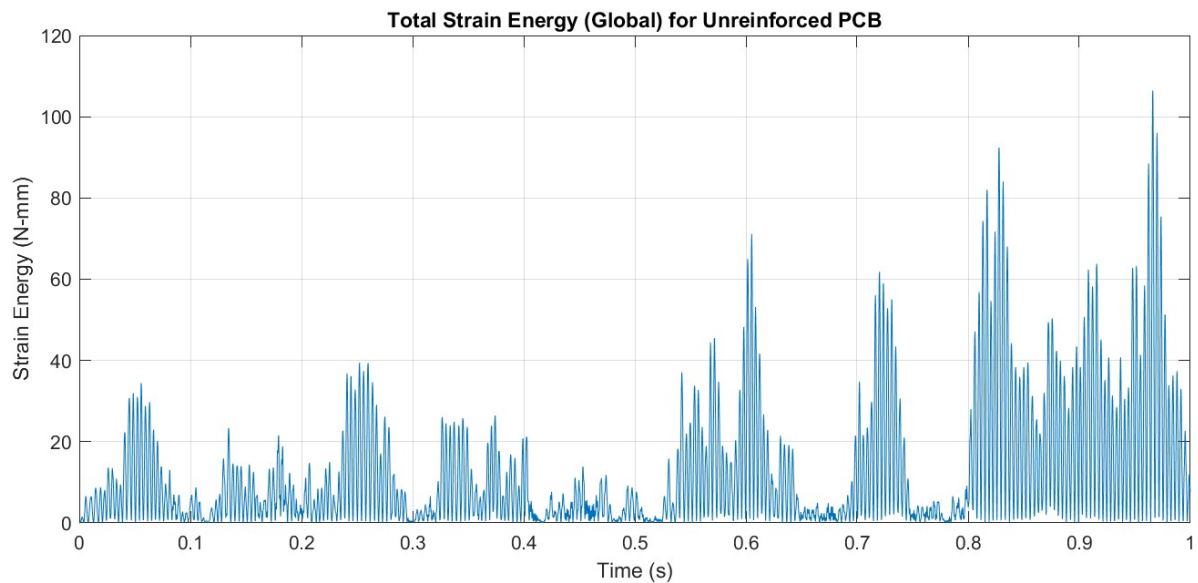


Fig. 15 Total strain energy over time (global) for initial MDOF model without reinforcement

17. Yang, Ping & Li, Wei. (2009). Numerical analysis on thermal characteristics for chip scale package by integrating 2D/3D models. *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*. 22. 43 - 55. 10.1002/jnm.694.
18. Gu Jie, Lim, C. T., & Tay, A. A. O. (2004). Modeling of solder joint failure due to PCB bending during drop impact. *Proceedings of 6th Electronics Packaging Technology Conference (EPTC 2004) (IEEE Cat. No.04EX971)*, 678–683. <https://doi.org/10.1109/eptc.2004.1396694>
19. Crowder, J.: Vibration fatigue of leaded solder joint interconnections for PCB electronics. M.S. thesis. Clemson University (2023)
20. AISI Type 316L Stainless Steel, annealed bar. ASM Material Data Sheet. (2022, March 22). <https://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MQ316Q>
21. Tümer, D., Güngörürler, M., Havıtcıoğlu, H., & Arman, Y. (2020). Investigation of effective coating of the Ti–6Al–4V alloy and 316L stainless steel with graphene or carbon nanotubes with finite element methods. *Journal of Materials Research and Technology*, 9(6), 15880–15893. <https://doi.org/10.1016/j.jmrt.2020.11.052>
22. NASA, G. S. F. C., “General Environmental Verification Standard (GEVS) STD 7000B,” Tech. rep., NASA GSFC-STD-7000B, 28 April, 2021.
23. Irvine, T. (2001). *Unit12* [PDF]. Vibrationdata. https://www.vibrationdata.com/Course_Units/UNIT12.pdf
24. Irvine, T. (2005). *THE HALF POWER BANDWIDTH METHOD FOR DAMPING CALCULATION* [PDF]. Vibrationdata. https://www.vibrationdata.com/tutorials2/half_power_bandwidth.pdf



Chapter 4

Towards a Digital Twin for the Condition Based Maintenance of a Hawk T1A Aircraft

Matt Tipuric, Matthew S. Bonney, Robin Mills, and David J. Wagg

Abstract Condition based maintenance for aircraft has major potential benefits for both reduction of cost and extension of operational hours. As this strategy relies on using live data to update a virtual representation of a system in order to make operational decisions, the relevance of digital twins is clear. This paper discusses the early-stage development of such a digital twin using vibration data collected from an ex-service Hawk T1A aircraft. The primary output of this research – the dataset itself as a benchmark for structural health monitoring – is detailed elsewhere. However, for digital twins to be successfully developed and employed, the holistic development of the entire cyber-physical system must be considered. The practical aspects of methodologies and frameworks for all parts of the system must be developed and refined. This paper discusses a number of these practical implementations within the context of the Hawk data, including sensor choice and placement, data pipelines, virtual models of legacy systems, and platform design. Based on this discussion, recommendations are made, both for immediate improvements of the data collection process and for the longer-term development of a practical operational digital twin.

Keywords digital twin · aircraft · SHM

Introduction

Digital twins are a technology with the potential to be transformational across multiple domains, both within engineering disciplines and more broadly. Such domains include manufacturing, healthcare, asset management, smart cities, and earth sciences. While the multiple definitions of a digital twin have been proposed over the years since its conception, a broadly applicable definition is *a virtual representation of a physical system with the capacity for bidirectional information exchange in relevant time between the digital and physical components*. Digital twins find use both during the design of new assets and across the lifecycle of existing assets - during the *product design phase* and the *asset management phase*, respectively [1].

One common research field that can utilise digital twins in both the product design and asset management phases is structural dynamics. The verification and validation of models using prototypes is vital for the design of critical components and assemblies, while condition monitoring allows for effective maintenance scheduling, life-time extension, and crisis management. None of these areas inherently require the use of digital twins. However, if well implemented, digital twins can act as enablers for all these areas by reducing siloing, simplifying workflows and allowing for live monitoring of assets.

These uses of digital twins make them particularly attractive in the aerospace sector. Aircraft have complex design requirements, with aerodynamic, lightweight geometries and components being exposed to hostile conditions. In addition to this hostile environment, these systems experience long service lives, often more than 30 years, and as a result are expensive to purchase and repair. They have the potential to fail catastrophically and so are required to undergo frequent and rigorous inspection and maintenance. Digital twins have the potential to improve each of these areas, improving the validation of

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models at the design phases and increasing the efficiency of in-service condition monitoring. However, much work is required before the practical implementation of digital twins for structural health monitoring at scale.

This paper focuses on an ex-service Hawk T1A aircraft, which underwent dynamic testing in the Laboratory for Verification and Validation (LVV) in the University of Sheffield. The primary focus of this work was the generation of high quality, publicly available data sets to support the future development of condition monitoring techniques. The data sets themselves - both of the starboard wing [2] and of the whole structure [3] - are described in detail in other papers which will not be duplicated here, beyond the small amount which is required for comprehension. A secondary goal of this work was to capture the difficulties in collecting and working with datasets at this scale, in support of the practical development of digital twins. These ancillary parts of the process are the focus of this current paper.

The Structure and Models

The BAE Systems Hawk T1A aircraft ('the Hawk') is a two-seater aircraft which was previously used by the British Royal Air Force and was previously used for advanced pilot training. After its retirement from service, it was donated to the University of Sheffield by the Defence Science and Technology Laboratory and is now housed at the LVV. It is 12.4m (40.7 feet) in length, with a wingspan of 9.1m (29.9 feet).

For safety and security reasons, a number of components were removed before the Hawk's arrival on site. These include the cockpit canopy, seats and instrumentation, the engines, and all electronics. Additionally a number of panels are missing, including some of the port wing, the flaps and rudder, and a substantial amount of the upper, rear fuselage, as shown in Fig. 1. The Hawk rests on its wheels on the strong floor of the LVV.

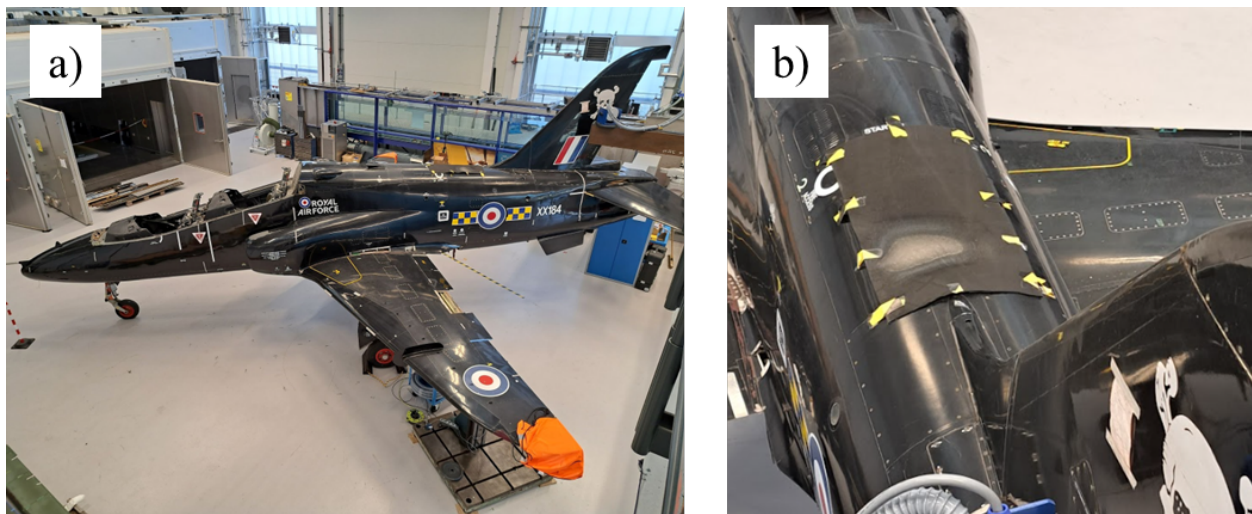


Fig. 1 a) The Hawk in situ and b) detail showing the missing upper fuselage, covered with fabric

The hawk was provided without documentation or a service history. All measurements and models included in this paper

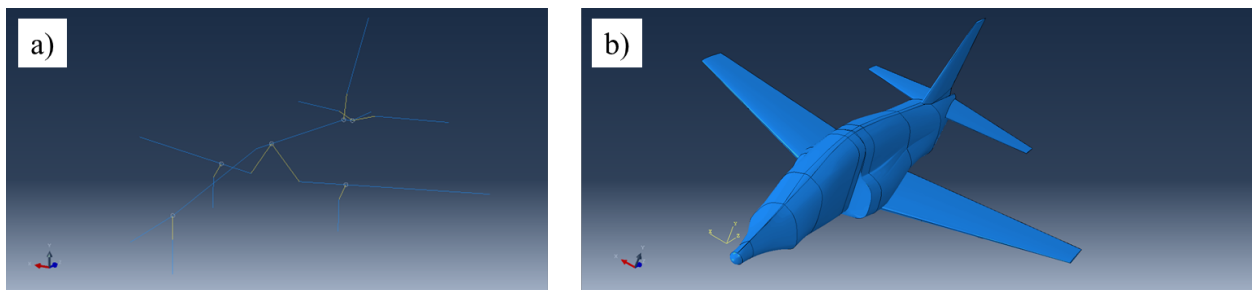


Fig. 2 a) The beam and b) shell CAD models of the hawk

are either from open sources or were collected during the project. To demonstrate digital twin capabilities, two computer aided design models were created of the Hawk in ABAQUS - a beam model and a rough shell model, shown in Fig. 2. The latter was also used as the basis of a 3D printed physical model of the Hawk, which aided discussion in the testing design phase.

Test Design

The Hawk was equipped with five shakers: one on each wing, one on each stabiliser, and one on the rudder. A total of 82 uniaxial accelerometers were applied to the fuselage, wings, stabilisers, rudder, and landing gear, as well as three triaxial accelerometers at the nose, cockpit and tail. Ultimately, the limitations were the number of channels available and the availability of the sensors themselves. An additional triaxial accelerometer was attached to the floor below the centre of the aircraft to detect ground vibration, while ambient temperature and sound were captured using a resistance temperature detector and microphone, respectively. Full detail about the test setup, including schematic diagrams of the locations of all sensors and shakers, can be found in Reference [3].

Fibre Bragg grating (FBG) was also applied along the lengths of the wings, stabilisers, and rudder. The reasoning for this was that FBG is lighter and less bulky than accelerometers. If damage could be successfully detected using the signals either the FBG and a reduced number of accelerometers (or even from the FBG alone) then this might present a more practical option for in-operation sensing.

The placement of the accelerometers was motivated by sensor placement optimisation [4]. Two separate methods were trialled: in this case effective independence [5] was found to be too computationally expensive and so the modal kinetic energy method [6] was used. The approximate nature of the underlying model (especially the fact it only represented the shell of the Hawk) meant that the applicability of the actual sensor locations calculated through this method was inherently limited. However, it did provide value by indicating a suitable distribution of the available sensors (i.e. how many should be placed on the wings compared to the tail). Expert knowledge, both from general experience with vibration testing and specific experience from the wing-only tests then guided the final placement of the sensors.

Data Collection, Storage, and Dissemination

The testing regime consisted of five phases, detailed in Ref. 3:

1. The Hawk in its normal, undamaged condition, using both white and pink noise inputs.
2. Single site pseudo-damage, using mass addition at one of fifteen locations and white noise input.
3. The Hawk in its undamaged condition, using odd random-phase multisine excitation.
4. Multi-site pseudo-damage with mass addition at up to three of the fifteen sites at once, using white noise input.
5. Damage test through the removal of panels on the port wing

A schematic of the test setup is shown in Fig. 3. The control equipment was operated using National Instruments (NI) Labview on the test PC. An analog output (A/O) card interfaced to the shaker amplifiers. The shakers themselves were then connected to the Hawk through stingers and load cells, with the load cells connecting back to the test PC through an analog input card. Analog input cards were also connected to the accelerometers, temperature and microphone, allowing the data from these sources to be captured through Labview synchronously with the output. A separate interrogator unit digitised the data from the FBGs and outputted it to the PC, where it was captured in a separate file to the NI data (although it was still acquired by the labview code). A synchronisation pulse was captured in all data files to ensure data alignment.

The multiple input, multiple output (MIMO) testing required significant effort at the start of each series of tests in order for each shaker to deliver the required signal. First, a breakpoint table was derived for each test sequence, describing the amplitude of the required excitation at each location. This was then resampled to create a breakpoint mask for each frequency line which, alongside a random phase, defined the sine wave for each frequency line, which combined to form the time domain signal. Each shaker was then excited, the response of the load cell compared to its expected value, and the gain mask updated accordingly. This process was then iterated until the target and measured values were within the specified tolerance of 4% and this final drive signal was used to gather the test data. New drive signals were generated using this method whenever any significant change might have occurred to the structure (i.e. at the start of each day and whenever masses or panels were added or removed) or whenever a different input signal was required.

Tests were logged using a Google sheet, in order to keep an up-to-date list across the multiple researchers involved, along with a Google form for redundancy and to capture any research notes. Data was then uploaded to the shared Google drive at the end of each day, as a backup and to allow for verification.

A total of 216 tests were conducted, creating in excess of 500 GB of raw data. In order to ease the practical challenges associated with working with a dataset of this scale, the dataset is packaged using the *hierarchical data format* (.hdf5), which allows the data and metadata to be stored concurrently. In addition, a simple interface was developed in Python to aid users. This API first checks for local versions of the requested files and then, if not available, accesses the data from a central repository. This means that users only download the exact data required and avoids data erosion through repeated sharing of the data.

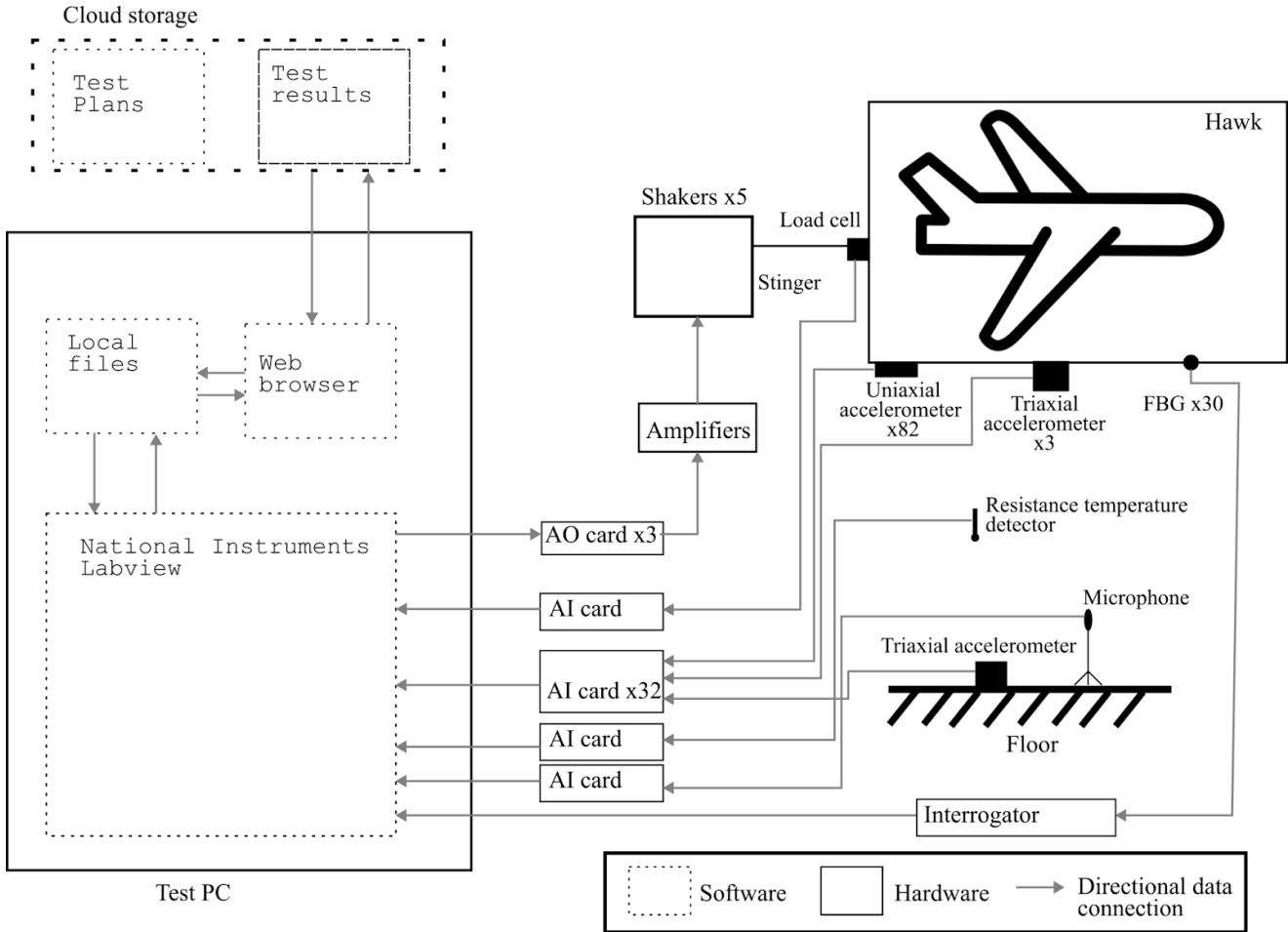


Fig. 3 A schematic of the test setup, showing both software and hardware components

Digital Twin Online Platform

An implementation of the Digital Twin Operational Platform (DTOP) [7] was used to demonstrate some digital twin capabilities. DTOP is an open source, browser-based digital twin architecture developed using Python and Flask. The DTOP platform enables the creation of a web-page or web-site, containing data and models of the physical twin, to act as a single source of truth. Currently these web-pages are created and deployed locally. In this instance, the DTOP interfaces with the data asynchronously, with real-time data linkage being a topic for future work.

The specific implementation of the DTOP created for the Hawk is nicknamed DTHive and is shown in Fig. 4. The DTHive webpages offer the user two capabilities. The first is access to the data from the experimental tests. The second capability is interfacing with the ABAQUS finite element analysis software to investigate and update the CAD models. This



Fig. 4 The landing page of DTHive

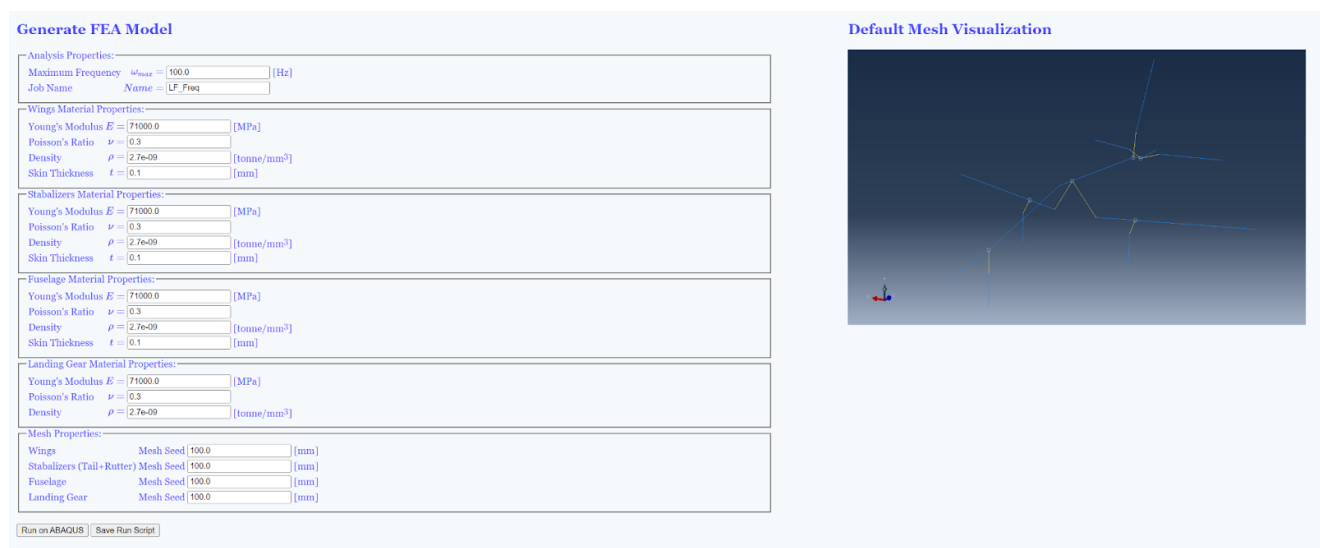


Fig. 5 The beam model generation page of DTHive

requires the user to have a local, licenced copy of ABAQUS. Running the script through the interface in Fig. 5 allows the user to open either the beam or the shell models from the DTOP, edit the material properties and load the resulting file into ABAQUS.

Data

Figure 6 shows an indicative sample of the data collected using the accelerometers and FBG sensors, respectively. An example of some pseudo damage data is shown in Fig. 7, which shows the response of an accelerometer on the starboard stabiliser to white noise excitation with the addition of the largest mass (M1, 245.3g) and smallest mass (M7, 64.4g), as well as without mass addition. Further detail can be found in Reference [3], which also includes a number of motivating research

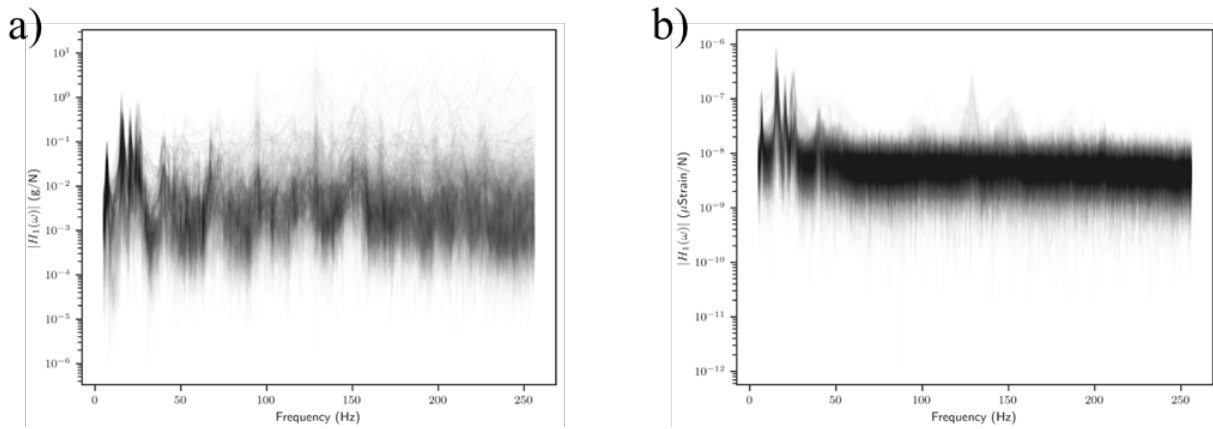


Fig. 6 Superimposition of the frequency response function magnitudes for white noise healthy state tests from a) all accelerometers and b) all FBGs

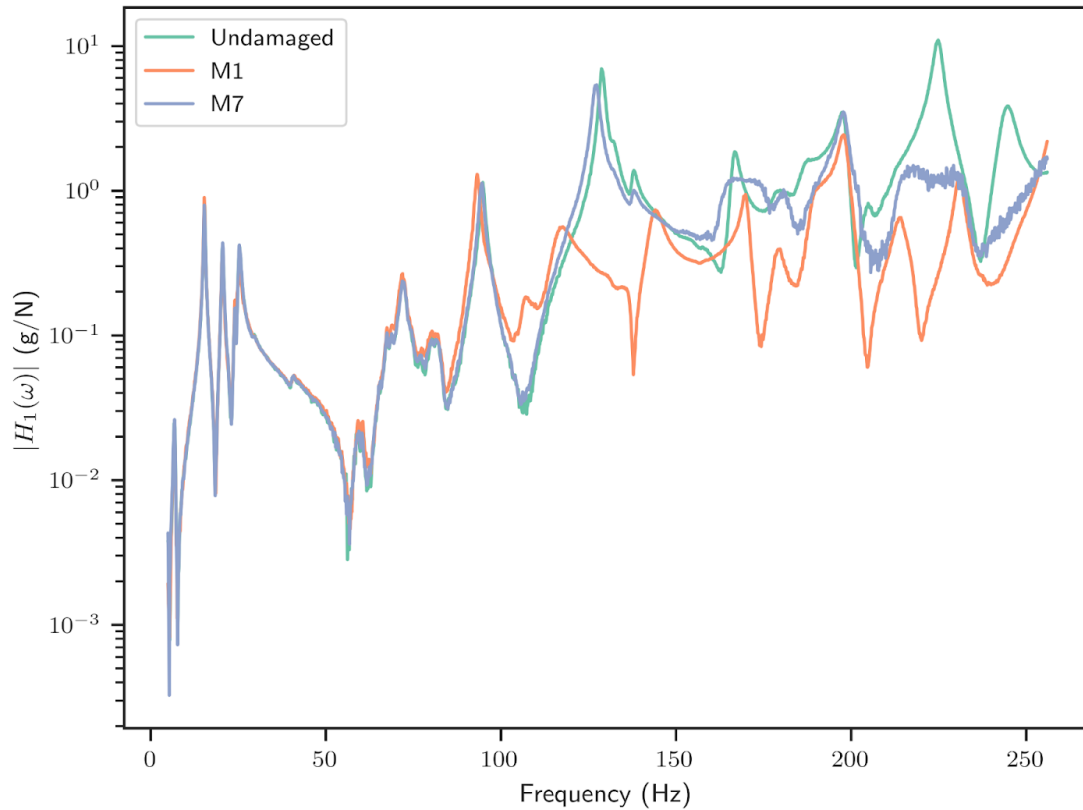


Fig. 7 Frequency response function magnitude of a single accelerometer on the starboard stabiliser in response to white noise input in an undamaged state and in the presence of two different masses

challenges. Of particular relevance to the discussion here are Challenge 11: Detecting pseudo-damage using subsets of the available data, Challenge 13: Detecting pseudo damage with exclusively the FBG data, and Challenge 14: Establishing the extent to which higher quality data is useful in the training phase, when only lower quality data is available during operation.

Discussion

The collection of the Hawk dataset highlights a number of key issues relating to the creation of digital twins. These largely relate to the nature of the purpose of the digital twin, and the identities of the users and data owner. While many aspects of digital twin design are widely applicable, as we move towards more industrially applicable demonstrators, the specifics of each of these aspects become more relevant and we are required to make assumptions about each. These assumptions radically affect the specifics across the design, including the sensor selection, data pipelines, modelling, and data visualisation.

For land-based structures, such as bridges and wind turbines, accelerometers and strain gauges are standard tools for monitoring structural dynamics. However, for aircraft, weight reduction and aerodynamics are key considerations. Attaching over 100 sensors to the external structure of an operational aircraft without significantly degrading performance would be at minimum a significant design challenge. Equally, attaching such a large quantity of sensors, along with their associated cabling, requires multiple days to achieve, which makes gathering as dense a dataset between flight hours impractical. These concerns motivated the inclusion of the fibre bragg grating in these tests, as these lower profile, lighter sensors might be more practical. However, resilient damage detection methods need to be shown using this FBG data. Similarly, if data from subsets of the accelerometers can be used for damage detection, then this would present a significant step towards the use of sparse data. The specific requirements of the digital twin matter here: if in-operation damage detection is required, then integration of light weight sensors is key; if instead the goal is to support the scheduling of on the ground maintenance, then effective sensor placement optimisation or investigation of other methods, such as the use of robotic laser vibrometry must be prioritised.

In both cases, the data collection and modelling are best supported by a high level of confidence in the knowledge of the structure in its undamaged state. The limitations of conducting sensor placement optimisation using the approximate models of the Hawk have already been discussed. High fidelity models also have value in other areas, such as interrogating the causes of nonlinearities in the underlying data. However, the generation of such models is not trivial, even with recent advances in 3D scanning, as is gaining a bill of materials for a sensitive legacy asset like the Hawk. If the involvement of the original equipment manufacturers in the creation of the digital twin can be assumed, then these issues become trivial. Otherwise, however, decisions will need to be made regarding the amount of effort to be spent on generating high quality computer models, and so the benefits need to be understood and well evidenced.

As is often the case with prototyping and experimental work, the data pipeline used here was low in automation, using immensely more human labour than would be practical in a real-world context. This includes both time in the set-up stage and during the testing itself. The requirement to check the attachment of the sensors manually, followed by a lengthy generation of the drive signals before each series of tests required a level of human involvement that precluded the automation desirable from a digital twin. Inclusion of a SCADAS style sensor integrity testing of accelerometers, as an example, could massively decrease the effort involved. Including data verification earlier in the pipeline would also be of major benefit, allowing for errors to be caught and tests to be re-run more quickly. However, given the nature of collecting a dataset that, by design, includes damage that is hard to distinguish, only so much verification can be automated. The number of edge cases mean that expert knowledge to distinguish when data “looks a bit off” is still required. These issues are included in Reference [3] as Challenge 2: Aiding (or reducing the reliance on) operator interpretation when collecting large datasets.

Many improvements were made in the testing protocol, building on the lessons learned from the single wing tests. One example is the inclusion of the microphone and resistance temperature detector. Having a comparable record of ambient testing conditions massively reduced the number of subjective, hard to quantify comments included alongside the data, as testers were no longer required to judge what levels of additional ambient sound were notable. As digital twin tools develop, this kind of ability for continuous improvement in workflows should be kept in mind, with tools designed to encourage them rather than lock in suboptimal workflows.

A final comment concerns data visualisation. In this project, relatively conventional data visualisation techniques have been used, as the dataset is designed to be used by subject matter experts. However, a major promise of digital twins is the reduction of silos and the greater dissemination of data across organisations. The ways in which structural dynamics data might be best conveyed to non-expert audiences will need to be studied, especially where it relates to uncertainty quantification and conveying the limits of what the data *can* show (i.e. avoiding overconfidence). As ever, the users and purposes need to be considered in all this - a ground technician crew will have very different understanding and requirements to a pilot in flight.

Conclusion

This paper has described the collection of the Hawk structural dataset from the perspective of digital twins. The structure, test design, test equipment and data collection were described in detail. A version of the digital twin online platform software representing the Hawk was also described. Finally, some discussion was given to the limitations of this testing as it relates to practical implementations of digital twins. This discussion focused on the ways in which digital twin design is necessarily motivated by the purpose of the twin, the identity of the user, and the identity of the data owner and covered aspects of sensor selection, the difficulties of creating digital twins of legacy structures, data pipeline design, test automation, the need for continuous improvement, and data visualisation.

Data Availability

All data collected is made freely available at: orda.shef.ac.uk/articles/dataset/BAE_T1A_Hawk_Full_Structure_Modal_Test/24948549. In addition, the authors make a python interface for interacting with the data available at: <https://github.com/MDCHAMP/hawk-data>.

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References

1. D. J. Wagg, K. Worden, R. J. Barthorpe, and P. Gardner, "Digital Twins: State-of-the-Art and Future Directions for Modeling and Simulation in Engineering Dynamics Applications," *ASCE-ASME journal of risk and uncertainty in engineering systems, Part B. Mechanical engineering*, vol. 6, no. 3, 2020, doi: [10.1115/1.4046739](https://doi.org/10.1115/1.4046739).
2. M. Haywood-Alexander et al., "Full-scale modal testing of a Hawk T1A aircraft for benchmarking vibration-based methods," *Journal of sound and vibration*, vol. 576, p. 118295, 2024, doi: [10.1016/j.jsv.2024.118295](https://doi.org/10.1016/j.jsv.2024.118295).
3. J. Wilson, M. D. Champneys, M. Tipuric, R. Mills, D. J. Wagg, and T. J. Rogers, "Multiple-input, multiple-output modal testing of a Hawk T1A aircraft: A new full-scale dataset for structural health monitoring," 2024, doi: [10.48550/arxiv.2406.04943](https://doi.org/10.48550/arxiv.2406.04943).
4. S. Hassani and U. Dackermann, "A Systematic Review of Optimization Algorithms for Structural Health Monitoring and Optimal Sensor Placement," *Sensors (Basel, Switzerland)*, vol. 23, no. 6, p. 3293, 2023, doi: [10.3390/s23063293](https://doi.org/10.3390/s23063293).
5. D. C. Kammer, "Sensor placement for on-orbit modal identification and correlation of large space structures," in *Proceedings of the American Control Conference*, 1990, pp. 2984–2990. doi: [10.23919/acc.1990.4791265](https://doi.org/10.23919/acc.1990.4791265).
6. M. Meo and G. Zumpano, "On the optimal sensor placement techniques for a bridge structure," *Engineering structures*, vol. 27, no. 10, pp. 1488–1497, 2005, doi: [10.1016/j.engstruct.2005.03.015](https://doi.org/10.1016/j.engstruct.2005.03.015).
7. M. S. Bonney et al., "Development of a digital twin operational platform using Python Flask," *Data-Centric Engineering (Online)*, vol. 3, no. 1, 2022, doi: [10.1017/dce.2022.1](https://doi.org/10.1017/dce.2022.1).



Chapter 5

Operational Modal Analysis of Satellites using Direct-field Acoustic Test Data

Bart Peeters, Alberto Garcia de Miguel, Umberto Musella, and Ivan Ngan

Abstract During acoustic qualification testing of satellites, the structural responses are being monitored and measured. The main objective of this paper is to explore the application of Operational Modal Analysis (OMA) to this structural response data. Acoustic qualification testing happens either in a reverberation chamber or by using so-called direct field testing where the test object is surrounded by many loudspeakers. This paper reports on a study involving the Structural and Thermal Model (STM) of the PROBA-V satellite. Several impact, compact shaker and direct field acoustic noise (DFAN) tests have been executed on this structure to study the influence of different test conditions on the control quality of a direct field acoustic environmental test and to identify the dynamic properties of the structure.

The purpose of the paper is to track and compare the modal parameters of the specimen during various testing phases for an enhanced understanding of the structural behaviour under acoustic loading. Classical Experimental Modal Analysis (EMA) will be applied to data from preliminary tap and compact shaker tests and Operational Modal Analysis will be applied to data from DFAN tests. During these DFAN tests, the structure is excited by a well-controlled acoustic field, but the forces acting on the satellite are not measurable and therefore structural Frequency Response Functions are not available which means that output-only OMA needs to be applied instead of input-output EMA.

Finally, the modal test results are compared with the modal parameters from a Finite Element Model of the satellite.

Keywords Operational modal analysis · satellite testing · direct-field acoustic test

Introduction

During a recently organized DFAN Master Class which had as objective to gather and disseminate know-how and experience related to spacecraft direct field acoustic testing, ESA made available the Structural and Thermal Model (STM) of the PROBA-V satellite. Several acoustic and tap tests have been executed on this structure to study the influence of different test conditions on the control quality of a direct field acoustic environmental test and to identify the dynamic properties of the structure.

Environmental testing is a necessity to ensure the survivability of hardware during launch of space vehicles. Particularly during the ignition and liftoff launch stages the main source of dynamic loading is characterized by a non-deterministic acoustic environment. Hence, the environmental acoustic test becomes a fundamental qualification test to evaluate if an aerospace structure (e.g. payload), system or component can withstand the vibratory load it will encounter during its operational life. DFAN testing is a well-established technology to perform acoustic qualification of satellites and large components such as antennas or solar panels [1]. The background, motivation and advanced possibilities of DFAN testing can be found in [2, 3].

This paper is not related to DFAN testing methodologies as such, but to the exploitation of the structural vibration data acquired during these types of tests by means of Operational Modal Analysis. Together with the pressure responses measured

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for acoustic control purposes, the structural responses, typically accelerations and strains, are also acquired to ensure the safety of the structure and validate its performance against the acoustic excitation. Often, the spacecraft is instrumented with a large number of sensors, requiring large efforts in terms of test preparation. The application of OMA creates an additional use case for this data by gaining more insights in the structural behaviour of the spacecraft. Recently, OMA was successfully applied to an antenna reflector during a DFAN test [4]. The objective of the present paper is to explore in how far this technology can be expanded towards a full satellite.

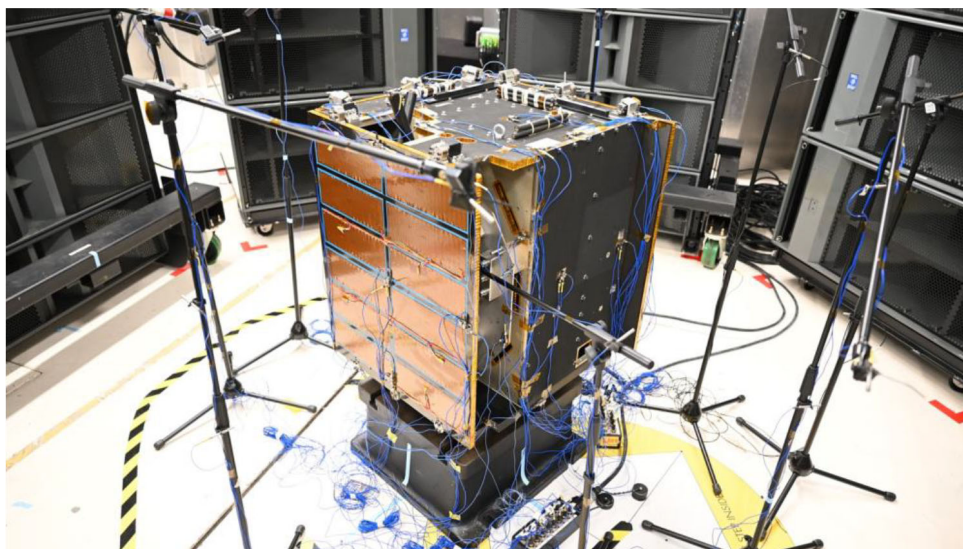


Fig. 1 PROBA-V STM instrumented with sensors and surrounded by microphones for capturing and controlling the sound field at the test facilities of Siemens Industry Software NV, Leuven, Belgium. Courtesy of ESA and Redwire Space NV

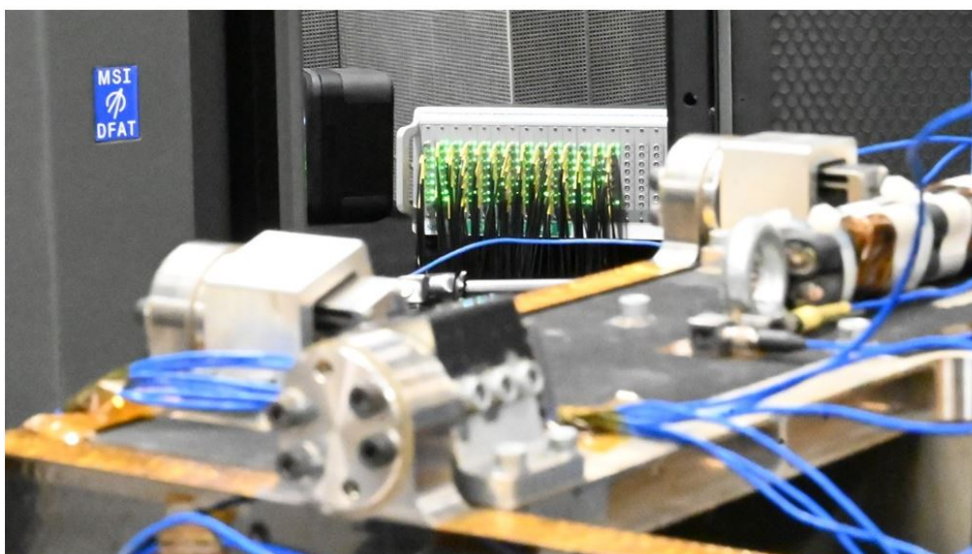


Fig. 2 In Background: Simcenter SCADAS Lab controller and data acquisition system; and MSI-DFAT loudspeakers. Courtesy of ESA and Redwire Space NV

Test Set-up

The DFAN test campaign was conducted in March 2024 in the Siemens facilities in Leuven, Belgium (Fig. 1). The loud-speaker equipment, or Noise Generation System (NGS), was provided by MSI-DFAT (Fig. 2) and installed inside a large semi-anechoic chamber. It consisted of 4 stacks of 3 subwoofers and 4 stacks of six 3-way cabinets distributed in a circle of 4.5 m diameter. Two Simcenter SCADAS Lab systems have been used, one providing DFAN MIMO acoustic control as implemented in Simcenter Testlab software, and the other acquiring structural response data (Fig. 2).

A total of 25 free-field microphones were installed around the test specimen, 22 of them used as control candidates (Fig. 3). The latter were placed at random positions between the loudspeakers and the satellite surfaces following the recommendations of [1]. The DFAN tests considered in this paper were executed using 18 control microphones and the 4 remaining ones served as additional monitoring microphones. The final selection of control microphones was done based on a pre-test analysis aiming at maximizing the uniformity of the acoustic field [5].

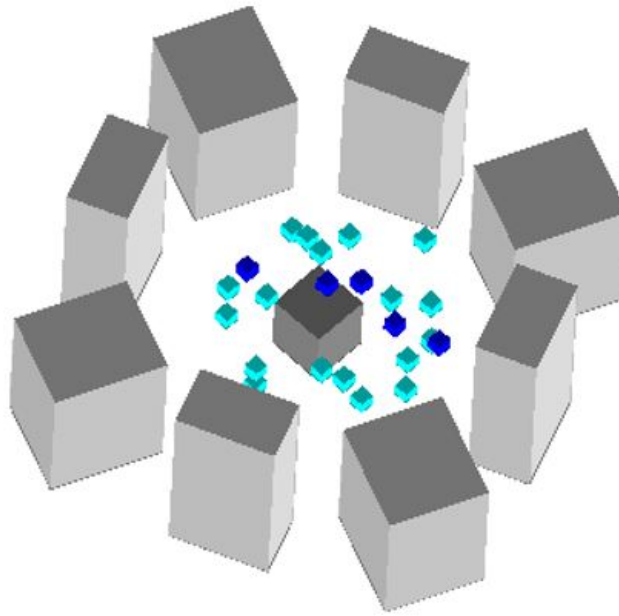


Fig. 3 DFAN test configuration. The blue markers indicate the microphone positions

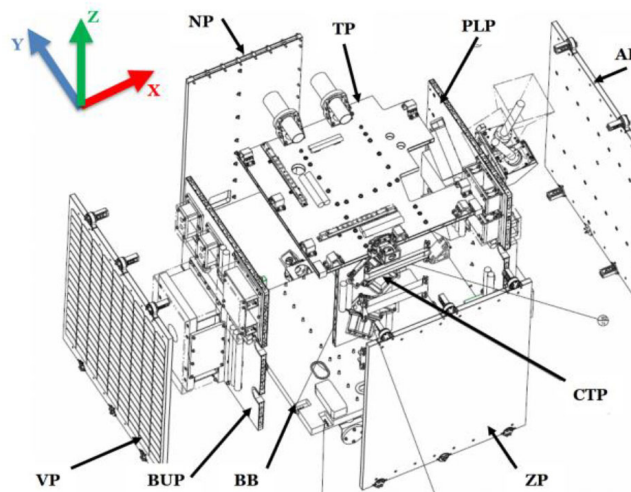


Fig. 4 Exploded view of the PROBA-V STM [6]. Courtesy of ESA and Redwire Space NV

The satellite was instrumented with more than 100 accelerometer channels, which will be heavily exploited in this paper. An exploded view of the PROBA-V with the naming of the different panels is provided in Fig. 4, reproduced from [6]. More details on the DFAN test set-up can be found in [2].

Impact (tap) Testing, Compact Shaker Testing and Experimental Modal Analysis

After the satellite instrumentation was completed and before the DFAN tests were run, preliminary impact testing (sometimes also called tap testing) was performed. In this study, the objective of these tests is to obtain a baseline set of satellite modal parameters for comparison with the results obtained during the DFAN tests. In addition, also shaker testing was performed, not with a large shock & vibration shaker, but a compact Simcenter Qsources Integral shaker attached to the satellite.

A (gentle) impact was applied at a “robust” location, i.e. at the intersection of 3 panels (Fig. 4): TP, NP, PLP, of the satellite in both the X- and Y-direction (Fig. 5, Left). For each direction, the impact was repeated 15 times, allowing for data averaging and coherence calculations. The Simcenter Qsources Integral shaker was attached at approximately the same location as the impact location (Fig. 5, Right). The shaker has a miniature size, ideal for hard-to-reach locations. It is self-supporting and self-aligning; has an integrated force cell and accelerometer; and covers a wide excitation frequency range: 20-2000Hz. The highly controllable shaker forces are attractive in cases where one may not be comfortable applying impact excitation to a satellite.

The driving point FRFs (in X-direction) and corresponding coherences are represented in Fig. 6. For the case of shaker excitation (Fig. 6, Right), also the shaker force PSD is shown. A band-limited (20-300 Hz) pseudo-random excitation signal

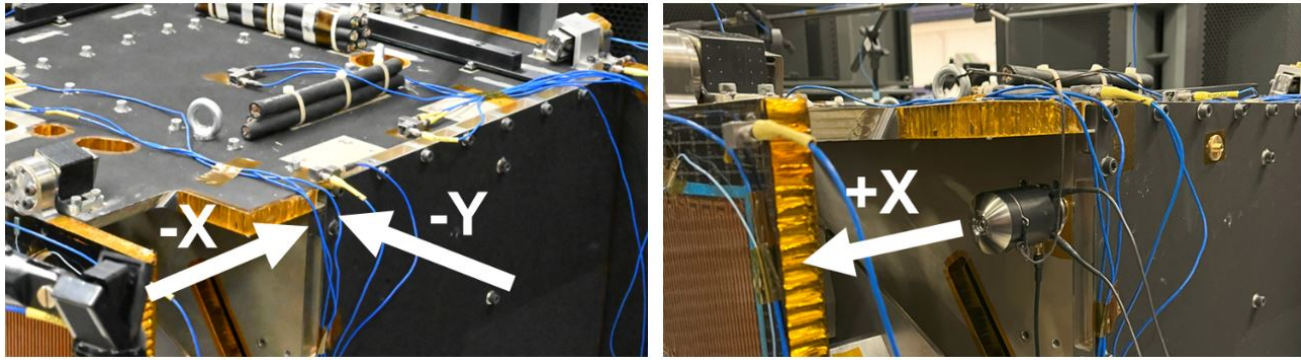


Fig. 5 (Left) Location and direction of the impacts; (Right) Location and direction of the Simcenter Qsources Integral shaker. Courtesy of ESA and Redwire Space NV

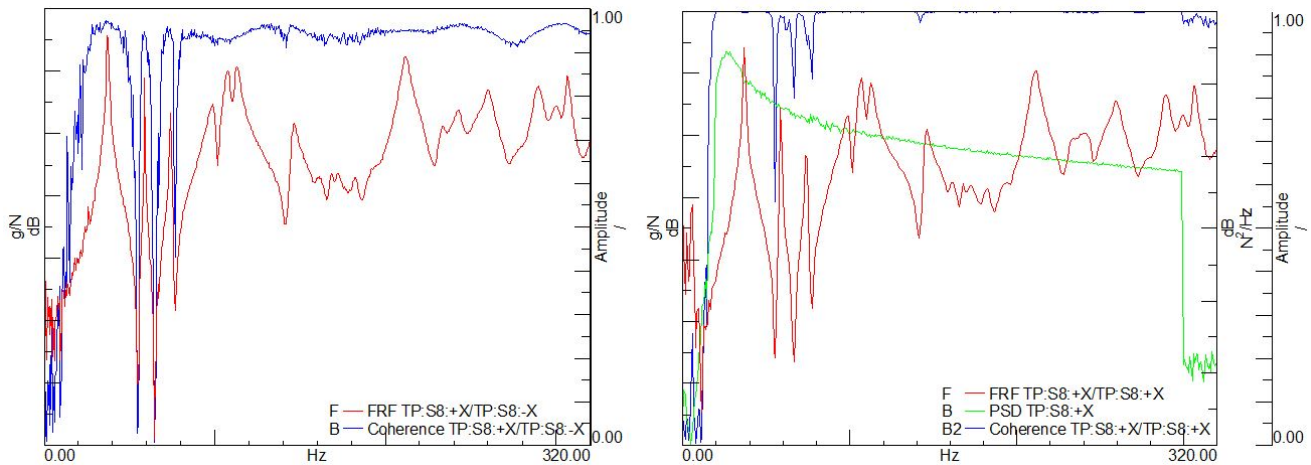


Fig. 6 PROBA-V modal testing. (Left) Impact testing: driving point FRF (red) and corresponding coherence (blue); (Right) Qsources testing: driving point FRF (red), corresponding coherence (blue), and shaker force input PSD (green)

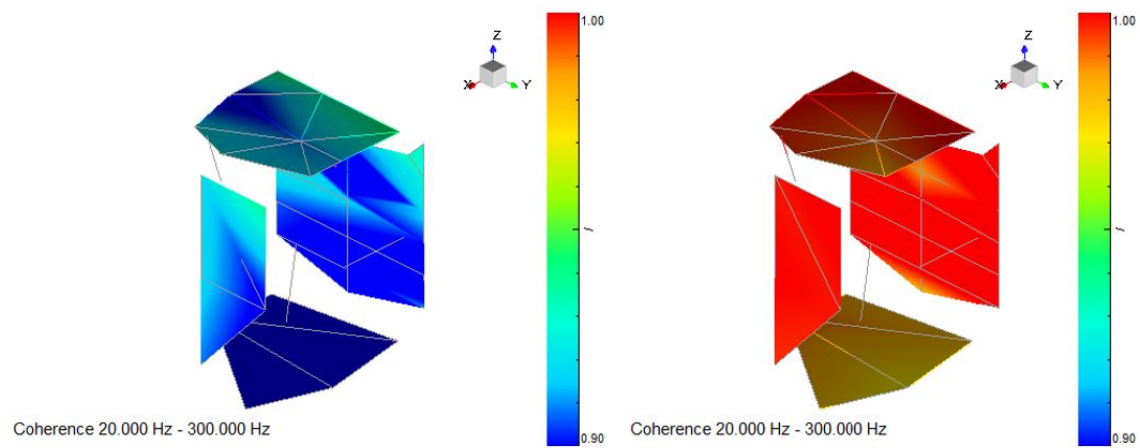


Fig. 7 PROBA-V modal testing – averaged coherences 20-300Hz plotted on the geometry (color scale is between 0.9 and 1). (Left) Impact testing; (Right) Qsources testing

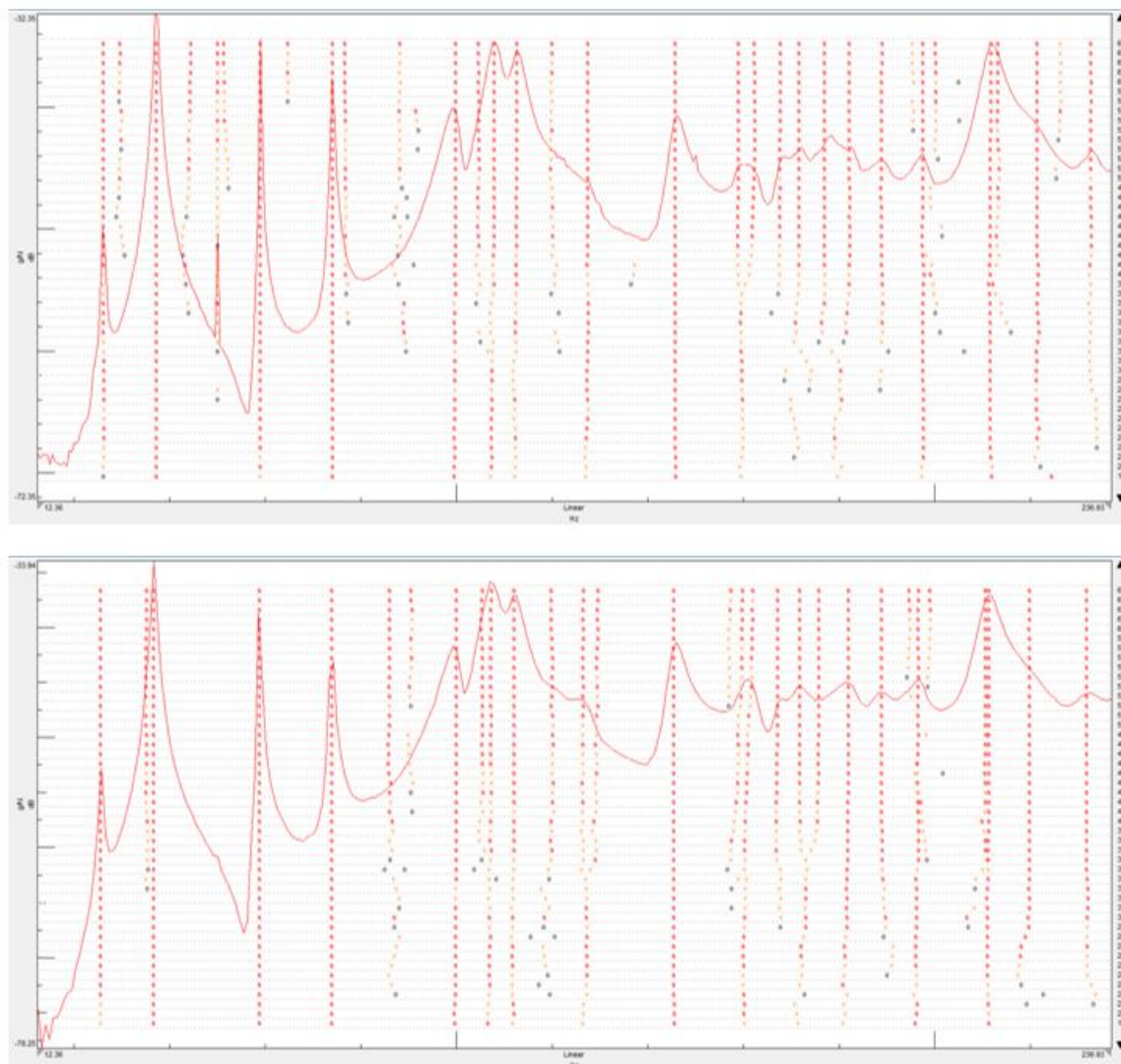


Fig. 8 PROBA-V modal testing – Polymax stabilization diagrams. (Top) Impact testing; (Bottom) Qsources testing

was used. Resonances are clearly visible in the FRFs and the coherences are good in the frequency range of interest, although there is a clear difference between impact and shaker excitation as can be seen in Fig. 7: for all measurement locations, the shaker coherences are superior. Polymax [7] was applied in a frequency range from 12 to 237 Hz. Fig. 8 shows the stabilization diagrams that are used to select the modes. Afterwards, the MLMM method [8] was run for 30 iterations to further improve the initial modal parameter estimates. MLMM is an optimization method in which the difference between measured and synthesized FRFs is minimized. Both Polymax and MLMM are implemented in Simcenter Testlab. The shaker tests took place 15 days after the impact tests. Overall, the resonance frequencies slightly decreased (on average with 0.5%).

A typical result is presented in Fig. 9 where a very good fit between measured and synthesized driving point FRFs can be observed; both for the impact and the shaker data. The conclusion of this section is that Experimental Modal Analysis (EMA) applied to the impact and shaker FRFs yields a high-quality baseline set of modal parameters. The modes themselves will be presented in next section together with the Operational Modal Analysis (OMA) results (Fig. 20, Table 1).

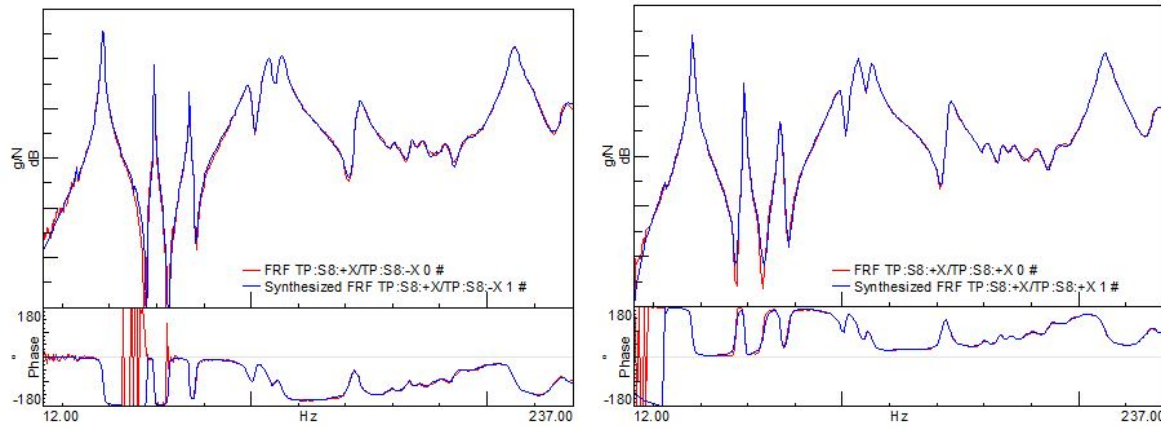


Fig. 9 PROBA-V modal testing – measured (red) and synthesized (blue) driving point FRF. (Left) Impact testing; (Right) Qsources testing

Direct Field Acoustic testing and Operational Modal Analysis

DFAN Tests with Different Number of Drives

DFAN tests have been conducted with different number of drive signals: 1, 2, 4 and 8. In this manner, different acoustic fields are generated to excite the structure, i.e. from a fully correlated field (1 drive) to a highly diffuse field (8 drives). The acoustic field can be assessed using established metrics for DFAN testing [9]. Whereas all 4 tests were able to maintain a stable OASPL of 125 dB over time and achieved a $1/3^{rd}$ octave spatially averaged SPL close to the reference profile and within the set tolerances (± 3 dB) in all bands, the acoustic field is quite different as indicated in Figure 10 and Fig. 11. The dispersion plot (Fig. 10) includes the responses of all the 22 microphones selected as control candidates, i.e. the 18 finally selected via pre-test analysis plus 4 remaining ones left as monitor sensors to check the acoustic fields in uncontrolled locations. The “dispersion” is the difference in dB between the individual microphones and the spatial average of the control microphones. It can be observed that the uniformity of the acoustic field improves when increasing the number of drives from 1 to 8. Figure 11 shows the “Sinc Indicator Function” (SIF) introduced in [10] as a diffusivity metric. The SIF has a value between 0 (bad) and 1 (good) and is based on the deviation of the measured pairwise microphone coherences from the theoretically expected coherences in a perfectly diffuse field. When increasing the number of drives from 1 to 8, the SIF, averaged over the frequency range, significantly improved from 0.15 to 0.84.

Also, the structural responses are quite different in the 4 different tests (Fig. 12). Following observations can be made:

- Results from tests with 1 and 2 drives are relatively similar to each other, as well as results from tests with 4 and 8 drives, but both groups differ considerably from each other.
- There is for instance a 6 dB difference in level at the frequency of the 1st bending mode of the AP panel.
- Despite the amplitude level differences, the frequency content is quite similar across the 4 tests. This qualitative observation is confirmed by the OMA analysis of the 4 tests: the same structural modes are found back for all cases. In the following, we will then only focus on the 8-drives case.

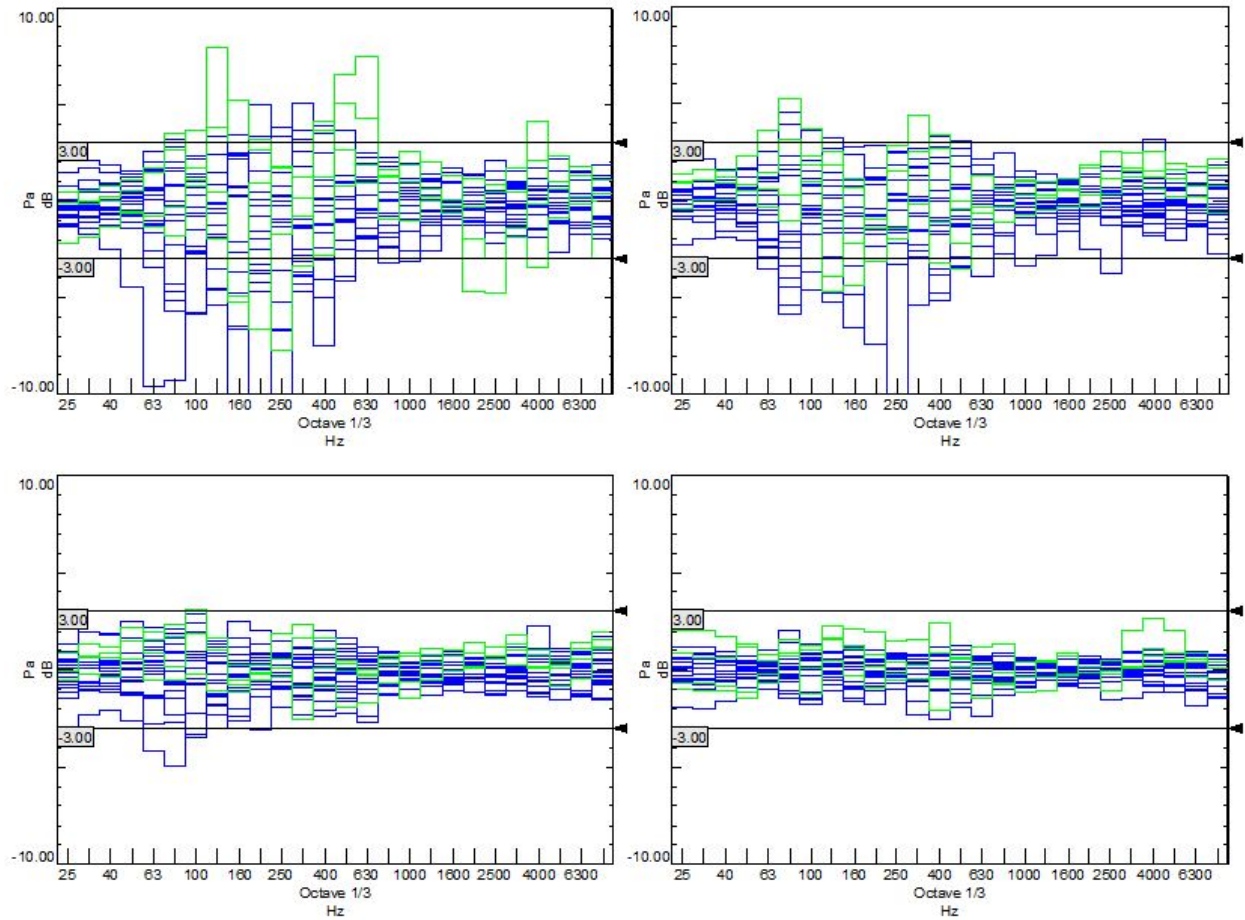


Fig. 10 Dispersion at control (blue) and measurement (green) microphones. (Top) 1 and 2 drives; (Bottom) 4 and 8 drives

Some more observations on the influence of the number of drives on the PROBA-V structural responses can be found in [2].

Operational Modal Analysis

The Operational Modal Analysis (OMA) variant of Polymax [11] as implemented in Simcenter Testlab will be used to estimate the modes from the DFAN data. A detailed discussion on data pre-processing for Operational Modal Analysis can be found in [11] and references therein.

First auto- and cross correlations are computed from the time domain data. In an OMA setting, the response correlations behave like impulse responses. So, just like in impact testing, an exponential window can be applied to the correlations before computing the Fourier transform. The exponential window reduces the effect of leakage and the influence of the higher time lags, which have a larger variance. Moreover, the application of an exponential window to impulse responses or correlations is compatible with the modal model and the pole estimates can be corrected.

As part of the pre-processing, a selection needs to be made on a subset of the sensors that will serve as so-called “reference sensors” for the cross-correlation and cross-spectra calculations. For a successful OMA, it is not needed that the full cross spectrum matrix between all available sensors is calculated. It suffices to calculate a subset of columns of this matrix: the cross-spectra between all sensors and the selected reference sensors. It is important that the ensemble of reference sensors can observe all modes of interest. For the PROBA-V, the X-, Y-, and Z- channel of a triaxial sensor at the TP panel were selected as reference channels (Fig. 13).

Figures 14 and 15 illustrate the pre-processing for OMA. 120 s of full-level DFAN data (Fig. 14, Top) is reduced to a correlation function that decays in less than a second (Fig. 14, Bottom). A single-block Fourier transform is then applied to these correlations to obtain the so-called “half spectra” represented in Fig. 15.

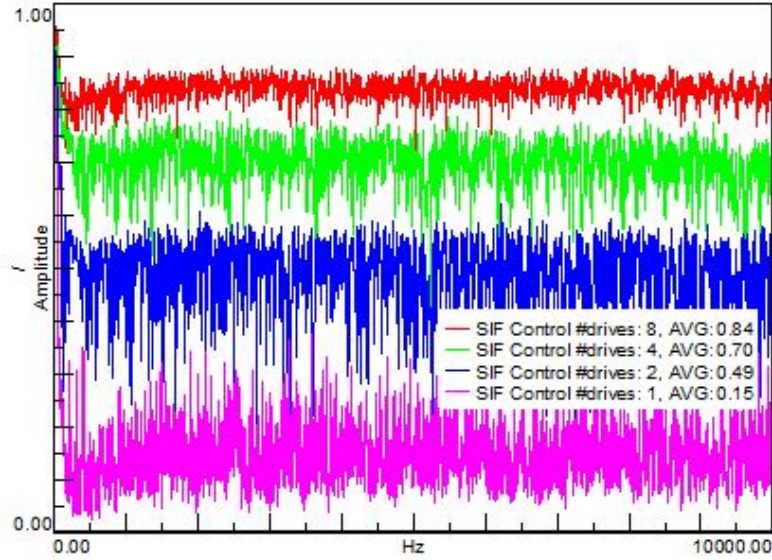


Fig. 11 SIF for the DFAN tests with different number of drives: 8, 4, 2 and 1

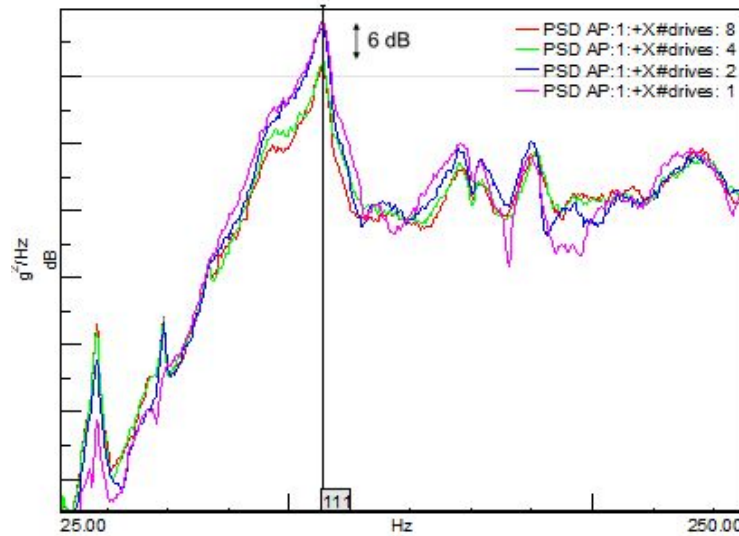


Fig. 12 Acceleration PSD at centre location of the AP panel for different number of drives: 8, 4, 2 and 1. The cursor indicates the frequency of the 1st bending mode of the AP panel

Theoretically, OMA relies on the assumption that the excitation is white noise. In such cases, it is not needed to measure the excitation sources, and the response measurements are sufficient to characterize the system (up to the input-output scaling). In practice, the white noise assumption can be relaxed a bit and OMA still provides good results when the (unknown) excitation spectrum is relatively “smooth”. A closed-loop test like DFAN typically does not satisfy the white-noise assumption, but nevertheless, by comparing a DFAN OMA half spectrum with an FRF obtained from impact testing (Fig. 16), it seems that roughly the same resonance peaks can be observed, which is a promising starting point for applying OMA.

The half spectra are fed into the (Operational) Polymax method to yield estimates for eigenfrequencies, damping ratios and mode shapes. As part of this process a so-called stabilization diagram (Fig. 17) is computed that allows to discriminate between physical and computational modes. As a quality measure for the modal analysis process, the measured spectra are compared with the synthesized spectra computed using the estimated modal parameters (Fig. 18). The correspondence is very good in this case, indicating that the measured data is well explained by the identified modal parameters. Some of the identified mode shapes are shown in Fig. 19. Please note that in this paper, we mainly focus on the OMA modes that are

included in the impact testing baseline set of modal parameters.

Modal Correlation

Impact Test (EMA) versus DFAN Test (OMA)

The modes retrieved from the DFAN test using OMA are compared to the baseline modes found by applying EMA to the impact test data:

- The eigenfrequencies and damping ratios agree very well (Table 1).
- Also, the mode shape estimates are in good agreement as indicated by the MAC values (Fig. 20). A MAC value expresses the correlation between 2 modal vectors.

Test versus Finite Element Model (FEM)

It is also interesting to compare how well the modes identified from Tests correspond to the Finite Element Model (FEM) modes. Hereto, Simcenter Testlab Modal Correlation was used. Figure 21 shows the Test – FE comparison for 6 modes of which the mode shapes agree very nicely. This indicates that a rather quick and easy impact test and / or a more detailed analysis of available DFAN data could be used for validating and improving the FEM of a satellite.

Conclusions

This paper showed the successful experimental identification of the modal parameters of the PROBA-V satellite by applying Operational Modal Analysis to DFAN data. The OMA results agreed very well with the EMA modal parameters obtained



Fig. 13 Location of triaxial reference sensor for OMA. Courtesy of ESA and Redwire Space NV

Table 1 Comparison of eigenfrequencies and damping ratios estimated using EMA (Impact data) and OMA (DFAN data)

Mode #	EMA – Impact		OMA – DFAN		Mode #	EMA – Impact		OMA – DFAN	
	f [Hz]	ξ [%]	f [Hz]	ξ [%]		f [Hz]	ξ [%]	f [Hz]	ξ [%]
1	26.2	1.1	25.5	1.3	8	162.4	1.8	161.9	2.4
2	37.2	1.1	36.6	1.4	9	167.6	1.0	166.7	1.2
3	59.0	0.2	58.8	0.3	10	171.6	1.2	171.5	1.2
4	74.1	0.4	73.7	0.8	11	176.9	1.3	177.3	1.6
5	112.7	1.5	111.7	1.2	12	182.5	1.4	181.8	1.2
6	128.2	2.6	126.4	2.1	13	188.7	2.2	185.5	1.0
7	158.7	1.1	158.9	1.3	14	232.7	1.0	231.0	0.8

from impact testing and shaker testing. Such modal tests are not significantly adding to the environmental test schedule. Simcenter Qsources shaker testing is very repeatable and benefits from highly controllable force levels, leading to superior coherences compared to impact testing. Nevertheless, both impact and shaker testing yield reliable modal parameters after applying Polymax combined with MLMM.

There are several advantages associated to the use of OMA in a DFAN context:

- OMA allows an enhanced exploitation of test data that was measured for another purpose (acoustic qualification).
- The modal parameters are obtained under realistic loading and with realistic boundary conditions.
- The interaction between the structure and the acoustic environment can be investigated.

Acknowledgments The authors would like to acknowledge the contribution of MSI-DFAT (Baltimore, USA) and DV2 (Metz, France) in the preparation and execution of the DFAN test campaign. Also, the authors would like to thank ESA and Redwire Space NV (Kruike, Belgium) for facilitating the use of the PROBA-V STM and FEM.

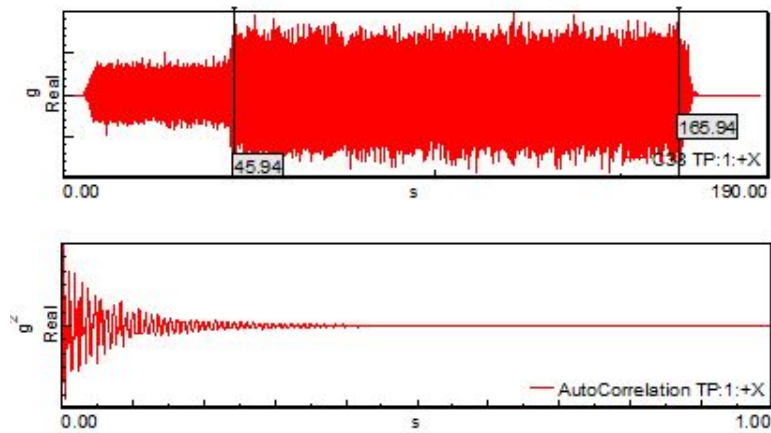


Fig. 14 (Top) Acceleration time recording during DFAN testing. A 2-minute segment at full level is selected for further processing. (Bottom) Acceleration auto-correlation computed from the time data

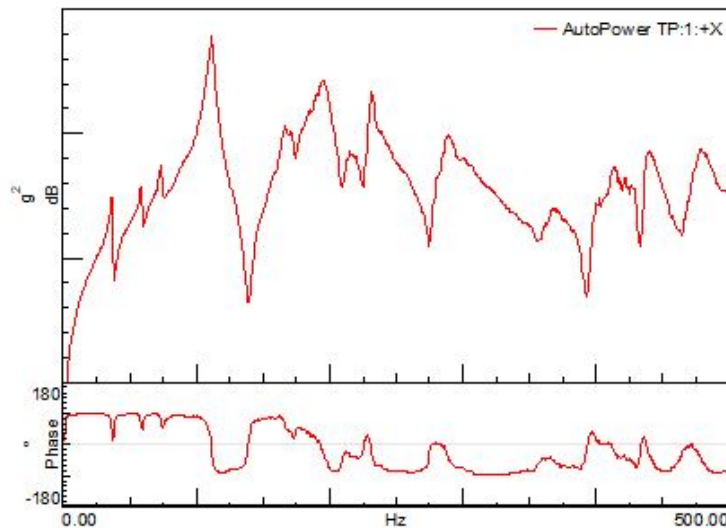


Fig. 15 “Half” power spectrum computed as the DFT of the (windowed) autocorrelation window of Figure 14 (Bottom)

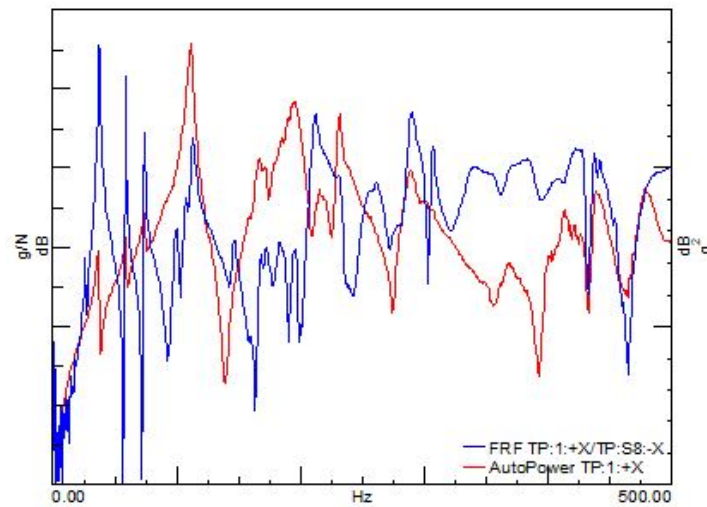


Fig. 16 Comparison of impact testing FRF (Left axis / blue) and DFAN half spectrum (right axis / red)

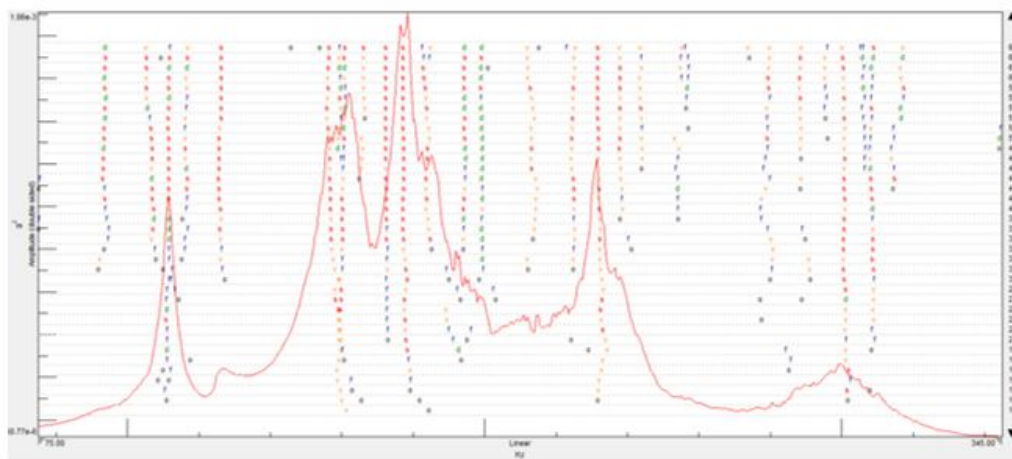


Fig. 17 PROBA-V DFAN testing – Operational Polymax stabilization diagram

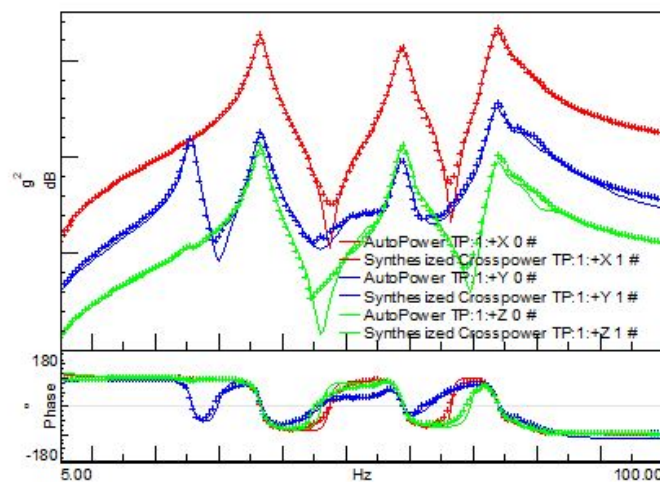


Fig. 18 PROBA-V DFAN testing – Measured (crosses) and synthesized (line) power spectra of the 3 reference sensors

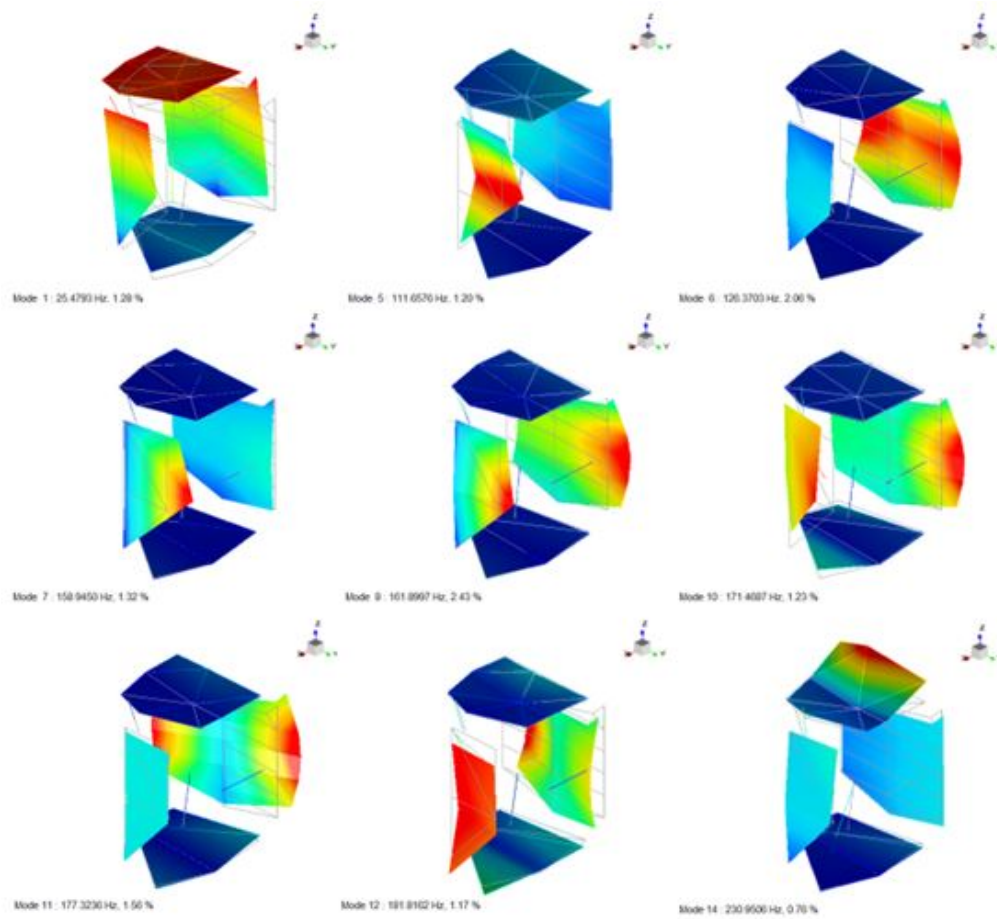


Fig. 19 PROBA-V DFAN testing – mode shapes obtained by OMA (Mode # 1, 5-8, 10-12, 14 of Table 1)

References

1. NASA (2016). *Direct field acoustic testing (DFAT)*, NASA Technical Handbook NASA-HDBK-7010.
2. de Miguel, A.G., Mastrodicasa, D., Grozavu, S., Musella, U., Choukri, M., Manzato, S., Peeters, B., Chimenio, M. & Ngan, I. (2024). Analysis of the vibro-acoustic interaction in direct field acoustic noise tests via optical measurement techniques. In Proc. ECSSMET, Noordwijk, Netherlands.
3. Musella, U., de Miguel, A.G., Carrella, A. & Peeters, B. (2024). Adopting direct field acoustic noise testing technology for the replication of directional acoustic fields: challenges and results. In Proc. ECSSMET, Noordwijk, Netherlands.
4. Peeters, B., Alvarez Blanco, M., Musella, U., Waimer, S., Di Lorenzo, E. & Carrella, A. (2017). Identifying the structural behaviour of a specimen during spacecraft acoustic testing by operational modal analysis. In Proc. 7th Int. Operational Modal Analysis Conf. (IOMAC), Ingolstadt, Germany.
5. Alvarez Blanco, M., Janssens, K., Bianciardi, F., Fabries, C. & Carrella, A. (2016). Industrial verification of projection algorithms and optimization routines for acoustic field uniformity enhancement in MIMO direct field acoustic control. In Proc. ECSSMET, Toulouse, France.
6. Yanez, A. (2021) *ProbaV STM RFAT/DFAT Instrumentation Plan*, ESA-TECMSS-PL-022053, ESA-ESTEC, Noordwijk, Netherlands.
7. Peeters, B., Van der Auweraer, H., Guillaume, P. & Leuridan, J. (2004). The Polymax frequency-domain method: a new standard for modal parameter estimation?, *Shock and Vibration*, **11**, 395-409.
8. Elkafafy, M., Peeters, B., Guillaume, P., & De Troyer, T. (2016). Constrained maximum likelihood modal parameter identification applied to structural dynamics. *Mechanical Systems and Signal Processing*, **72-73**, 567–589.
9. Carrella, A. & Musella, U. (2023). What's behind the plots of a direct field acoustic test (DFAT). In Proc. ECSSMET, Toulouse, France.
10. Carrella, A. & Mayne III, W. (2022). Insight into the sound field during a direct field acoustic test (DFAT). In Proc. Internoise, Glasgow, UK.
11. Peeters, B., Van der Auweraer, H., Vanhollenbeke, F. & Guillaume, P. (2007). Operational modal analysis for estimating the dynamic properties of a stadium structure during a football game. *Shock and Vibration*, **14**(4), 283-303.

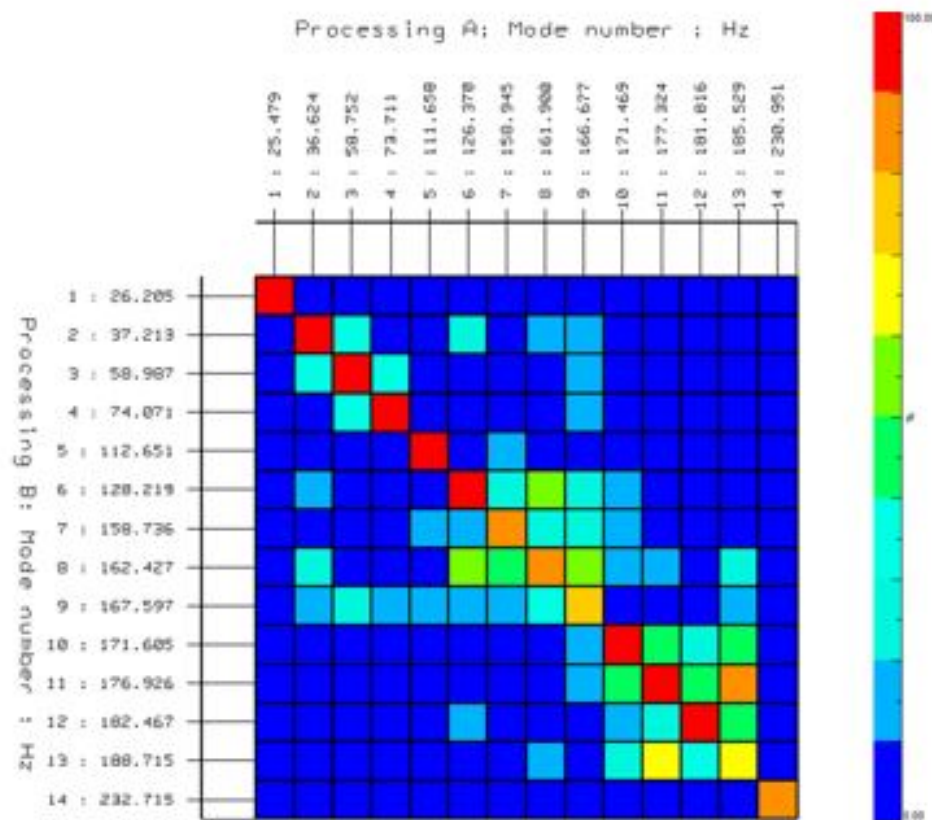


Fig. 20 MAC between DFAN – OMA (columns) and Impact – EMA (rows) mode shapes

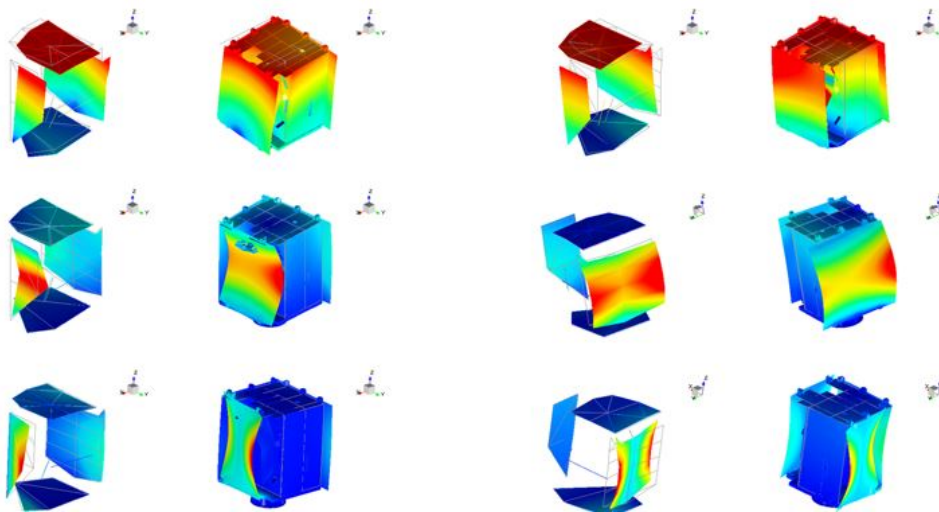


Fig. 21 Side-by-side comparison of Test (left) and FEM (right) mode shapes. 6 mode pairs are shown (Mode # 1-2, 5-7, 11 of Table 1)



Chapter 6

The Impact Calibration Technique for Piezoelectric Force Sensors

Michael Mains, Derrick Hollins, and Chad Kallmeyer

Abstract This paper is an expansion upon a previous IMAC paper which compared calibration techniques for piezoelectric force sensors [1]. In this paper, the dynamic impact calibration technique will be further explored and verified with additional testing. We will retake some of the same data taken in the original paper with the same sensor models and test procedures tested to investigate repeatability of the technique.

In the original paper force was applied to the sensor under test similar to the way an impact hammer is calibrated, this is similar in many ways to how an impact test is performed for modal testing. The use of an impact hammer is convenient and commonly used for modal testing, but known to not provide the best data for modal analysis. Many of the same issues that affect impact modal testing, such as alignment of the hammer to the surface of the structure to be impacted are known to affect accuracy. We will look at using the mass extenders that come with the impact hammer and using a larger hammer that what was used in the original paper. Additionally, a MATLAB app using the DigiDAQ was used in the original paper, we will also acquire data with a higher quality lower noise DAQ card and use The Modal Shop (TMS) 9961 software the is traditionally used for impact hammer calibration.

All results will be compared back to the tradition hydraulic force sensor calibration that is currently used for calibrating force sensors.

Keywords Force · Sensor · Calibration · Impact Hammer

Background

The available methods of calibrating piezoelectric force sensors have limitations. The simple to use and accessible Gravimetric Drop Calibration may not adequately test a high range sensor while the use of deadweight and hydraulic systems is set up intensive and costly. The Impact Calibration technique provides both a simplified set up and higher force input for calibration of higher measurement range force sensors.

The Gravimetric Drop Calibration rig is shown in Figs. 1 and 2 was discussed in detail in the original paper. A typical time history is shown in Fig. 3. This measurement is repeated with several weights. The limitation of this method is limited to the maximum weight that can be supported by the monofilament line. In The Modal Shop calibration lab weights of approximately 1, 2 and 3 lbs. are used.

A representative hydraulic force sensor calibration system is shown Fig. 4. This method was discussed in detail in the original paper.

In this paper we continue to investigate the impact method as a third option to calibrate force sensors. Impact testing had been used in trouble shooting and impact hammer calibration for a number of years. It is also receiving renewed interest in the research of National Metrology Institutes (NMI's) like Germany's Physikalisch-Technische Bundesanstalt (PTB) [2] and the United States' National Institute of Standards and Technology (NIST) [3]. Figures 8 thru 10 show their respective test systems for hammer calibration. They have not yet tried to extend this to general purpose force sensors and impedance heads as we do in this paper. It is interesting to note that both systems implement a pendulum setup for there hammer to mitigate operator error that might occur during testing. PTB's system specifically notes the use of a scale to help maintain a consistent application of force. TMS is evaluating is currently evaluating these designs as well as investigating a dual

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Fig. 1 The Modal Shop Gravimetric Calibration rig

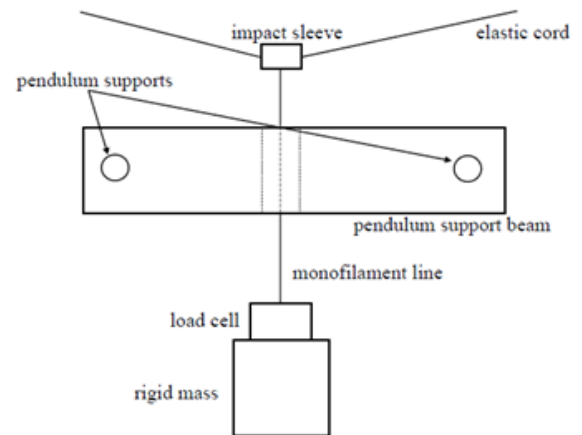


Fig. 2 Gravimetric Load Cell Calibration

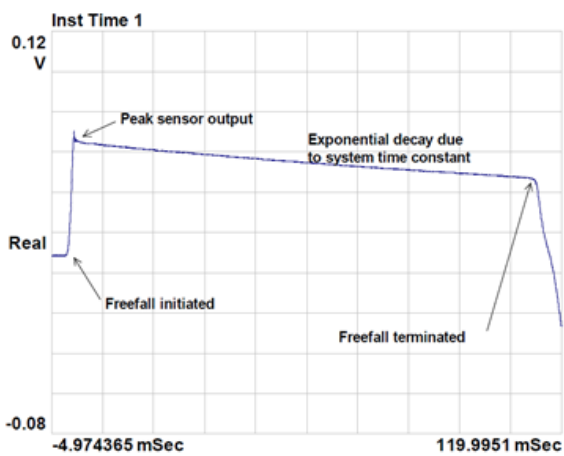


Fig. 3 Time history of a gravimetric force cell calibration

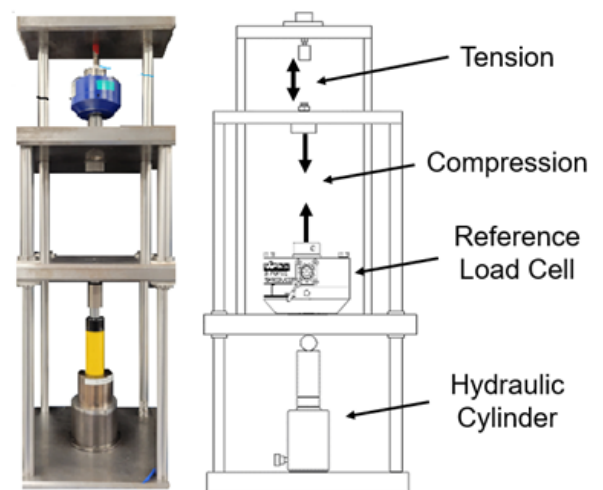


Fig. 4 Force Calibration Rig

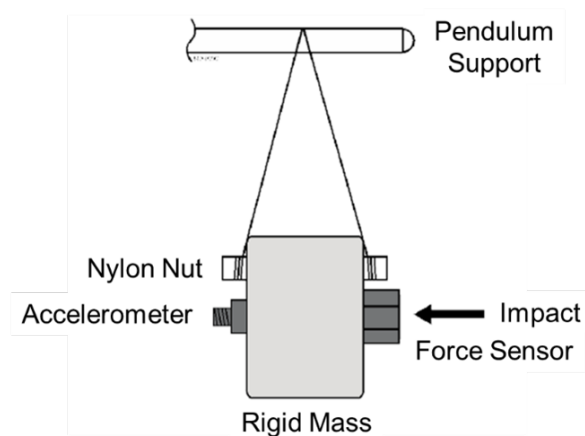


Fig. 5 Impact Force Sensor Calibration

pendulum impactor that would use two weights with both pendulums supported by monofilament line. Figure 5 illustrates the setup using the pendulum of the Gravimetric Calibration rig that is typically used for calibration of impact hammers. With this method, instead of calibrating the impact hammer we mount the force SUT on the rigid mass and install an impact cap on the force SUT. The impact hammer is used to apply the force to the SUT and the acceleration is measured using a reference accelerometer. The force is calculated multiplying the measured acceleration by the mass of the rigid mass of the pendulum. Several impacts were made at varying force levels.

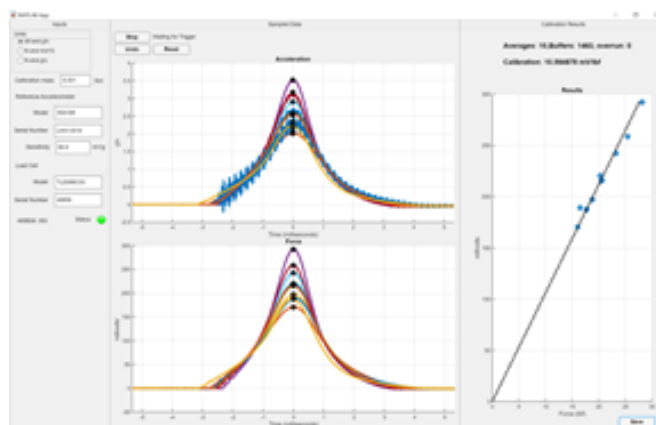


Fig. 6 MATLAB application



Fig. 7 DigiDAQ Model 485B39

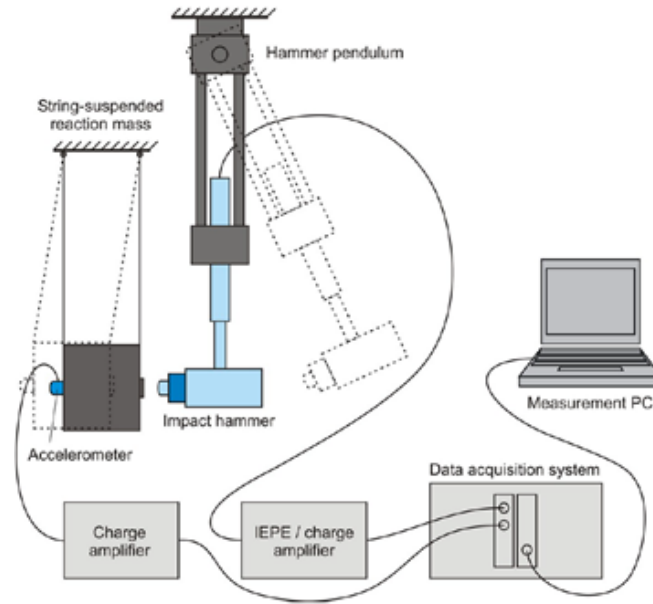


Fig. 8 Impact Hammer Calibration Device at PTB

Testing

Application and DAQ

The measurements were made with two different acquisition setups. The first is a continuation of what was used in the previous paper and was comprised of a custom MATLAB app (Fig. 6) written for the DigiDAQ Model 485B39 (Fig. 7). The second setup was the TMS Impact Hammer Calibration System 9155-961 software option which uses a National Instruments (NI) 4461 DAQ card. Acceleration measurements were made with 353B04 ICP Accelerometers. Force impacts were applied with a PCB TLD086C03 or TLD086D05 impact hammer with and without mass extender. Hammer tips included the red (super soft), black (soft), white (medium) and steel (hard) hammer tips included with TMS and PCB impact hammers. The steel hammer tip was not always used with the MATLAB DigiDAQ app because the DigiDAQ is limited to a 48kHz sample rate. Typically, 10 to 16 impacts were made taken per test. A test consisted of a specific force sensor, hammer tip and either a slow, rapid or mixed application of force. Unlike hammer calibration, hammer tips don't seem to bias the results of calibrating the force sensor. For hammer calibration a different sensitivity is often specified for different hammer tip and extender configurations.

Table 1 Table of Sensors

Range (lbf)	Sensitivity (mV/lbf)	Type	Normalized Error E_N		Normalized Error $ E_N $		Sample Size (Sensors)		Model
			2025	2024	2025	2024	2025	2024	
10	500	Force	-1.27	0.07	1.35	0.80	72(3)	8(2)	208C01
100	50	Force	-0.45	-0.20	0.46	0.20	98(2)	10(2)	208C02
500	10	Force	-0.66	-0.82	0.67	0.82	98(3)	10(2)	208C03
1000	5	Force	-1.16	-1.74	1.17	1.65	98(3)	10(2)	208C04
5000	1	Force	0.36	2.10	0.38	2.10	72(3)	8(2)	208C05
50	100	Impedance Head	-	0.07	-	0.73	-	10(2)	TLD288D01
100	50	Impedance Head	-	0.30	-	0.37	-	10(2)	TLD288M25

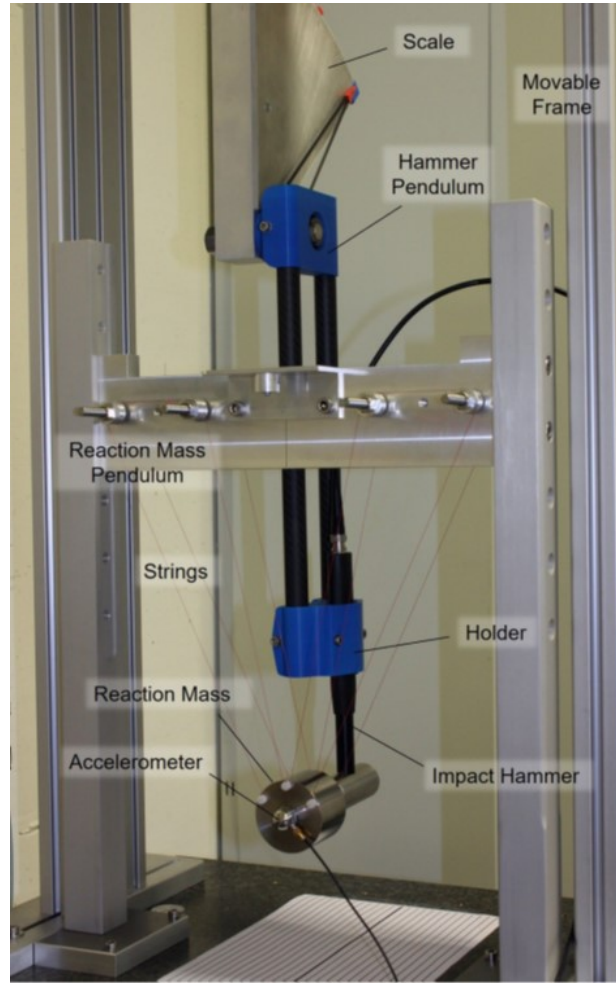


Fig. 9 Photo of PTB Device

Sensors

Two to three sensors each for seven different models of sensor were tested for a total of 18 different sensors pulled from the TMS rental pool. The maximum force level for each sensor model varied from 10 lbf to 5000 lbf. Two of the force sensors used were impedance head type sensors which include force and acceleration measurement capability. These sensors are typically used for making driving point FRF measurements for modal testing where shakers are used for excitation.

Analysis

To determine the force sensor's sensitivity a simple least squares regression is performed. Since the y-intercept between the known acceleration and the unknown force is zero for ICP sensors the least-squares regression simplifies to:

$$x_n = \frac{\bar{V}_p}{m\bar{a}_p} \quad (1)$$

where: x_n = sensitivity of the new method

$\bar{V}_p$ = is the mean of the peak Voltages measured by the force sensor

m = mass of the pendulum

$\bar{a}_p$ = is the mean of the peak accelerations measured by the accelerometer

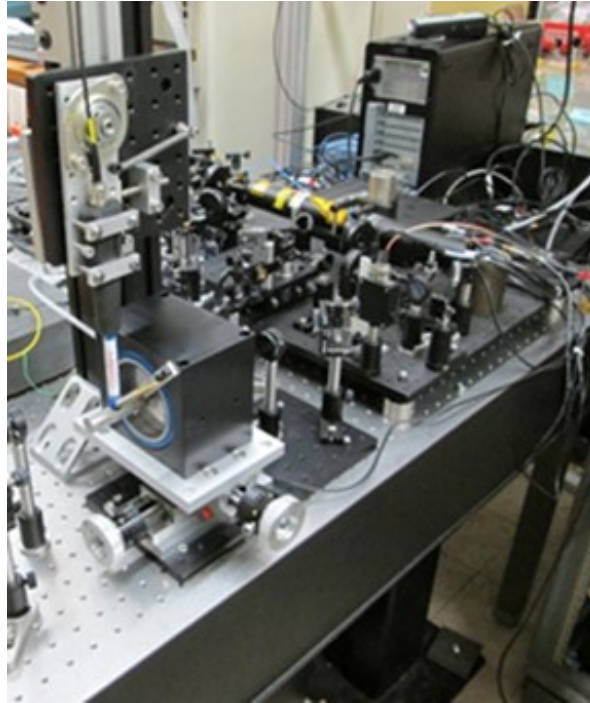


Fig. 10 NIST Impact Force Standard

Normalized Error

It is common when comparing calibration results between laboratories to use normalized error [4]. This is often done with laboratories participating in proficiency tests or interlaboratory comparisons. Normalized error is a statistical evaluation where the uncertainty in measurement result is included.

$$E_n = \frac{\frac{x_n}{x_R} - 1}{\sqrt{U_n^2 + U_R^2}} \quad (2)$$

where: x_R = sensitivity of the existing method

U_n = Uncertainty percentage of the new calibration procedure

U_R = Uncertainty percentage of the existing calibration procedure

For this analysis, similar to coherence, values between ± 1 are considered acceptable.

Results

The Normalized Error calculation was applied to 457 of 462 tests that were performed on 18 different sensors. 4 of 6 data sets for 5000 lbf. sensor were omitted from this study because the sensitivities were over 20 mV/lbf for a sensor with a nominal sensitivity of 1 mV/lbf. Since the other 2 data sets were reasonable, they were retained. 1 of 64 data sets for a 10 lbf. were omitted because the sensitivity was 171 mV/lbf when the nominal is 500 mv/lbf.

The difficulty with using Normalized Error is assigning an Uncertainty percentage to the new calibration method. This is primarily because there hasn't been any data acquired for this method. The first attempt was to assume that the Uncertainty percentage of the new method would be equivalent to PCB's uncertainty for their hydraulic calibration method which is stated at $\pm 1\%$. When applying 1% to both Uncertainties the histogram shown in Fig. 11 with a range of -5 to 10 can be plotted. Realizing that the Uncertainty for impact testing is typically stated at 5% the histogram was replotted and shown in Fig. 12 with a more reasonable range of -1.58 to 2.94 . Noting that there a positive bias in data, the question was raised as to whether the mass of the reference accelerometer and force sensors included in the overall system mass. Realizing that this wasn't the case the nominal mass of the Force, Impedance Head and Reference accelerometer were included and the sensitivities for each sensor were recalculated. The mass of each item in the system are listed in Table 2.

Table 2 Masses

	Mass (lbm)	Mass (grams)	Model
Pendulum	1.94	880.0	N/A
Force	0.050	22.7	208C
Impedance Head	0.042	19.2	TLD288
Accelerometer	0.023	10.5	353B04

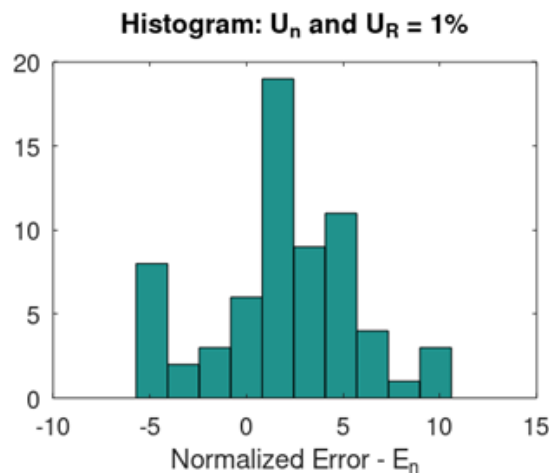
Looking back at Table 1 we see we have added the 2025 data to the 2024 data and presented the 2024 data for comparison. For the 2024 data the force sensors in the 10 to 500 lbf range have the best average normalized error of -0.07 , -0.20 and 0.82 respectively with the worst results for the 1000 and 5000 lbf sensors, of -1.58 and 1.31 respectively. This can be partially explained by trying to impact these sensors that hardest, trying to introduce higher force and thus introducing more human error in impacting perpendicular to the sensor and to the center of the sensor. The average of the absolute value of normalized error gives similar result with 10 to 500 lbf force sensors ranging from 0.20 to 0.82 . Again, the 1000 and 5000 lbf sensors have the worst average of the absolute value of normalized error of 1.65 and 1.31 . Adding the 2025 data to the 2024 data we see that the 10 lbf sensors an excellent normalized error of -0.07 to -1.27 , for the worst change. The 100 lbf sensors went from -0.20 to -0.45 in normalized error, but not a significant difference. The 500 lbf sensor improved slightly from -0.82 to -0.66 . The 1000 lbf sensor also improved from -1.58 to -1.16 . The 5000 lbf sensor improved the most from 1.31 to 0.36 in normalized error. We see similar trends with the absolute value of the normalized error. Note that no additional data was taken in 2025 for Impedance Head sensors.

Comparing the 2024 data sets with 66 samples (Fig. 13) with the combined 2024 and 2025 data sets of 457 samples (Fig. 14) we see a normalized error still between -2 and 2 . Most of the new data sets are falling between normalized errors of between -1.5 and 0.5 . If we remove the data from 2024 from Fig. 14 we produce a sample size of 391, shown in Fig. 15. Here we accentuate the negative Bias in the data seeing no normalized errors above 0.5 in the new data. Since we introduced a new DAQ and Software Figs. 16 and 17 make the comparison between the 257 MATLAB/DigiDAQ and TMS 961/NI 4461. Note the negative bias in the uncertainty is present in both, so the negative bias is not created by the introduction of the new software and hardware combination.

Figures 18 through 20 group the data by operator. In 2024 Operator A performed all of the tests and Operator B and C conducted the tests in 2025. It seems that Operator B is contributing more to the negative bias than Operator C that we see 2025 data. Operator A seems to have a more Gaussian-like distribution in their Normalized Error. Perhaps operator error is the biggest cause of variation. An obvious error in not allowing the accelerometer to settle was show in the previous paper.

Figures 21 through 25 group the data by sensor model. These histograms display some of the most Gaussian-like distributions suggesting that there be some dependency on the sensor models. The 10 lbf (Fig. 21) and 1000 lbf (Fig. 24) sensor are the biggest contributors to the negative bias in the data. The 5000 lbf sensor (Fig. 25) has the most positive bias with the largest positive outliers from the 2024 data.

Figure 26 shows the data from 2024 with Impedance Heads. We simply need to take more data for these sensors.

**Fig. 11** Histogram with Uncertainty for both methods set to 1%

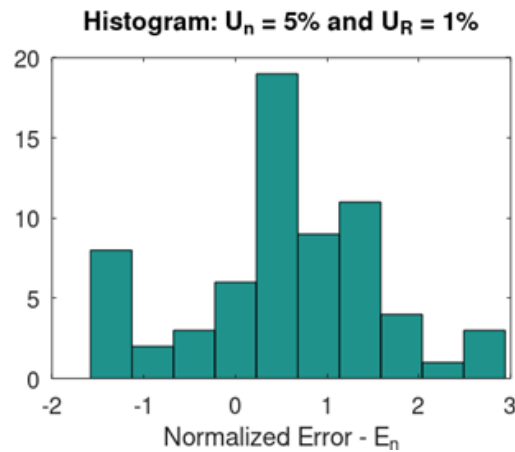


Fig. 12 Histogram with New method uncertainty set to 5%

Table 3 gives a general overview of how force is affected as we harden the tip and increase mass of the hammer. As expected, the harder the tip, the harder the force and heavier the hammer the more force is applied. For the hardest tip and larger hammer with extender mass, we achieved a max measured force of 320.80 lbf, getting much closer to the full mass range of the 500 lbf, Model 208C03 force sensor. While this was a “well selected” example, this was the general trend for all the sensors involved in this study. It’s believed that a proper setup could be developed to optimize the force applied to various model sensors. Perhaps optimizing a setup for a 5000 lb force sensor could be dialed back for lower capacity sensors so as not to damage them and apply optimal force. Again a pendulum used for applying the force could be setup to swing at different heights for different loads.

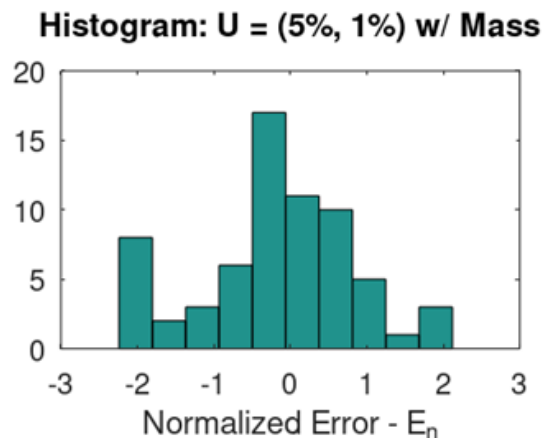


Fig. 13 Histogram with values adjusted for mass

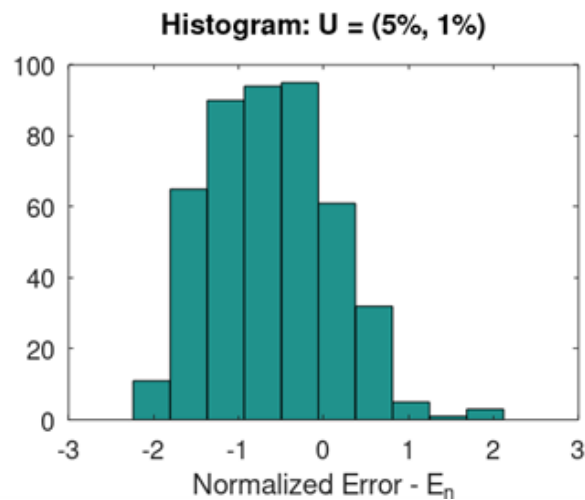


Fig. 14 2024 and 2025 Data, Sample Size 457

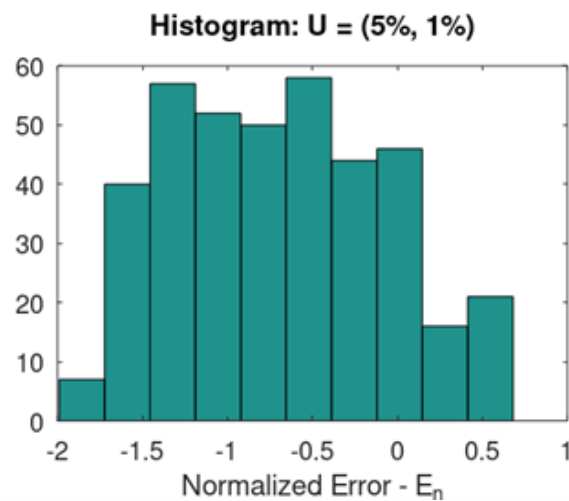


Fig. 15 2025 Data, Sample Size 391

Conclusions

The Impact Calibration Technique for Piezoelectric Force Sensors continues to show promise as a less expensive method to calibrate sensors than what has traditionally been done with a force calibration rig. This method can also apply more force than what can be accomplished with the gravimetric (drop) calibration method. The force applied via impact is also more representative of how the sensor is used in actual testing than the static hydraulic method or gravimetric method.

Operator error and model type (force capacity) have been shown in the data as areas that introduce uncertainty. Certainly, the setups at PTB and NIST where a pendulum is used to reduce operator error merit attention. Interestingly enough an inexpensive setup with the DigiDAQ provides adequate data quality compared to the more expensive NI 4461, with its only limitation being sample rate to accommodate the hard tip hammer.

The Impact Calibration Technique for Piezoelectric Force Sensor has shown that appropriate forces can be applied to sensor under test (SUT) and can be controlled with various hammer tips, hammer size and addition of a mass extender. The more massive the hammer and the harder the tip allows for larger forces to be applied to the sensor.

Future research should be done to reduce the uncertainty in the measurement and more data needs to be taken with more sensors in the specific model range and sensor type.

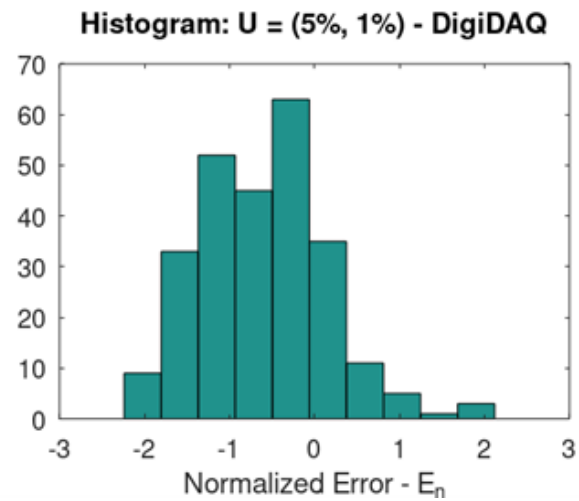


Fig. 16 MATLAB App, Sample Size 257

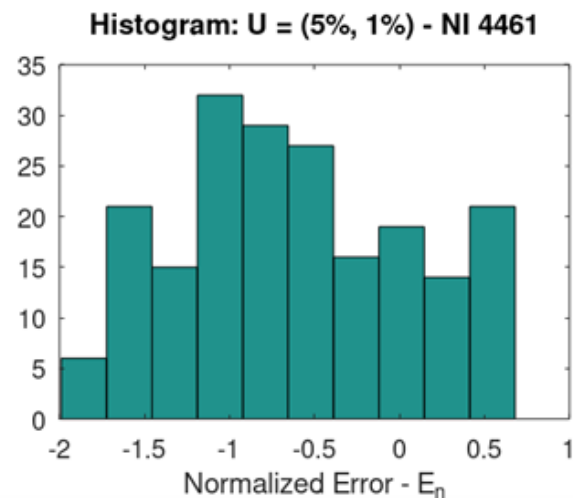


Fig. 17 TMS 9961 Application, Sample Size 200

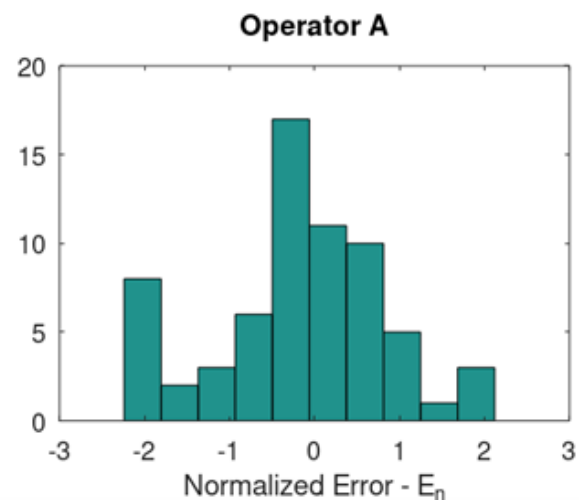
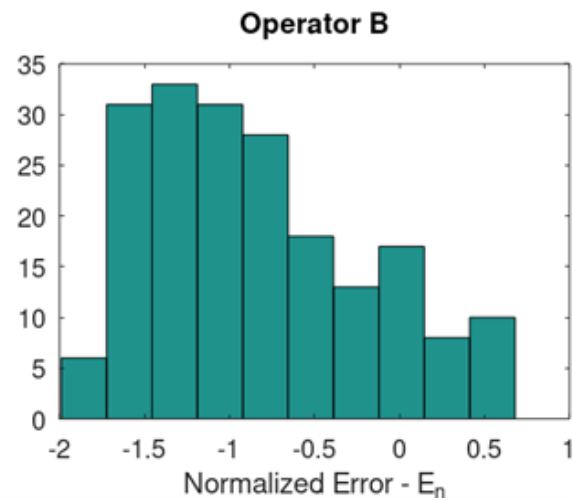
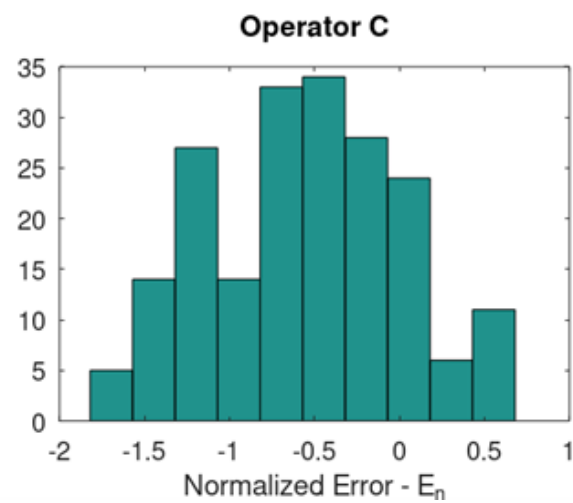
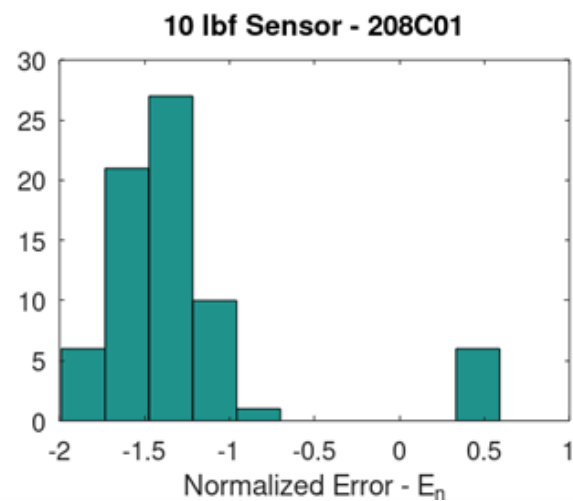


Fig. 18 Sample Size 66

**Fig. 19** Sample Size 195**Fig. 20** Sample Size 196**Fig. 21**

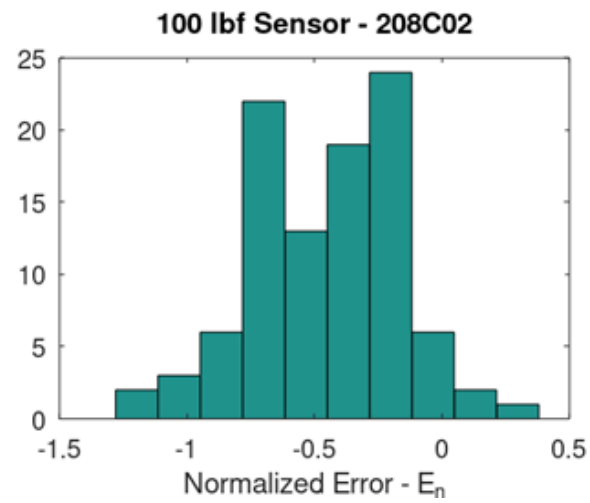


Fig. 22

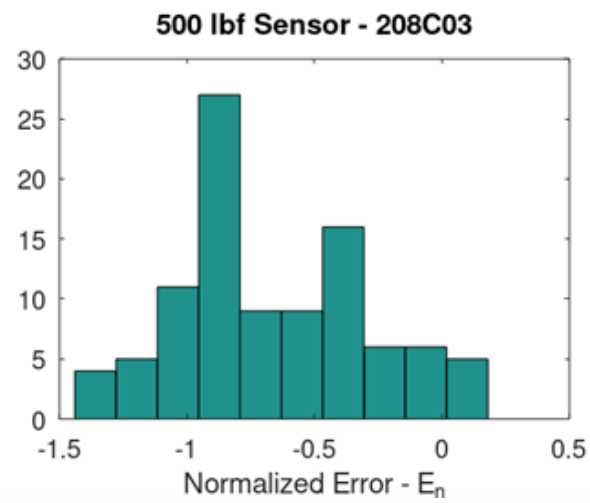


Fig. 23

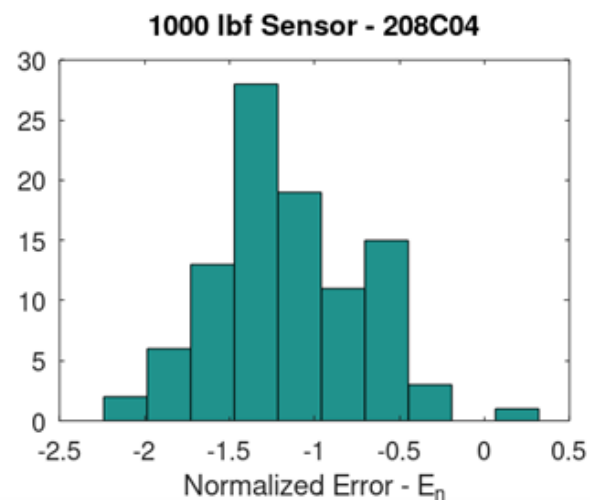


Fig. 24

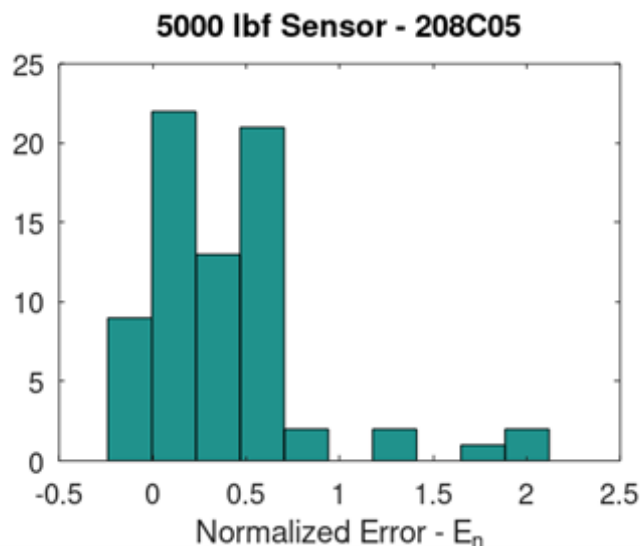


Fig. 25

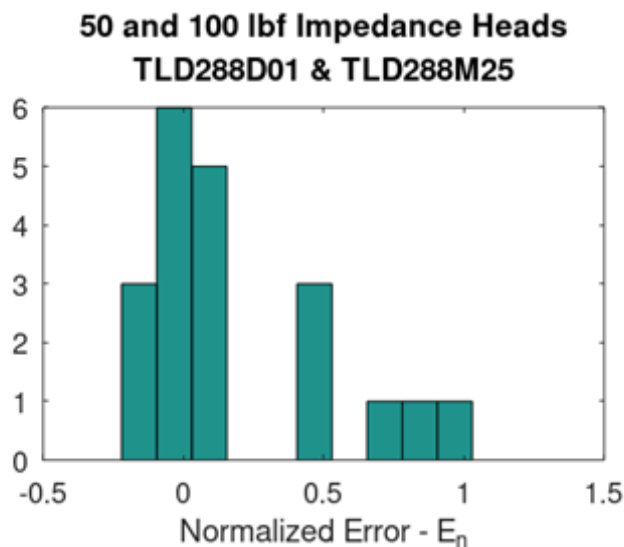


Fig. 26

References

1. Mains, M., Peres, M., Kallmeyer, C., Dillon, M., Weaver, C., Bono, R., "Comparison of Calibration Techniques for Piezoelectric Force Sensors", International Modal Analysis Conference (IMAC) XLII, January 2024.
2. Kobusch, M., Klaus, L., "Uncertainty in Measurement for the Calibration of The Shock Sensitivity of Impact Hammers", XXIV IMEKO World Congress, August 2024.
3. The NIST Impact Force Standard, <https://www.nist.gov/programs-projects/dynamic-mechanical-metrology-acceleration-force-and-acoustics/impact-force>
4. Hogan, R., "How to Calculate Normalized Error", www.isobudgets.com/how-to-calculate-normalized-error/

Table 3 Tip and Hammer Effects on Force: 500 lbf sensor - 208C03

Hammer Tip	Hammer Model	Extender Mass	Normalized Error E_N	Mean Force	Min Force	Max Force
red	086C03	No	0.038	8.82	4.58	12.69
red	086C03	Yes	-0.121	12.93	6.42	17.09
red	086D05	No	0.177	14.76	4.08	28.07
red	086D05	Yes	0.137	17.81	3.24	27.79
black	086C03	No	-0.181	15.39	7.65	21.54
black	086C03	Yes	-0.339	25.02	21.32	32.02
black	086D05	No	0.058	25.92	6.01	36.14
black	086D05	Yes	-0.062	20.80	1.02	38.13
blue	086C03	No	-0.121	24.63	4.67	38.54
blue	086C03	Yes	-0.161	35.45	14.97	46.00
blue	086D05	No	-0.101	24.43	6.39	44.18
blue	086D05	Yes	-0.081	44.00	13.07	61.08
white	086C03	No	-0.379	97.10	64.41	135.70
white	086C03	Yes	-0.399	97.04	65.09	119.75
white	086D05	No	0.177	125.38	35.04	174.96
white	086D05	Yes	-0.220	146.53	61.26	188.90
steel	086C03	No	-0.637	152.04	91.28	211.69
steel	086C03	Yes	-0.399	199.78	115.44	274.35
steel	086D05	No	-0.200	212.74	120.15	290.87
steel	086D05	Yes	-0.181	249.96	145.48	320.80



Chapter 7

USB Connectivity for Harnessing High-Quality Digital Sensor Data

Marco A. Peres, Tom Clary, and Mike Mains

Abstract In recent years, an increasing number of digital sensors and signal conditioner devices have adopted the USB Audio format [1] as their core communication protocol. The USB device naming introduces a novel method for retrieving sensor calibration making it easier to identify devices, scale, and acquire accurate sensor data. Software applications across various platforms can be optimized for seamless communication with these digital sensor products, eliminating the need for proprietary drivers as the USB audio protocol is either native or readily available on all popular operating systems such as Windows, Linux, macOS, Android, and iOS. Last but not least, the USB device name is readily available in all these applications while other USB parameters can be more complicated to access. While there are multiple possible technological implementations to accomplish digital sensing, the USB connectivity is unique on its natural plug-and-play capabilities facilitating the interface with the sensors while revolutionizing the way in which data is captured, calibrated, shared, and utilized across multiple platforms.

Keywords Digiducer · SmartSensors · Calibration · USB · DigiDAQ

Historical Context: TEDS and IEEE 1451.4

To appreciate the current landscape of digital sensors, a brief examination of Transducer Electronic Data Sheets (TEDS) as per IEEE 1451.4 is warranted. Two decades ago TEDS provided the first framework for storing sensor data in a standardized format, allowing data acquisition systems to automatically retrieve sensor model number, serial number, and calibration information. This groundwork laid the path and inspiration for the most recent generation of digital products featuring embedded calibration information.

The USB Audio Format and its Impact

Digital sensing products such as the 333D01 DigiducerTM and the 485B39 DigiDAQTM embody the principles of USB audio technology by embedding sensor calibration within the device name. These products utilize **digital counts** as the output unit that provides 24-bit high-resolution for data samples consistent with the sensor output.

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The following figure illustrates conceptually how the digital accelerometer sensitivity is defined:

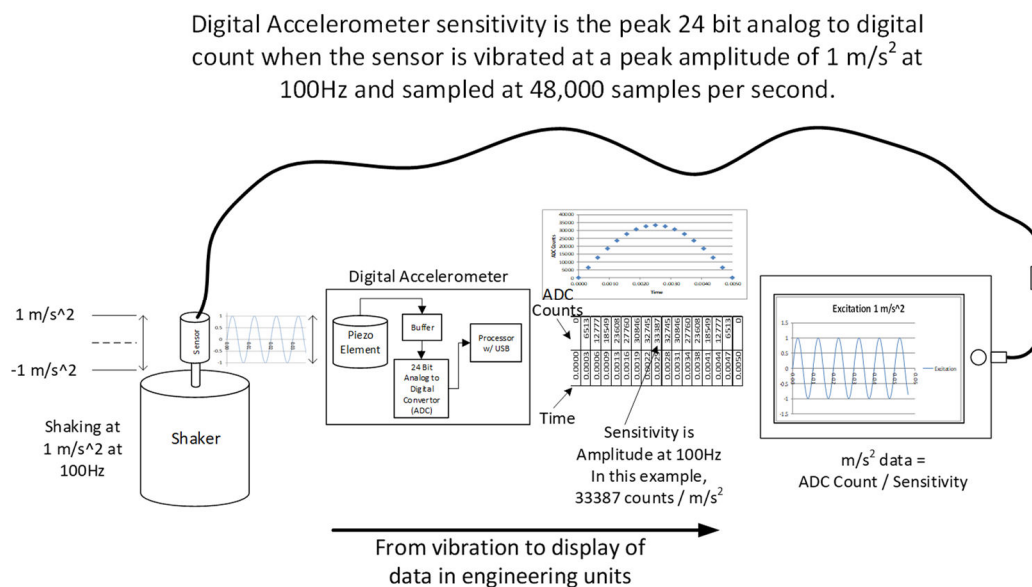


Fig. 1 Digital Accelerometer Sensitivity

The sensitivity information for the original model 333D01 Digiducer, for example, can be retrieved by simply decoding the data embedded in its device name. During calibration of the sensor, the sensitivity information is programmed into the name field of the USB descriptor. Specifically, for digital accelerometer products (Version 1 Format), the device name field format is:

MMMMMMs1NNNNNNAAAAABBBBYYMMDD

Where:

- MMMMMM is 6 digit model number (333D01, 633A01, 333D04, ...)
- s is space
- 1 is single character format definition. This indicates an accelerometer is being defined
- NNNNNN is 6 digit serial number
- AAAAA is 5 digit sensitivity value in counts / (m/s^2) for channel A
- BBBB is 5 digit sensitivity value counts / (m/s^2) for channel B
- YY is the last two digits of the year (00-99) with actual year being 20YY.
- MM is the month number (1-12).
- DD is the day (1-31)

Essentially, software applications can access this information and use it to scale the acquired data into calibrated engineering units for display and analysis.

The 485B39 DigiDAQ on the other hand converts ICP® (or IEPE) sensor signals from voltage to digital samples. Instead of being limited to only a single axis of acceleration data, the digital signal conditioners interfaces to the wide family of ICP sensors. Sensor types include piezoelectric accelerometers, force, microphone, and tachometer sensors. The device has 2 channels of phase coherent data. The implementation and processing are similar to the digital accelerometers. The applicable units are, in this case, Digital Counts / Vpeak, instead of Digital Counts / m/s^2 . The 485B39 can serve as the digital interface for a large range of dynamic sensors offering high-quality dynamic data for a variety of applications. It should be noted that the DigiDAQ only has calibration to volts. The sensitivity for the sensor connected to the DigiDAQ needs to be known and used to scale from volts to engineering units. A Version 2 Format (Voltage) is defined and used by the digital signal conditioner products [2].

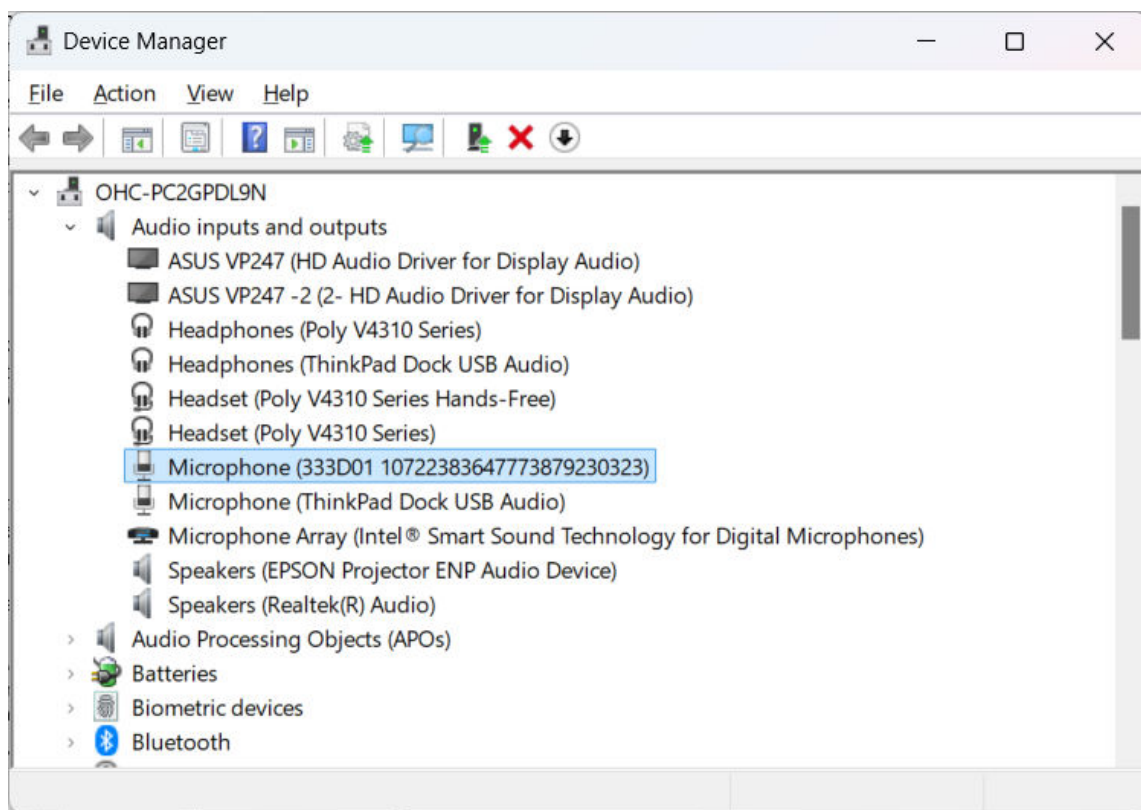


Fig. 2 A USB Digital Accelerometer shows up as an Audio Input Device under Microsoft Windows Device Manager

Conversion to Acceleration

The table below has typical conversions from digital sensitivity to the acceleration in g's:

From	Factor	Decimal	To
Integer Sample -2^{23} to $(2^{23}-1)$	$\frac{1}{\text{sensitivity} \left(\frac{\text{counts}}{m} \right) * 9.80665 \left(\frac{m}{s^2} \right) \left(\frac{m}{g} \right)}$	$\frac{0.10197}{\text{sensitivity}}$	g
Normalized-1.0 to 1.0MATLAB, M+P Smart Office	$\frac{2^{23} \left(\frac{\text{counts}}{\text{signed full scale}} \right)}{\text{sensitivity} \left(\frac{\text{counts}}{m} \right) * 9.80665 \left(\frac{m}{s^2} \right) \left(\frac{m}{g} \right)}$	$\frac{855400}{\text{sensitivity}}$	g

There are similar formulas to convert digital counts to volts for the digital signal conditioner [2].

Implementation and Practical Considerations

In implementing USB-connected digital sensors, developers must consider several factors, including phase synchronization and signal conditioning. The 485B39 DigiDAQ, for instance, is designed to minimize phase mismatch, making it suitable for applications where channel synchronization is critical. Unfortunately, getting consistent phase response between multiple USB devices remains a challenge.

Power consumption is another critical requirement when operating with mobile devices such as phones and tablets. It is important to keep the current and the power consumption under 100mA at 5V while working with Apple mobile devices. The low power and compact design implementation of the Digiducer and the DigiDAQ devices makes it compatible with the power requirements of all portable Apple iOS and Android based products in the market today.

This new class of digital products drastically reduces the implementation effort for enabling customer solutions. It is easy for existing application developers to add support, or for customers to use their programming environment and language of choice to develop their own custom applications.

Conclusion

The integration of USB Audio format connectivity in digital sensors offers a remarkable transformation in acquiring high-quality sensor data. These advancements provide robust, scalable, portable, and straightforward solutions for data collection, ensuring seamless integration across diverse operating systems. As technology progresses, digital implementations like the 333D01 Digiducer and 485B39 DigiDAQ underscores the ongoing journey towards simplicity and precision in sensor technology. The future undoubtedly holds further innovations in this realm, promising to refine how sensors are utilized in various applications.

References

1. “Universal Serial Bus Device Class Definition for Audio Devices” <https://www.usb.org/document-library/audio-device-document-10> March 1998
2. Clary, Tom “USB Audio Interface Guide – MAN-0343” <https://www.modalshop.com/docs/themodalshoplibraries/software/usb-audio-interface-guide-man-0343.pdf> October 2022



Chapter 8

Investigation of Experimental Modal Substructuring for Constrained Modal Testing of Aerospace Structures

Adam Johnson and Ivan Kurakin

Abstract With the rapid expansion of the commercial aerospace industry, there is an increased need for more efficient and cost-effective approaches for component modal testing. Modal testing is typically used to reduce uncertainty in finite element model (FEM) predictions, proving to be a useful risk reduction tool in many programs whether it's a satellite preparing for a vibration qualification test or characterization of component structural dynamics for use in control loop simulation. Despite this, modal testing is often not a part of the workflow for smaller programs due to budgetary and schedule constraints. Relative to "free-free" tests, constrained modal testing poses significant challenges implementing a boundary condition that reveals accurate constrained modes for the unit under test (UUT). However, empirical substructuring methods exist for correcting an imperfectly constrained boundary condition in post processing of test data. These methods can be used to reduce the requirements on a test fixture, leading to shorter set-up times and higher confidence in the test results. However, many of these methods are very sensitive to measurement error and can involve logistical challenges. The authors of this report previously performed a study that identified experimental modal substructuring (EMS) to be a strong candidate for reducing boundary effects during a constrained modal test subject to practical budgetary and schedule constraints. This method is primarily selected due to its ease of implementation into an existing modal test plan. This paper represents a follow-on study whose focus is to better understand the reliability of using modal substructuring constraints (MSCs) to calculate constrained mode frequencies and identify post-processing techniques that improve confidence in the corrections. Since this study is focused on revealing the efficacy of MSCs under a typical modal test plan, no special accommodations are made to an otherwise typical modal test procedure outside of adding a few additional triaxial accelerometers to the fixture. The impacts of various UUT configurations & boundary conditions are studied. Corrected results are compared to a "truth" reference based on the same test articles grounded to a seismic table. In general, the use of MSCs is found to be highly effective at reducing the boundary effects for a constrained modal test. However, the approach is most beneficial for tests where a larger correction is needed. This study shows that test set-ups requiring finer corrections $<5\%$ may require a test plan that emphasizes enhanced measurement sensitivity and tighter requirements on test fixturing beyond the scope of this study. Finally, a workflow is proposed as to how substructuring uncertainty margins can be used to make decisions during a modal test campaign for FEM correlation.

Keywords Experimental Modal Substructuring (EMS) · Experimental Modal Analysis (EMA) · FEM Correlation · Modal Testing · Spacecraft Testing

Introduction

Constrained modal testing is often desired for improving the accuracy of FEA models such that accurate predictions of a structure's dynamics can be made. In the aerospace industry, some practical examples include:

1. Component level modal testing before integration into spacecraft or launch vehicle
2. Spacecraft modal screening ahead of a high-risk vibration test
3. Modal test of large appendage, such as solar array, for the purposes of guidance navigation and control (GNC) simulations
4. Aircraft component testing prior to integration (wing, blade, rotor, etc.)

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The largest challenge in constrained modal testing is properly controlling or quantifying the boundary condition. Various approaches exist to attempt to isolate the dynamics of the unit under test (UUT) from the fixturing to ensure accurate correlation of key properties in a FEM. However, all these approaches have their own associated uncertainties resulting in varying levels of sensitivity to detect relevant modeling errors in the FEM. The most common approach is to incorporate the fixturing structure into the FEM and include its properties in the model correlation. However, in doing this, the complexity of the model increases and a large part of the correlation often depends on the fixture properties and interface to the UUT, reducing the test's sensitivity to modeling errors within the UUT itself. Another approach is to perform a “free-free” test where the UUT is suspended by comparatively “soft” cables. This approach has a similar draw-back in that most of the modal strain energy is concentrated in joints or parts of the UUT that have little relevance to the constrained modes [2]. Recently, a family of substructuring approaches have been used to remove the effects of a flexible boundary or fixture in post-processing. These approaches fall into two primary categories: frequency domain substructuring and modal substructuring. All these approaches have robust mathematical frameworks allowing them to function well on idealized datasets. In practice, however, many of these approaches are highly sensitive to measurement error or require numerous drive points resulting in increased test scope. The authors of this report previously identified EMS as strong candidate for extensive and practical applications in the industry based on its ease of implementation into an existing test plan. This approach utilizes a synthesized modal parameter model reducing its sensitivity to measurement error and requires nothing more than a few additional accelerometers to measure the dynamic response of the fixture. However, preliminary findings published by the authors [2] show that the applied MSCs do not always reveal the correct constrained modes in practice. The motivation of this study is to quantify the uncertainty in predicted constrained modal frequencies when using MSCs and propose ways to improve confidence in the constrained modes. A MATLAB algorithm is developed to reduce some of the impacts of measurement noise on the results. It is important to note that this study applies typical modal test best practices to UUTs mounted to non-ideal boundary conditions. No special modifications are made to the modal test setup to minimize or optimize the error. This is done to reflect common industry applications of the approach in which budgetary or schedule constraints are present. In some industry applications, the need for boundary correction is not realized until after a test is complete.

Background

EMS is a substructuring technique that leverages modal parameters extracted during a modal test to build and modify a modal model. In the case of constrained modal testing, we are often interested in applying a constraint to the developed modal parameter model to remove the influence of the fixture on the UUT's modal response. In this study, this is referred to as a MSC. To do this, a simple requirement must be added to the scope of the modal test: the instrumentation set must include a set of accelerometers on the fixture itself that reveal the motion of the fixture that is to be constrained. The modal parameters that are extracted from the test include natural frequencies (ω_n), modal damping fractions (ζ), and unity-mass-normalized modes shapes (ϕ) of the UUT and the fixture. These parameters can be derived in various ways but are most commonly derived by performing EMA transfer function curve fitting techniques. Ultimately, a modal model must be derived that consist of “N” normal modes that approximate the measured system drive point transfer functions using the form shown in Eq. (1). An example curve fit is shown in Fig. 1.

$$\frac{Y_k(\omega)}{f_j(\omega)} = \sum_{n=1}^N \frac{\phi_{j,n} \phi_{k,n}}{\omega_n^2 - \omega^2 + 2\zeta\omega_n i\omega} \quad (1)$$

Once the modal parameters are extracted, a transformation matrix must be found and applied to the modal model that satisfies a compatibility condition as explained in [1]. As outlined by Allen and Mayes [3, 5], this is done by finding a matrix, B , that spans the null space of a set of physical equations transformed to modal coordinates. In practice, this can be done using various approaches, however in this study, B is found in the null space of a set of constraint shapes transformed to modal coordinates as shown in Eq. (2). The constraint shapes, U_c , are a set of orthogonal motions that describe the unique motions of the fixture calculated from a singular value decomposition of the extracted mode shapes at the fixture degrees of freedom [3]. The transformation matrix B can then be applied to the modal parameters as shown in Eq. (3) and Eq. (4). Note that the matrices M and K in these equations represent the modal mass and modal stiffness matrices such that M is unity and K is a diagonal matrix with entries of ω_n^2 . To find the constrained modal parameters, an eigenvalue decomposition is performed on the transformed K and M matrices as in Eq. (5). The constrained modal frequencies are found directly from this eigenvalue decomposition, and the constrained mode shapes are found as shown in Eq. (6).

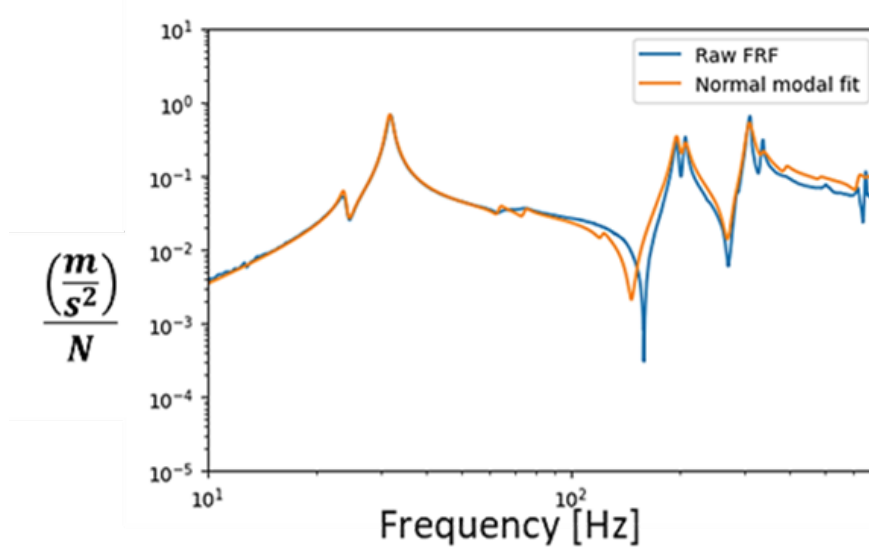


Fig. 1 Example Fit of Drive Point Transfer Function Using Normal Modes

$$B = \emptyset(U_c^T \phi) \quad (2)$$

$$M_c = B^T M B \quad (3)$$

$$K_c = B^T K B \quad (4)$$

$$\omega_c^2, \hat{\phi}_c = \text{eig}(K_c, M_c) \quad (5)$$

$$\phi_c = \phi B \hat{\phi}_c \quad (6)$$

In the ideal case, this approach achieves zero motion at the constraint points and a well-formed constrained mode shape. However, in practice, various types of measurement error accumulate to make the method imperfect. An example of this can be seen by constraining the first bending mode of an idealized FEA model of one of the test items examined in this study. In this example, an FEA model of a UUT 3 (later described and shown in Fig. 5a) is joined to a constrained flexible plate such that the interface is allowed to move. A Guyan Static Reduction followed by an Eigenvalue Decomposition is performed to achieve a mass normalized modal model at a set of response locations for mode shape visualization. The flexible interface response (before correction) can be seen in Fig. 2. Note the presence of unwanted motion at the base interface (Fig. 2a). Figure 3 shows the results of applying a MSC to twelve degrees of freedom (DOFs) at the fixture interface (three directions at four locations). The response of the first bending mode is then fully corrected and matches what we expect in the ideally constrained structure overlaid in red (18-bolt ring fully constrained). Adding additional constraint points would improve agreement at higher frequencies, but this example is focusing only on 1st bending. Figure 4 demonstrates that adding measurement noise to the mode shapes and truncating the number of retained modes corrupts the process resulting in a constrained mode frequency that is considerably higher than the true frequency as well as an irregular appearing mode shape. In a real test scenario, similar behavior is observed when applying MSCs. Because of this error accumulation, care must be taken in selecting the minimum number of measurement degrees of freedom that need to be constrained for the modes of interest. Further, it can be advantageous to add constraint shapes one at a time to determine if any of the shapes are creating spurious solutions. However, even when exercising these cautions, a considerable amount of uncertainty can remain in the true constrained mode frequency. Because of this, one of the primary objectives of this study is to quantify uncertainty as it manifests in practical scenarios when applying MSCs to constrained tests.

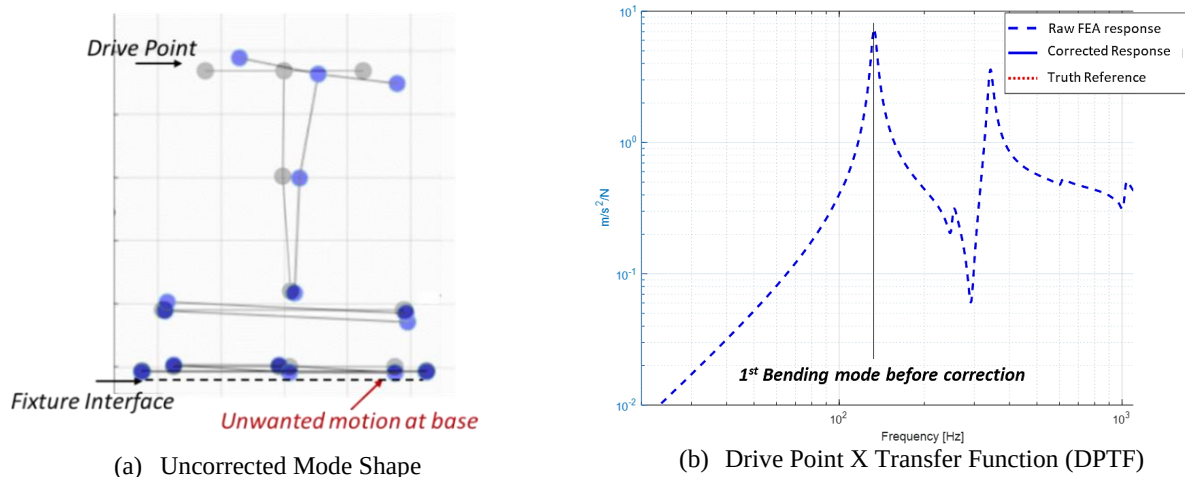


Fig. 2 FEA Lateral Bending Response with Flexible Fixture Interface Motion (Before Correction)

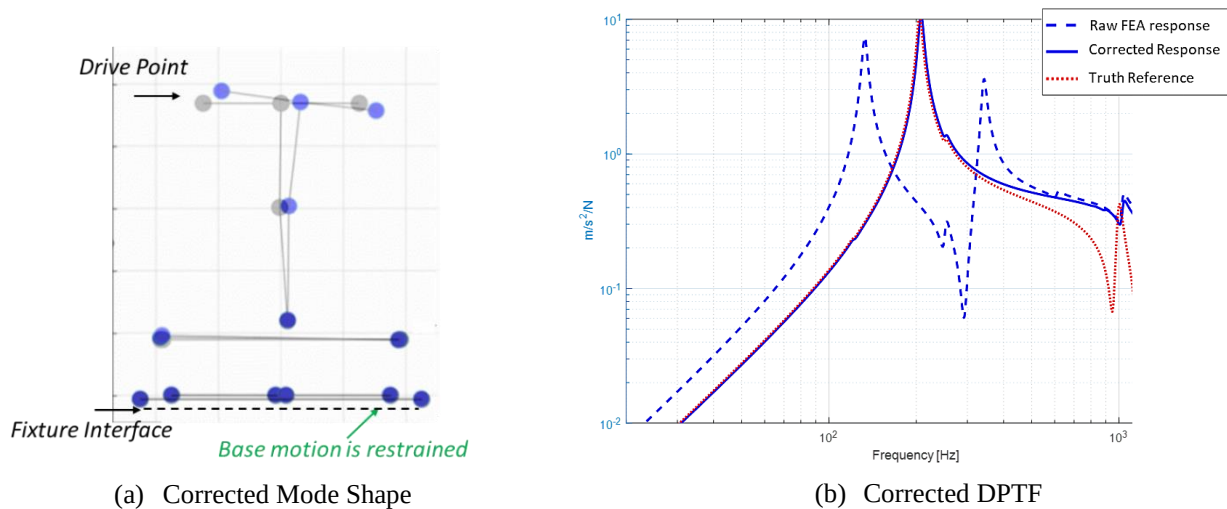


Fig. 3 Idealized Correction of 1st Bending Mode of FEA Model

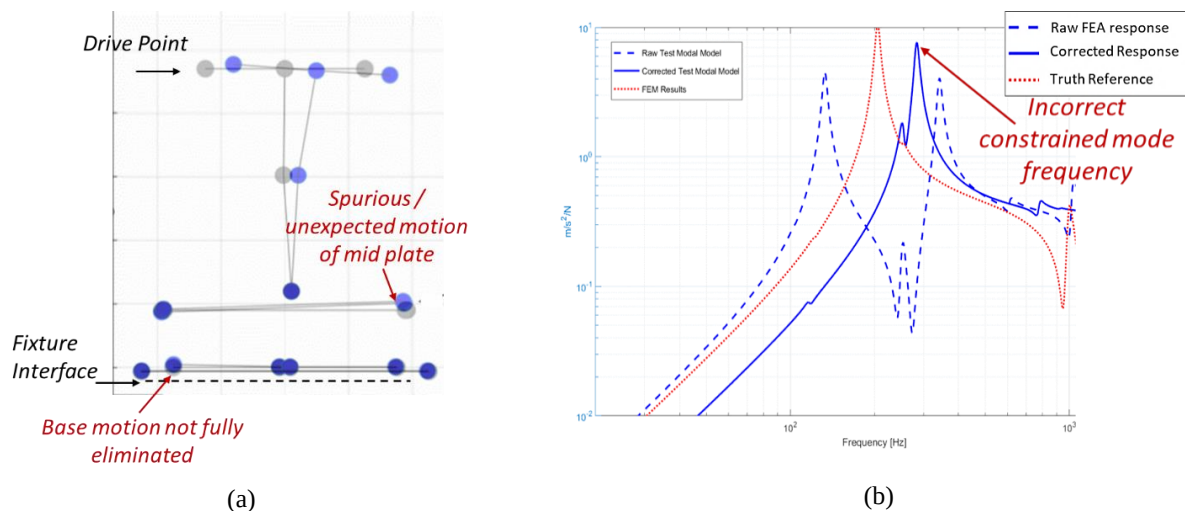


Fig. 4 1st Bending Mode Correction of FEA Model with Gaussian Noise and Truncated Modal Model

Test Setup

The authors of this study developed four UUTs with various boundary conditions resulting in eight distinct modal tests summarized in Table 1. For each test, the constrained mode frequency is estimated based on the algorithm discussed in the previous section. The calculated frequencies are compared to a “truth” reference to assess the resulting accuracy. The “truth” reference is based on the observed structural responses when the UUTs are clamped to a seismic table. For this study, no corrections are performed on the data received from the UUTs mounted to the seismic table, but it is expected that its efficacy as a “truth” reference below 200Hz is appropriate. For this reason, this study focuses on extracting constrained first bending modes in each axis of the UUTs. This provides fifteen samples of “primary modes” for comparison with the “truth” reference since one of the test items only has pronounced bending modes in a single axis. Some “Secondary Modes” (second bending and axial modes) were also processed during this study, but it was expected that the estimates of their constrained frequencies are significantly more erroneous.

Table 1 Executed Test Matrix

Test Number	DUT	Boundary Condition
1	Small Sat 1	1” Vibe Plate on Floor
2	Small Sat 2	1” Vibe Plate on Floor
3	Small Sat 3	1” Vibe Plate on Floor
4	Small Sat 1	Heavy Vibe Fixture on Springs
5	Small Sat 2	Heavy Vibe Fixture on Springs
6	Small Sat 3	Heavy Vibe Fixture on Springs
7	Small Sat 1	Heavy Vibe Fixture on Floor
8	Composite Blade	Steel Fixture with Elastomeric Element

UUTs 1-3 are various mass configurations of a small satellite dynamic simulator with adjustable ballast as shown in Fig. 5a. This test item is intended to identify logistical and technical challenges of performing constrained tests of smaller satellites. For this reason, a realistic separation ring simulator is incorporated into the design. The inclusion of a separation ring, is easily instrumented for EMS. However, for many frequency domain substructuring approaches, drive points would need to be measured in the narrow clearance of the separation ring which is found to be impractical. Additionally, the first three resonant modes and the CG of the satellite simulator are tuned to match a typical small satellite of that size. UUT 4 is a light composite blade clamped to a steel fixture, shown in Fig. 5b. This test item is chosen to add variance to the dataset and demonstrate an additional application for constrained modal testing such as identifying appropriate FEA properties for custom materials on aerospace components.



(a) Small Sat Dynamic Simulator with Adjustable Ballast: 3 Mass Configurations for 3 Unique Test Items



(b) Light Composite Blade Clamped to Steel Fixture on Elastomeric Elements

Fig. 5 UUTs used for this study

UUTs 1-3 are tested on two different flexible boundary conditions. The first, is bolted to a 1" plate that is resting on an uneven cement floor. This creates a quasi-linear flexible response of the plate boundary since the ground does not fully support the bottom of the plate. The second boundary condition for these test items is a 3" vibe fixture which rests on compression springs. This is selected to provide a repeatable linear boundary condition that is more controlled than the first. This boundary condition, with its springs, poses a challenge for the correction algorithm because it creates a nearly free-free response of the test item, but with rigid body modes within the measurable frequency range. A third boundary condition is tested on UUT1 where the 3" vibe fixture is placed directly on the uneven cement floor. UUT 4 is tested on a steel fixture on elastomeric elements.

Analytical Procedure

An analytical procedure is developed to reduce the influence of measurement noise on the resulting constrained frequencies. As discussed, when measurement noise is present, the extracted modal parameters and constraint shapes have inherent small errors. In these real scenarios, leveraging all the constraint shapes almost always results in an over-constrained response because of the inevitably imperfect orthogonalization of the mode shapes. In general, if the constraint shapes are removed one by one, at some point a "best estimate" of the constrained mode is found. Further, it was found in [2] that seemingly equivalent sets of constraint points result in slightly different solutions. The authors previously proposed in [2] that a proxy for accuracy of each result can be calculated by comparing the resulting shapes to an analytical or expected shape using modal assurance criterion (MAC) or cross orthogonality. If numerous perturbations of the constraint set are run in a sensitivity study, a repeatable constrained mode that matches the expected shape can indicate a "best estimate" solution. This procedure assumes that the influences of perturbations to the constraint during the sensitivity study are substantially larger than the true discrepancies between the expected and actual structural mode shape.

A MATLAB user interface is developed to implement this approach in an automated fashion. The primary motivation for the MATLAB tool is to prevent the user from selecting over-constrained solutions like that shown in Fig. 4 or solutions with insufficient constraint. The tool leverages a sensitivity-stabilization style approach by calculating all possible combinations of constraint points and constraint shapes and highlighting the calculated poles that match the expected analytical shape within a specified MAC tolerance. This approach toward exploring the solution space reveals repeatable solutions to the user to establishing a higher level of confidence in the selected solution. An example of this approach applied to real test data is shown in Fig. 6 applied to a test number four. In this case, the algorithm reveals a well-formed first bending shape is observable at the indicated frequency. The dotted blue line indicates the original test measurement of the modal response which is significantly softer than the "truth" reference shown in red. When the synthesized constrained modal model (blue solid line) is compared to the "truth" reference, it is found that the general shape of the frequency response function matches reasonably well, and the primary bending mode response frequency has an error of 4.5% compared against the "truth" reference.

Results

The procedure described in the previous section is repeated for all tests and all primary modes. Additionally, an attempt is made to extract the higher frequency "secondary" modes. The example shown in Fig. 6 is chosen as a representative example for two reasons: first, it reveals a level of error close to the average of all cases; second, it shows the power of MSCs to make extremely large corrections to a set of modal measurements. Test four places the UUT in a nearly free-free condition, with the first rigid body mode near 3Hz. Given this large dissimilarity from a constrained state, the algorithm is still able to resolve an approximate constrained response.

Figure 7 summarizes the accuracy of MSCs at resolving all primary mode frequencies. The grey data on the plot shows pre-correction results and the blue data shows the results after applying MSCs and estimating the constrained frequencies according to the process outlined above. On average, the mode frequencies are corrected to within 5% of the respective "truth" references and most cases fell within 8% (P80). Figure 8 examines the results from an uncertainty reduction perspective. It compares the needed improvement for each measured mode frequency to the amount of improvement that was actualized when the MSCs are applied. A key takeaway from this visualization is that applying an MSC to modal test results improves the estimate of the constrained frequency in almost all scenarios – even for secondary modes. Ideally, the data in Fig. 8 would follow a perfect 1:1 line that intersects zero (i.e. needed correction is perfectly achieved). However, the fit to the primary mode correction data shows an expected deviation from that ideal. This study finds that using MSCs to make corrections to modes that are already within 8% provides reduced benefit, but still improves results in most cases.

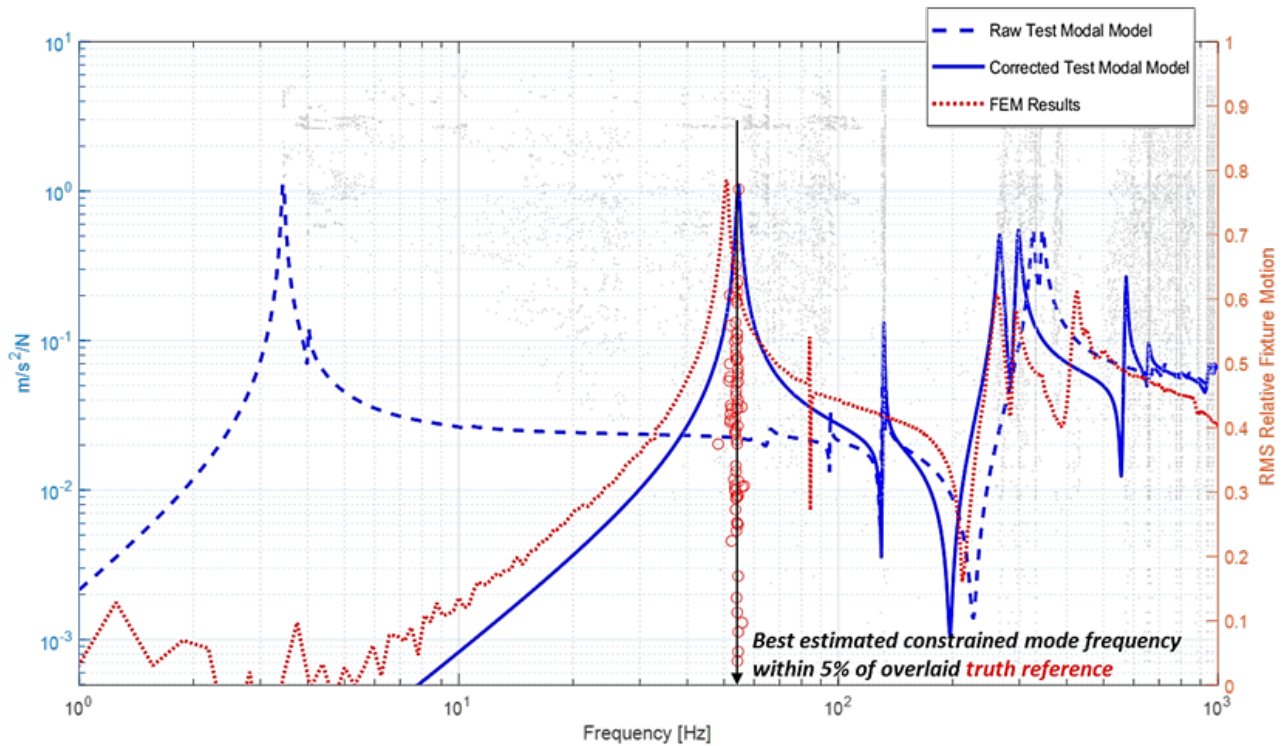


Fig. 6 MATLAB User Interface Results for UUT1 on Test 4, X Axis Drive Point Transfer Function

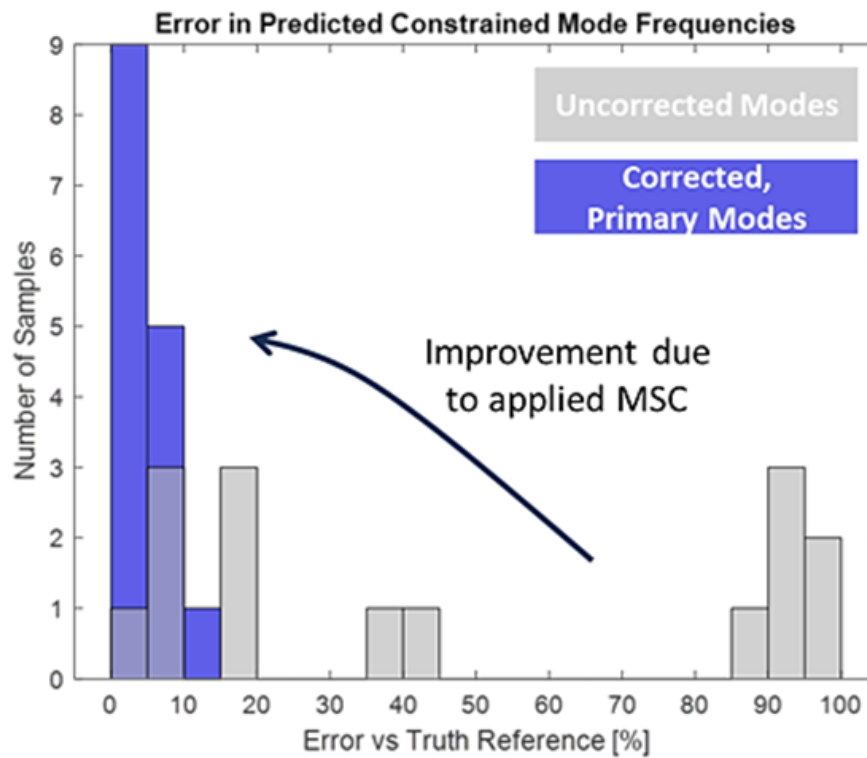


Fig. 7 Effect of Modal Substructure Constraints on Primary Modes Compared to Truth Reference

Further, making finer adjustments $<5\%$ may increase risk of “over-correcting” and incorporating more error into results (observed in only one out of thirty instances). It is important to note that in all cases MSCs improve the overall shape of the frequency response functions revealing the nature of the interaction with the test fixture (e.g. which response peaks are driven by fixture vs the UUTs natural response).

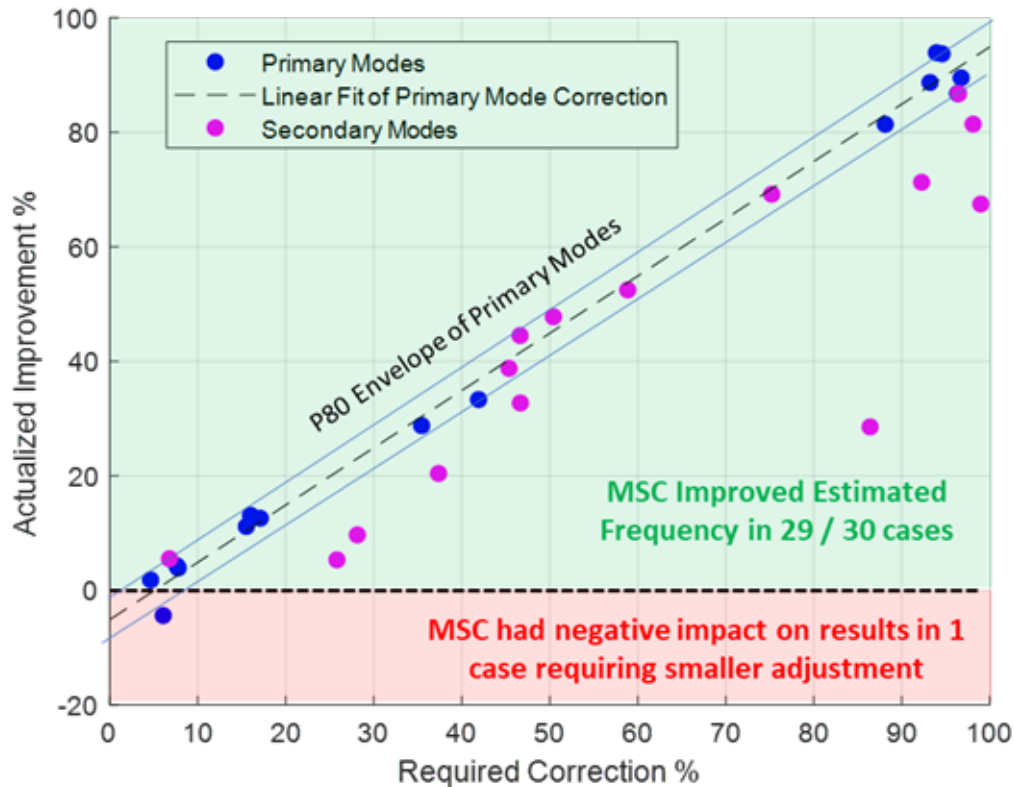


Fig. 8 Actualized Improvement vs Required Correction (%Relative to Truth Frequencies) when MSC’s were Applied to Test Results

Conclusion

This study investigates the efficacy of MSCs for removing flexible boundary effects during constrained modal tests of realistic aerospace structures. Four test items are investigated under various boundary conditions. This study finds that MSCs are a powerful tool for reducing risk and uncertainty during a constrained modal test with little additional expense or technical scope. They offer a practical solution when cost, schedule, or logistics limit the efficacy of a constrained boundary, making modal testing accessible to many programs that previously did not utilize it. In this study, MSCs prove capable of making significant corrections to experimentally measured modal parameters, substantially reducing the influence of an unwanted flexible fixture or boundary. However, the application of MSCs is most beneficial when large corrections are needed to primary modes $>8\%$ (P80). Further, the observed trends indicate that smaller, fine-tuning adjustments $<5\%$ may not add substantial value without special attention to fixture properties or measurement accuracy that fall beyond the scope of this study. A practical interpretation of these error margins is offered in Fig. 9. In this process, a modal test campaign must select a threshold “X%”. If the discrepancy between a corrected test mode and FEM mode is greater than that threshold, an adjustment to the FEM is likely to benefit the value of the FEM predictions after a model update is made. This study preliminarily determines that an appropriate value for that threshold lies between 5% and 8%. Using a value lower than 5% will often result in unneeded corrections; using a value higher than 8% will often miss needed corrections. Further research should be continued to investigate ways of reducing these error margins as well as comparing the efficacy of modal substructuring constraints to other leading approaches in a similar light.

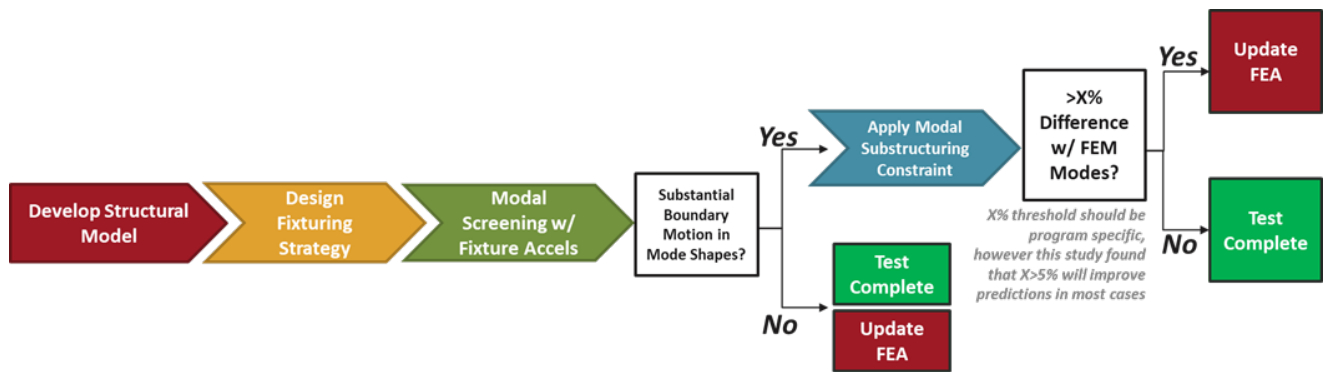


Fig. 9 Decision tree describing practical implementation of modal substructure constraints in a modal test for purposes of FEM updating

Acknowledgments Dr. Matthew S. Allen

References

1. Klerk, D. d., General Framework for Dynamic Substructuring: History, Review, and Classification of Techniques. AIAA, 2008
2. Johnson, A. Improved Vibration Test Predictions Leveraging a Pre-Vibe Modal Screening, 2024
3. Allen, Mayes, Experimental Modal Substructuring to Extract Fixed-Base Modes from a Substructure Attached to a Flexible Fixture, 2011
4. Kerrian, P., A Comparison of Different Boundary Condition Correction Methods, Topics in Modal Analysis and Testing, Chapter 34, 2021
5. Allen, Mayes, Experimental Modal Substructuring to Couple and Uncouple Substructures with Flexible Fixtures and Multi-point Connections

