

Dynamics of Civil Structures, Vol. 2

Matthew Whelan
P. Scott Harvey
Fernando Moreu



Proceedings of the 43rd IMAC, A Conference and
Exposition on Structural Dynamics 2025



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Matthew Whelan • P. Scott Harvey • Fernando Moreu
Editors

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Preface

Dynamics of Civil Structures represents one of twelve volumes of technical papers presented at the 43rd IMAC, A Conference and Exposition on Structural Dynamics, organized by the Society for Experimental Mechanics, and held February 10-13, 2025. The full proceedings also include volumes on Nonlinear Structures & Systems; Model Validation and Uncertainty Quantification; Dynamic Substructuring & Transfer Path Analysis; Special Topics in Structural Dynamics & Experimental Techniques; Computer Vision & Laser Vibrometry; Dynamic Environments Testing; Sensors & Instrumentation and Aircraft/Aerospace Testing Techniques; Topics in Modal Analysis & Parameter Identification I&II; Data Science in Engineering; Data Science in Engineering; and Structural Health Monitoring & Machine Learning.

Each collection presents early findings from analytical, experimental and computational investigations on an important area within Structural Dynamics. Dynamics of Civil Structures is one of these areas which cover topics of interest of several disciplines in engineering and science.

The Dynamics of Civil Structures Technical Division serves as a primary focal point within the SEM umbrella for technical activities devoted to civil structures analysis, testing, monitoring, and assessment. This volume covers a variety of topics including structural vibrations, damage identification, human-structure interaction, vibration control, computer vision, model updating, modal analysis of in-service structures, innovative measurement techniques and mobile sensing, and bridge dynamics among many other topics.

Papers in this volume cover testing and analysis of different kinds of civil engineering structures, including buildings, bridges, wind turbine towers, marine structures, historical constructions, and light poles.

The organizers would like to thank the authors, presenters, session organizers, and session chairs for their participation in this track.

Editors:

Matthew Whelan—*University of North Carolina-Charlotte*, NC, USA; P. Scott Harvey—*University of Oklahoma*, OK, USA; Fernando Moreu—*University of New Mexico*, NM, USA.

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Chapter 1

Ambient Vibration Testing and Modal Response Analysis of a Curved Concrete Bridge

Mehrtash Motamedi, Carlos E. Ventura, and Brook Robazza

Abstract This paper describes a series of ambient vibration tests conducted on a curved concrete bridge in British Columbia, Canada. Modal response analysis was performed to identify the dynamic properties of the structure, including predominant natural frequencies and the corresponding mode shapes to support seismic assessment and upgrading of the bridge. The bridge was built in 1956, and spans over Nelson Creek near Horseshoe Bay, British Columbia to carry traffic. After the construction of a highway nearby, it serves as a pedestrian and utility bridge. The Nelson Canyon bridge is a six-span bridge with a total length of 80m. The superstructure consists of a concrete deck, steel longitudinal girders and transverse floor beams. The substructure includes two concrete abutments at both ends, concrete pier at the end of the jump span and two steel truss towers.

The ambient vibration testing method was implemented using sophisticated methods of modal analysis. Vibration Tests were conducted at the bridge in order to determine the dynamic modal properties (modal frequencies and mode shapes) of the bridge. The testing program consisted of multiple measurement setups. Tromino® velocity/acceleration wireless sensors were used for these measurements which were placed on predetermined locations. The computer program ARTEMIS was used to perform the system identification of the structure. The software allows to develop a 3D model of the structure and test points; the resulting mode shapes are displayed using this geometry. Two different techniques were used for modal identification: the Enhanced Frequency Domain Decomposition (EFDD) and the Stochastic Subspace Identification (SSI). These two modal identification techniques were used to cross-validate the results. The joint analysis of the signals measured in various strategic points of the structure made it possible to identify the modal configurations and the corresponding natural frequencies.

As result, a total of 8 modes were identified in the 2 to 11 Hz frequency range. Modal frequencies, modal damping and mode shape were identified for each of the 8 modes. The modes associated with vertical response of the deck showed that the bridge deck is flexible.

Keywords Ambient Vibration Test · Modal Response · Concrete Curved Bridge · Frequency, Mode Shape

Introduction

Nelson Canyon Bridge (located at 49.36299, 123.26039) was built in 1956 and spans over Nelson Creek near Horseshoe Bay in West Vancouver, BC, Canada. This bridge was built to carry the traffic but after the construction of Highway 99, it serves as a pedestrian and utility bridge that was categorized as a lifeline structure. The bridge is a curved six span deck with a total length of 80 m including a 12 m concrete jump span on the west approach with the radius of 87 m for the inner edge, with width of 10.1 m and cross-fall of 8%. The superstructure consists of a concrete deck with steel transverse floor beams supported by two lines of steel longitudinal girders. The substructure includes two concrete abutments at both ends with spread footings on bedrock, concrete pier at the end of the jump span and two steel truss towers. These towers were

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constructed from W sections and built up lattice sections, which were built on the concrete pedestals cast directly onto the bedrock, ensuring a robust foundation. Fig. 1 shows the plan and elevation view of the bridge.

A targeted seismic evaluation of the bridge was planned in the District of West Vancouver. The purpose of this assessment was to determine whether the bridge has sufficient capacity to support the pedestrian and utility weight in earthquakes and, if required, quantify the extent of strengthening works required to increase the seismic capacity of the bridge [1].

A series of Ambient Vibration Test (AVT) were suggested for this bridge in order to determine the modal frequencies and the mode shapes of the bridge. Prior to the AVTs, a site visit to the bridge was conducted to determine the location of the measurements and develop a test plan. The test plan included ambient vibration measurements on the bridge deck, abutments and approaches. The vibration tests were then carried out using velocity sensors on the bridge. The main purpose of the AVTs was to determine the fundamental frequency and corresponding mode shape of the bridge. This information would help calibrate the computer model being used for the seismic evaluation of the bridge [2].

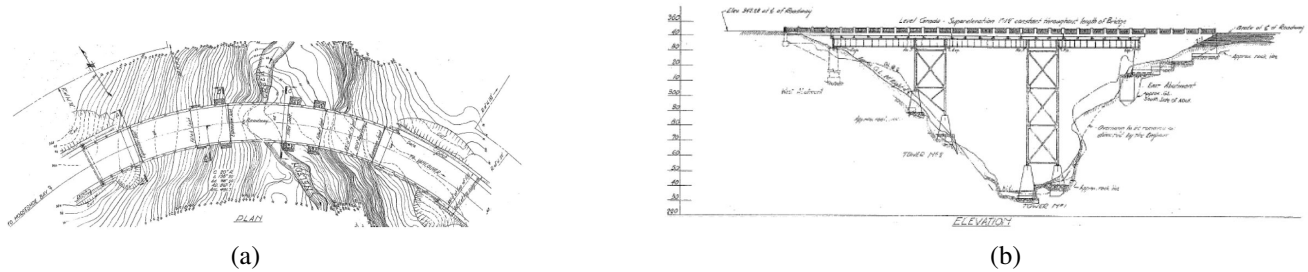


Fig. 1 Plan and elevation view of the bridge: a) Plan view; b) Elevation view

Ambient Vibration Tests

Vibration Tests (VTs) were conducted at the bridge in order to determine the dynamic modal properties (modal frequencies and mode shapes) of the bridge. The testing program consisted of six measurement setups. Tromino® velocity/acceleration sensors were used to carry out these VTs [3]. The collected records were time synchronized with a radio antenna and amplifier in each sensor. This allowed the synchronization of the recordings both within each measurement setup and between setups. The Tromino sensors are suitable for high-resolution ambient vibration tests as they are fully portable, wireless, compact, and light instruments. Each sensor is equipped with two sets of three orthogonal high-resolution electrodynamic sensors (high gain and low gain velocity meter) and one set of three orthogonal digital accelerometers with a frequency range of 0.1 to 300 Hz. For these tests the high-gain velocity data was used for the modal identification process. Fig. 2 illustrates the location of the sensors on the bridge during the tests.

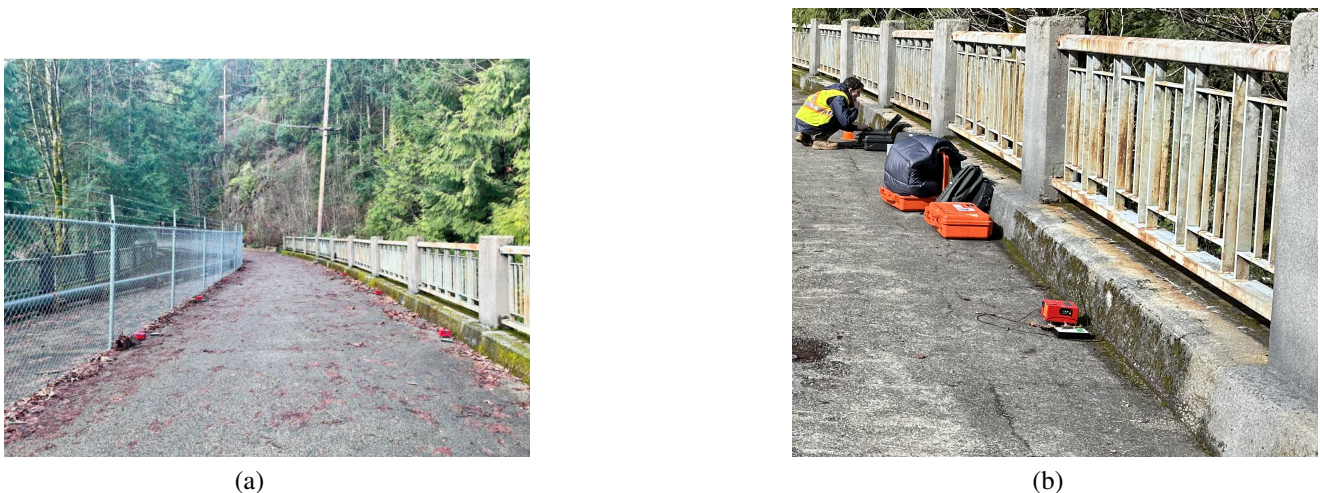


Fig. 2 Location of the sensors on the bridge during the tests: a) General view, b) Close up of a sensor

Modal Analysis

The computer program ARTEMIS Version 6, was used to perform the modal analysis of the specimen [4]. The software allows developing a 3D model of the structure and test points; the resulting mode shapes are displayed using this geometry. Two different techniques were used for modal identification: the Enhanced Frequency Domain Decomposition (EFDD) and the Stochastic Subspace Identification (SSI). These two modal identification techniques were used to cross-validate the results. The joint analysis of the signals measured in various strategic points of the structure made it possible to identify the modal configurations and the corresponding natural frequencies [5]. The Power Spectra Densities obtained from ambient vibration measurements are illustrated in Fig. 3 for both methods. The peak values show the natural frequency of the bridge in each mode.

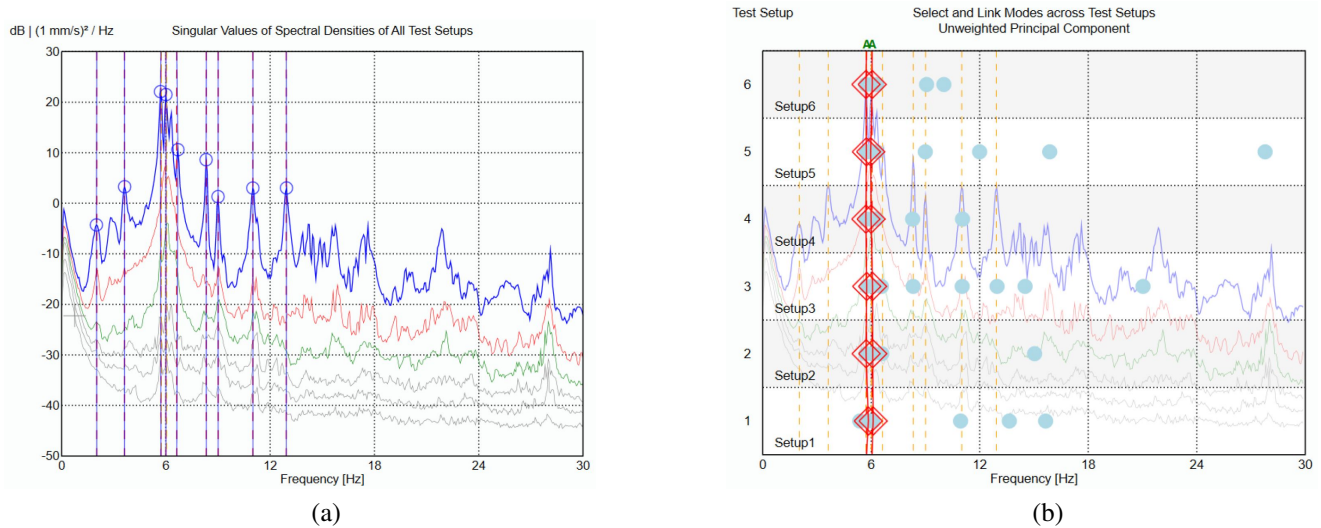


Fig. 3 Power Spectral Densities obtained from ambient measurements: a) Singular values of spectral densities of all test setups in EFDD method, b) Linked Modes for SSI UPC method

Based on this analysis, A total of 8 modal frequencies in the 2 to 11 Hz range were identified with confidence. The predominant frequency (the first natural frequency) of the structure is estimated to be 2.03 Hz, and it corresponds to the fundamental mode of the bridge in the vertical direction. The identified modal frequencies and associated damping values are presented in Table 1. The presented modal frequencies from the vibration data are those for which the confidence on the results is high from both the EFDD and SSI-UPC methods. Some of the associated mode shapes are presented in Fig. 4.

Table 1 Natural frequency, period and damping of combined analysis

Mode	Frequency (Hz)	Period (sec)	Damping (%)	Mode description
1	2.03	0.49	5.43	Vertical
2	3.63	0.28	0.00	Transverse
3	5.71	0.18	0.91	Torsional
4	6.00	0.17	0.86	Transverse and vertical
5	6.62	0.15	0.88	Transverse and vertical
6	8.33	0.12	0.51	Torsional and vertical
7	9.01	0.11	0.61	Torsional and vertical
8	11.01	0.09	0.73	Vertical

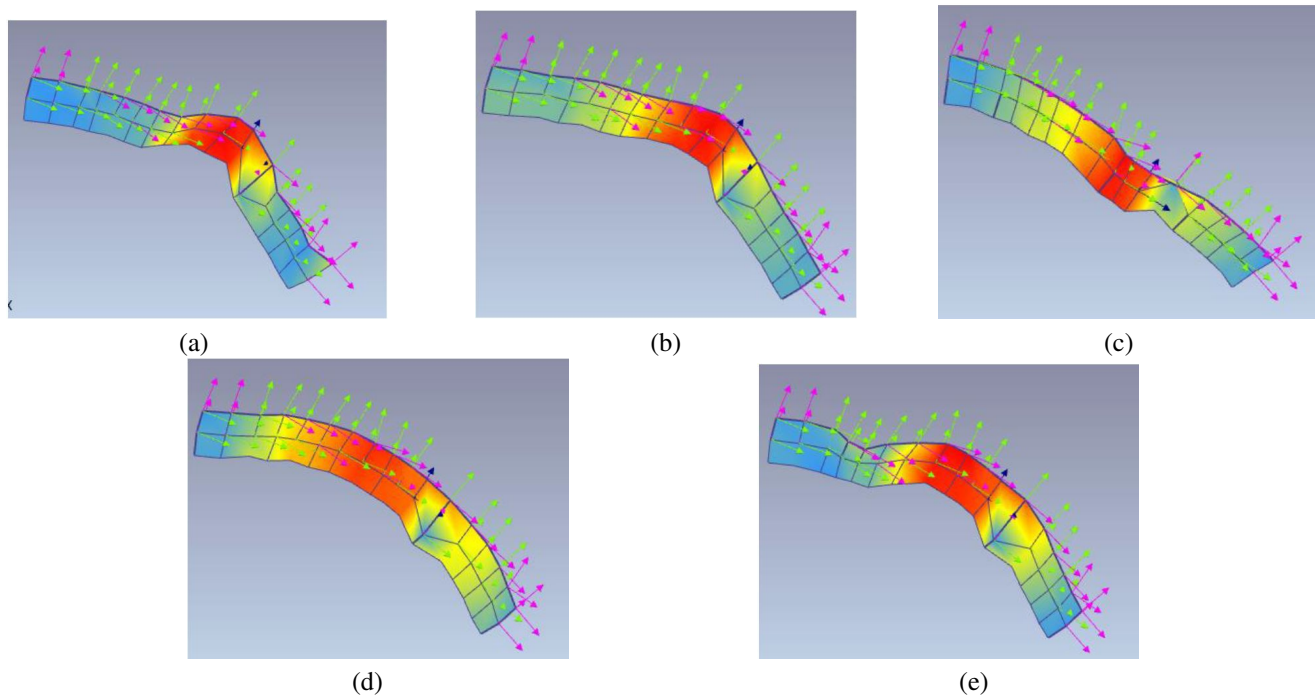


Fig. 4 Identified mode shapes of the bridge: a) 1st mode, b) 2nd mode, c) 3rd mode, d) 4th mode, e) 5th mode

Conclusion

Ambient vibration tests of a concrete curved bridge in British Columbia, Canada were conducted. The main objectives of these vibration tests were: 1) to determine the vibration characteristics of the bridge; and 2) to perform a modal analysis using the vibration data in order to determine the predominant natural frequencies and the corresponding mode shapes of the bridge. The testing program consisted of six measurement setups. The results from the modal analyses indicated that the fundamental frequency of the bridge in the vertical direction 2.03 Hz. The first torsional mode of the deck is at 5.71 Hz. A total of 8 modes were identified in the 2 to 11 Hz frequency range. Modal frequency, modal damping and mode shape were identified for each of the 8 modes. The modes associated with vertical response of the deck showed that the bridge deck is flexible.

Acknowledgments This paper has been extracted from a term project report prepared by the following students of the CIVL508 course in the Department of Civil Engineering at the University of British Columbia, class of winter 2024: Ehsan Ferdosi, Sony Fu, Dyllon Hunt, Amber Shen, Jonathan Steele, Weite Su, Alex Sun, Bowen Wu. Their input is really appreciated. This project was carried out for the purpose of determining the modal properties and seismic assessment of the Nelson Canyon Bridge in West Vancouver, BC as part of the Bridge and Pier Inspection Program. This collaboration with the City of West Vancouver is acknowledged.

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Chapter 2

Response Modification of Structures with Supplemental Rotational Inertia

Nicos Makris and Gholamreza Moghimi

Abstract Tall, multistory, buildings are becoming increasingly popular in large cities as a result of growing urbanization trends. As cities continue to grow, many of them along the coasts of continents which are prone to natural hazards, the performance of tall, flexible buildings when subjected to natural hazards is a pressing issue with engineering relevance. The performance of structures when subjected to dynamic loads can be enhanced with various response modification strategies which have been traditionally achieved with added stiffness, flexibility, damping and strength. Together with the elastic spring that produces a force proportional to the relative displacement of its end-nodes and the viscous dashpot that produces a force proportional to the relative velocity of its end-nodes; the inerter produces a force proportional to the relative acceleration of its end-nodes and emerges as the third elementary mechanical element (in addition to the spring and dashpot) capable for modifying structural response. Accordingly, in this lecture we examine the seismic performance of multistory and seismic isolated structures when equipped with inerters upon deriving selective frequency- and time-response functions of the inerter and other simple inertoelastic and inerto viscous network in association with the mathematical operations involved.

Keywords Inerter · Supplemental rotational inertia · Response modification · Seismic protection · Nonlinear analysis

Introduction

Response modification of civil structures has been traditionally achieved with added stiffness, flexibility damping and strength ([1–12] and references reported therein). Nevertheless, together with the elastic spring that produces a force proportional to the relative displacement of its end-nodes and the viscous dashpot that produces a force proportional to the relative velocity of its end nodes, the inerter produces a force proportional to the relative acceleration of its end-nodes and emerges as the third elementary mechanical element (in addition to the spring and dashpot) for the synthesis of mechanical networks [13–16]. While a driving spinning top as the one shown in Figure 1, which is a physical realization of the inerter, has been familiar to several generations; inerters were apparently first proposed for the response modification of buildings in the mid-1980s by Kawamata [17] and subsequently by Ishimaru [18] and Arakaki et al. [19, 20] in the mid to late 1990s in Japan.

Following the pioneering work in Japan and the subsequent theoretical and experimental studies by Smith and coworkers [13, 21–24] who established, within the context of linear networks, the analogy of the inerter to the electric capacitor; a growing number of publications have proposed the use of inerters for the response modification of buildings by installing them at all floor levels [25–28]; at selected levels [29–33] or at the ground level, either within the context of protecting a first soft-story [34–36], or within the context of enhancing a seismic isolation system [37–41].

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Fig. 1 A physical realization of the inerter in which the force output is proportional only to the relative acceleration between nodes 1 and 2.

Background

Supplemental rotational inertia can be achieved either with a rack-and-pinion- flywheel arrangement, a ball-screw assembly as shown in Figure 1 [19–21, 26, 28, 42–44] or fluid inerters [45–48]. The mechanical system shown in Figure 1 is a mechanical analogue of the electrical capacitor in a force-current/velocity-voltage analogy between mechanical and electrical networks. This missing analogy was first recognized by Smith [13] who coined the term inerter for any mechanical arrangement where the output force is proportional only to the relative acceleration between its end-nodes. The constant of proportionality of the inerter is coined the inertance $= M_R$ [13] and has units of mass [M]. Accordingly, with reference to Figure 1 if F_1 , u_1 and F_2 , u_2 are the forces and displacements at the end-nodes of the inerter with inertance, M_R , its constitutive relation is [15, 35, 37]

$$\begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix} = \begin{bmatrix} M_R & -M_R \\ -M_R & M_R \end{bmatrix} \begin{Bmatrix} \ddot{u}_1(t) \\ \ddot{u}_2(t) \end{Bmatrix} \quad (1)$$

The basic frequency and time-response functions of the mechanical element defined by equation (1) have been derived by Makris [14, 15] and are covered in the lecture together with those of simple inertoelastic and inerto viscous elements. The unique characteristic of the inerter is that it has an appreciable inertia mass as opposed to a small gravitational mass.

The potential unfavorable situation when the rotational inertia stored in the inerter may drive the structure was first recognized by Makris and Kampas [34] who introduced the implementation of two parallel inerters together with the use of a clutch (pair of clutching inerters), so that the rotating flywheels only resist the motion of the structure without inducing any deformations. The benefits of using a pair of counter-rotating clutching inerters were subsequently examined on a 2DOF elastic structure [35]. About the same time De Domenico and Ricciardi [39] examined the response of a multi-degree-of-freedom seismic isolated elastic structure where the isolation system was enhanced with a tuned mass damper inerter. In that study the nonlinear behavior of the isolator is accounted for, and subsequently, the study proceeds with a stochastic linearization technique.

A seminal contribution among the aforementioned studies is the work of Furuhashi and Ishimaru [25] who showed that when inerters are installed without being interrupted, starting from the first level, then the inertance of the inerters can be adjusted to eliminate the participation of higher modes. At the same time, the unique ability of the inerters to eliminate the contribution of higher modes vanishes when they are placed at a higher level without having inerters at all the floors down below to the first level. Following Furuhashi and Ishimaru's [25] seminal contribution, Takewaki et al. [28] explained that for inerters to suppress the induced ground acceleration they need to be installed without being interrupted, starting from the ground level; otherwise, the inerters which are installed above the “interrupted” level can no-longer suppress the ground induced acceleration. In spite of Takewaki et al. [28] important finding-that solitary inerters when placed at higher levels do not suppress the ground induced acceleration, several subsequent publications investigated the response of structures equipped with a solitary inerter at a floor level other than the first level [29–33] within the context of proposing an enhanced tuned mass damper. Soon after Takewaki et al. [28] publication, Lazar et al. [31] examined in detail the role of a tuned inerter

damper (TID)—which is essentially an inerter supported on a compliant support (a spring-dashpot parallel connection), as an alternative tuned mass damper (TMD) and they correctly concluded that the TID is more effective when placed at the ground level, reaffirming the findings of Takewaki et al. [28].

In view of the appreciable number of publications that examine the response of tall buildings when equipped at a higher level with a solitary mechanical network that involves inerters [32, 33, 49], in association that this concept has enjoyed full-scale implementations [29, 30], this lecture reviews, the dynamics of structures equipped with inerters and how inerters modify elastic and inelastic structural response.

Analysis

The Distinguishable Characteristics of the Inerter

a) Suppression of the induced ground acceleration

Figure 2 depicts a single-degree-of-freedom undamped structure with stiffness k and mass m . A stiff chevron frame supports a flywheel with radius R and mass m_W that can rotate about an axis O. In the interest of simplicity, we consider the case of a non-compliant chevron frame whose deformation is negligible to the translational displacement, $u(t)$, of the SDOF structure. The case of a compliant chevron frame has been examined in depth in past publications [34–36]. Concentric to the flywheel, there is an attached pinion with radius ρ engaged to a linear rack connected to the bottom of the vibrating mass m of the SDOF. With this arrangement when the mass m undergoes a positive displacement, $u(t)$, the flywheel is subjected to a clockwise rotation, $\theta(t)$. Given that there is no slipping between the rack and the pinion, $u(t) = \rho\theta(t)$, the internal force $F_I(t)$ that develops along the rack-pinion interface satisfies dynamic moment equilibrium

$$F_I(t)\rho = I_W\ddot{\theta}(t) \quad (2)$$

where $I_W(t) = (1/2)m_W R^2 =$ moment of inertia of the flywheel about point O. With reference to Figure 2, for a positive displacement, $u(t)$, to the right, the internal force, $F_I(t)$ given by equation (2) opposes the motion of the mass, m (to the left). Accordingly, dynamic equilibrium of the vibrating mass when subjected to a ground acceleration, $\ddot{u}_g$, gives [34]

$$\ddot{u}(t) + \frac{\omega_0}{1 + \frac{1}{2} \frac{m_W R^2}{m \rho^2}} u(t) = - \frac{1}{1 + \frac{1}{2} \frac{m_W R^2}{m \rho^2}} \ddot{u}_g(t) \quad (3)$$

where $\omega_0 = \sqrt{k/m}$ = natural frequency of the structure when the pinion-flywheel is disengaged and m_W is the mass of the flywheel.

Equation (3) indicates that the engagement of the flywheel lengthens the vibration period of the structure [50]; yet, most importantly, it suppresses the level of ground shaking given that the denominator in the right-hand side is always larger than

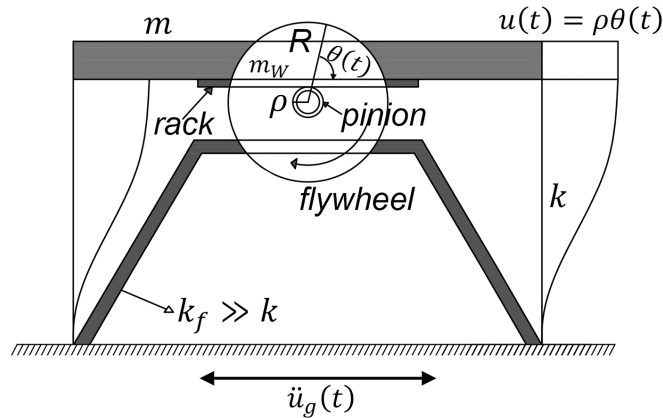


Fig. 2 Single-degree-of-freedom structure with mass m and stiffness k with supplemental rotational inertia from a flywheel with radius R supported on a chevron frame with stiffness k_f that is much larger than k .

unity [34]. The quantity $M_R = (1/2)M_W(R^2/\rho^2)$ is defined as the inertance of the inerter with units of mass $[M]$, whereas the ratio $\sigma = M_R/m$ appearing in the denominators of equation (3) is the inertance ratio.

b) Elimination of the participation of a higher mode

In a study with remarkable insight, Furuhashi and Ishimaru [25] showed that when the inertance of an inerter placed at the ground level is judiciously selected, the supplemental rotational inertia eliminates the contribution of a higher mode. In a recent study that investigated the implementation of inerters in seismic isolated structures, Makris and Moghimi [41] showed that while a small amount of supplemental rotational inertia is needed to eliminate the participation of the second mode of the 2DOF linear isolated structure; the effect of this elimination is marginal on the structure response, since the participation of the second mode is invariably small even when isolation systems without inerters are used. The nonlinear response analysis of the same 2DOF isolated structure is examined by adopting a bilinear behavior for the isolation system in association with a formulation that accounts for the compliance of the support of the inerter. Our study shows that supplemental rotational inertia aggravates superstructure displacements and accelerations at larger isolation periods ($T_b > 2.5\text{sec}$). In view of these findings in association with the small gains in reducing displacements above isolators, the use of inerters in isolation systems is not recommended.

Conclusion

In this lecture we first present the dynamics of single-story and multistory structures with supplemental rotational inertia and how inerters modify their elastic or inelastic response. Because of the participation of the inerter, some basic time-response functions of the examined mechanical networks exhibit a causal oscillatory response and this behavior complements the decaying-exponential response due to the participation of the dashpot. It is shown that the inerter emerges as an attractive response-modification element given that in some cases it “absorbs” the impulsive response of the solitary spring or dashpot present in the mechanical network. The basic response-functions derived for the inerters in association with the mathematical operations outlined, extends the well-established theory of linear viscoelasticity to the inertoelastic and inertoelastoviscoelastic behavior (combination of inerters, dashpots and springs) and introduces the subject of inertoelastoviscoelasticity.

Supplemental rotational inertia controls effectively the displacements of the first story of a 2DOF elastic structure along a wide range of the response spectrum. The proposed seismic protection strategy can accommodate large relative displacements without suffering from the issue of viscous heating ([51–53]) and potential leaking that challenges the implementation of fluid dampers under prolonged cyclic loading. When the chevron frame that supports the rotational inertia system is stiff, the use of two parallel rotational inertia systems offers improved results for the response of the 2DOF structure. However, as the compliance of the chevron frame that supports the inerters increases, the use of a single rotational inertia system offers more favorable response other than increasing the forces transferred to the chevron frame.

While a small amount of supplemental rotational inertia is needed to eliminate the participation of the second mode of the 2DOF linear isolated structure; the effect of this elimination is marginal on the structure response, since the participation of the second mode is invariably small even when isolation systems without inerters are used ([5, 54]). The nonlinear response analysis of the same 2DOF isolated structure is examined by adopting a bilinear behavior for the isolation system in association with a formulation that accounts for the compliance of the support of the inerter. Our study shows that supplemental rotational inertia aggravates superstructure displacements and accelerations at larger isolation periods ($T_b > 2.5\text{sec}$). In view of these findings in association with the small gains in reducing displacements above isolators, the use of inerters in isolation systems is not recommended.

The response analysis of a SDOF elastoplastic and bilinear structure reveals that when the yielding structure is equipped with supplemental rotational inertia (inerters), the equal-displacement rule is valid starting from lower values of the preyielding period given that the presence of inerters lengthens the apparent preyielding period. Furthermore, inerters suppress effectively the inelastic displacements of SDOF yielding structures; while the resulting base shears are systematically lower than when large values of supplemental damping ($\xi_d = 25\%$) are used. The forces transferred at the mounting of the inerters are appreciably lower than the corresponding forces originating from an elastic structure. Consequently, the implementation of inerters emerges as an attractive response modification strategy for elastoplastic and bilinear SDOF structures with larger preyielding periods. The use of a pair of clutching inerters does not offer any additional benefits compared to the case where a single inerter is used. Pair of clutching inerters are found to be attractive when suppressing the response of elastic structures.

Inerters are also effective in suppressing the inelastic response of 2DOF yielding structures with pre-yielding periods up to $T_1 = 1.5\text{sec}$ without aggravating the inelastic response of the superstructure. The effectiveness of inerters to suppress the inelastic response of the 2DOF yielding structure outperforms the effectiveness of large values of supplemental damping

($\xi_d = 25\%$) when the support frame of the response modification device is compliant. In view of these findings the use of inerters emerges as an attractive response modification strategy and is recommended for yielding structures. For larger pre-yielding periods (say $T_1 > 2.0\text{sec}$), the effectiveness of inerters to suppress the inelastic response of 2DOF yielding structures reduces; and for very flexible first stories; as in the case of isolated structures, the use of inerters at the first level (isolation system) is not recommended.

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Chapter 3

Effect of Bottom Flange Angle Blocking on the Dynamic Response of Two All-Steel Laboratory Floors

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Abstract This paper presents an experimental investigation into the effect of angle blocking on the dynamic behavior of all-steel laboratory floors. Two all-steel floor specimens, one unbraced and the other braced with angle blocking members, were subjected to dynamic testing to evaluate the impact of this design modification on the floor's dynamic behavior. The results demonstrate that angle blocking significantly enhances the dynamic behavior of the specimens, increasing both modal frequencies and stiffness. The findings indicate that angle blocking installation is an effective method for controlling beam torsion and increasing stiffness, thus improving vibration serviceability. The study's findings have significant implications for the design and construction of steel floors in modern buildings, particularly in office spaces where floor vibration complaints are prevalent.

Keywords Angle blocking · Dynamic testing · All-steel floor · Modal testing · Modular construction

Introduction

All-steel floor systems are distinguished by their low self-weight and limited inherent damping, which can make satisfying vibration serviceability criteria particularly challenging. This challenge is compounded by the need for longer beam and girder spans in modern building design [1]. In addition, the trend towards more open and flexible office layouts, featuring reduced partitioning and lighter furniture, has contributed to a growing concern about floor vibration serviceability in modern office buildings [2].

Furthermore, long unbraced steel beams supporting the floors are prone to torsional and sideways movement, which can be particularly problematic in terms of dynamic behavior and vibration serviceability when the diaphragm on top of the beams is slender. Connecting the bottom flanges of the W-beams, using angle blocking members provides lateral bracing to the floor and it is a promising solution to address these challenges. Angle blocking can help to reduce the torsional movement of beams and increase the stiffness of the floor, thereby improving its overall dynamic behavior. By providing additional lateral support to the floor, angle blocking can help to mitigate the effects of bottom flange sway and reduce the potential of floor vibration complaints. In addition, angle blocking can be easily applied to existing floor systems, making it a practical and cost-effective retrofit option for improving floor vibration serviceability.

The FastFloor system examined in this study eliminates the need for concrete on steel deck components, which are predominantly used in traditional composite floors [3]. FastFloor is a pioneering, modular steel floor system designed to meet the goals of the "Need for Speed" initiative by American Institute of Steel Construction (AISC). The FastFloor system aims to reduce steel construction time by up to 50%. By prefabricating modular units off-site and minimizing non-steel

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components, FastFloor facilitates rapid construction and erection on steel girders, featuring a reduced weight and carbon footprint [1]. A typical full-bay FastFloor is designed to be 30 ft $\times$ 40 ft on plan. However, the tested specimen configurations are 10 ft $\times$ 40 ft slices (modules) of the full-bay FastFloor.

This study aims to investigate the dynamic behavior of a scaled-down FastFloor specimen in a controlled laboratory environment through experimental modal testing on two FastFloor modules with unbraced and braced conditions. The bracing is achieved by welding angle blocking members to the bottom flanges of the beams with the goal of preventing the torsional movement of beams and increasing the stiffness of the floor. The experimental results provide valuable information on the vibration modes and frequencies of the specimens, and the contribution of the angle blocking to torsional beam movement. The findings of this study contribute to the understanding of the effectiveness of angle blocking installation for improving floor vibration serviceability and provide valuable insights into the design and construction of steel floors in modern buildings.

FastFloor Specimen Configurations

Two FastFloor specimens, each measuring 10 ft in width and 40 ft in length in plan, were prefabricated off-site and shipped to the West Virginia University's Structures Laboratory for vibration testing. Each specimen was tested with and without the angle blocking members to evaluate the effect of bracing on the overall vibration response. Fig. 1-a shows the bare conditions of a typical 10 ft $\times$ 40 ft specimen, while Fig. 1-b illustrates the sample specimen braced with angle blocking members welded to the beams' bottom flanges at $L/3$ and $2L/3$ along the specimen length (L).

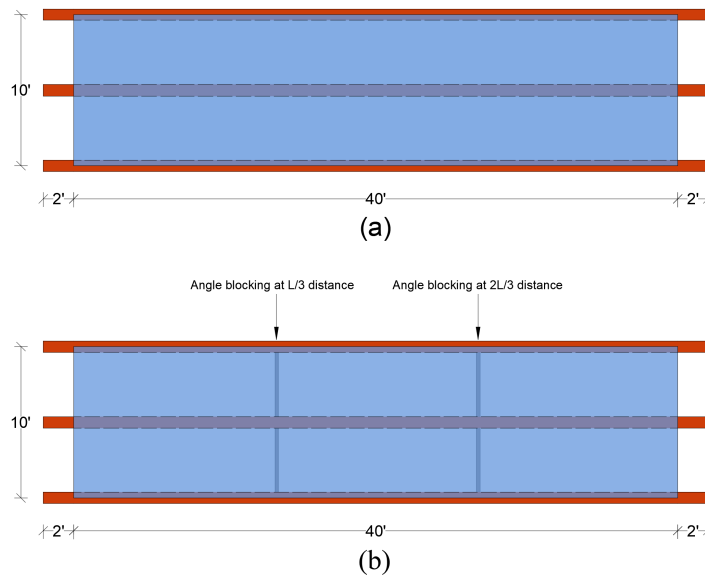


Fig. 1 Layout of a typical 10 ft $\times$ 40 ft single FastFloor module tested in this study. (a) Unbraced configuration, showing the beams without bracing. (b) Braced configuration, featuring angle blocking members installed at $L/3$ and $2L/3$ distances on the beam's bottom flanges.

Each specimen represents a single module of the full-bay FastFloor system. The specimen configurations are summarized in Table 1. The testing program consisted of four configurations: SM68_RAF, SM68_RAF_AB_L3, SM94_RAF, and

Table 1 Configurations of the FastFloor specimens.

Specimen	SM68_RAF	SM68_RAF_AB_L3	SM94_RAF	SM94_RAF_AB_L3
Beam Sections	W24 $\times$ 68	W24 $\times$ 68	W24 $\times$ 94	W24 $\times$ 94
Plate Thickness	3/8 in.	3/8 in.	1/2 in.	1/2 in.
RAF	Standard	Standard	Standard	Standard
Angle Blocking	NA	At $L/3$ and $2L/3$	NA	At $L/3$ and $2L/3$

SM94_RAF_AB_L3. These configurations were designed to evaluate the effect of beam size, plate thickness, and bracing conditions on the vibration response of the FastFloor system. Shuriken blind bolts were used to connect the plate to the beam's top flange on one side, while staggered welding was applied on the remaining beam-plate connections.

The pin-supported beams are spaced 5 ft apart, and the main span of the specimens is 40 ft. The raised access floor (RAF) panels are supported by 125 adjustable steel pedestals per specimen, each panel measuring 2 ft $\times$ 2 ft in plan. To limit the torsional behavior of the beams, L3 $\times$ 3 $\times$ 1/4 profiles are welded to the bottom flanges at L/3 and 2L/3, as shown in Figs. 1 and 2. Specifically, Fig. 2-a illustrates a section of the lighter specimen (SM68_RAF_AB_L3) with angle blocking, while Fig. 2-b shows a section of the heavier specimen (SM94_RAF_AB_L3) with angle blocking. For all dynamic tests discussed in this paper, RAF units were in place.

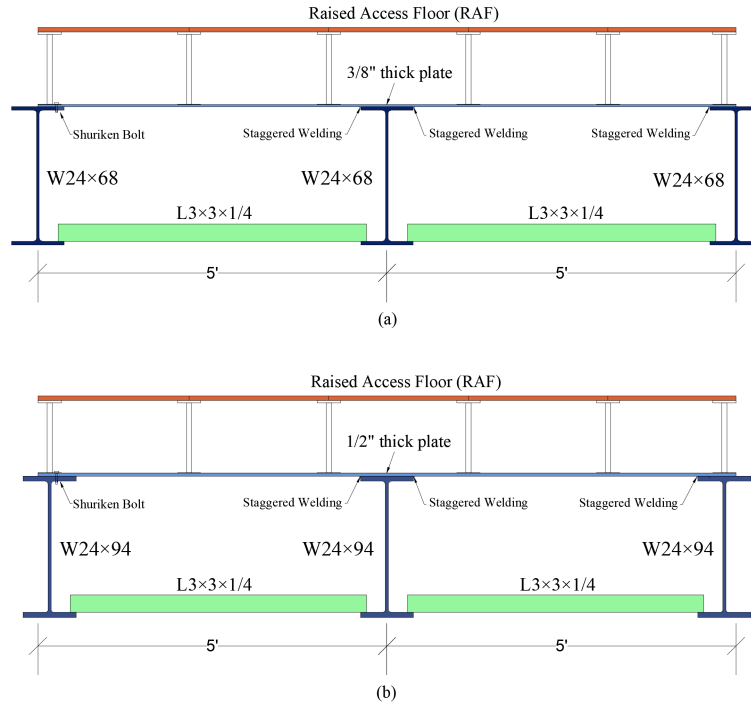


Fig. 2 Sectional view of the braced 10 ft $\times$ 40 ft FastFloor modules tested in the project. (a) Section of the lighter specimen (SM68_RAF) with angle blocking. (b) Section of the heavier specimen (SM94_RAF) with angle blocking.

Modal Testing of the FastFloor Configurations

Experimental modal testing was performed on the four configurations listed in Table 1 to determine the dynamic characteristics of the specimens. Acceleration response data in the gravity direction were collected from 99 points on each specimen. Accelerometers were placed on the top of the RAF panels to obtain accurate dynamic properties and high-resolution mode shapes. The points were spaced at 15-inch intervals in the short direction and 48-inch intervals in the long direction. The denser spacing in the short direction was intended to capture the critical deflection direction for most of the vibration modes. The vertical input excitations generated by a dynamic shaker were measured by an accelerometer located at point 19 (Fig. 3), which was positioned near the shaker. The input is represented by low-amplitude excitations (less than 5%g) at the shaker location.

The collected acceleration data were postprocessed using MEScope [4] to obtain the frequency response functions (FRFs) and mode shapes. For each test configuration, 99 FRFs were calculated using the input from the low-amplitude dynamic mass shaker excitations, with accelerations recorded at point 19 remaining below 5%g [3]. Fig. 4 presents the 99 merged FRFs from testing the unbraced light specimen (SM68_RAF), showing the natural vibration frequencies and damping ratios annotated. Subsequent testing was conducted after the installation of angle blocking members at L/3 and 2L/3 locations throughout the length to form SM68_RAF_AB_L3, and the resulting FRFs are shown in Fig. 5. The mode shapes of the first eight global vibration modes are illustrated in Fig. 6 and Fig. 7 for the unbraced and braced light specimens, respectively.

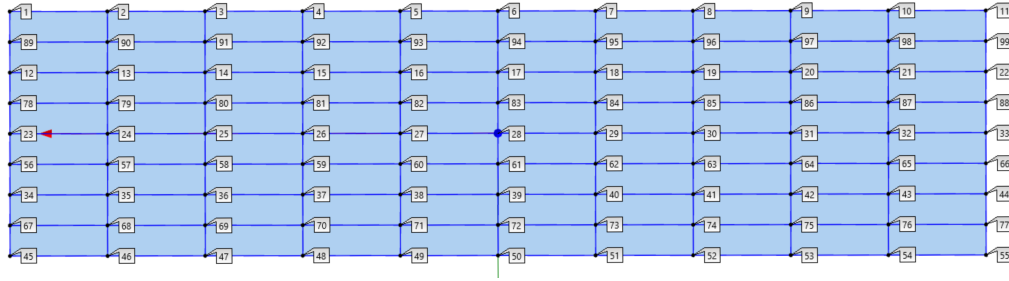


Fig. 3 The floor plan indicating the 99 points used for accelerometer positions. The dynamic mass shaker was located at point 19 for all dynamic excitations.

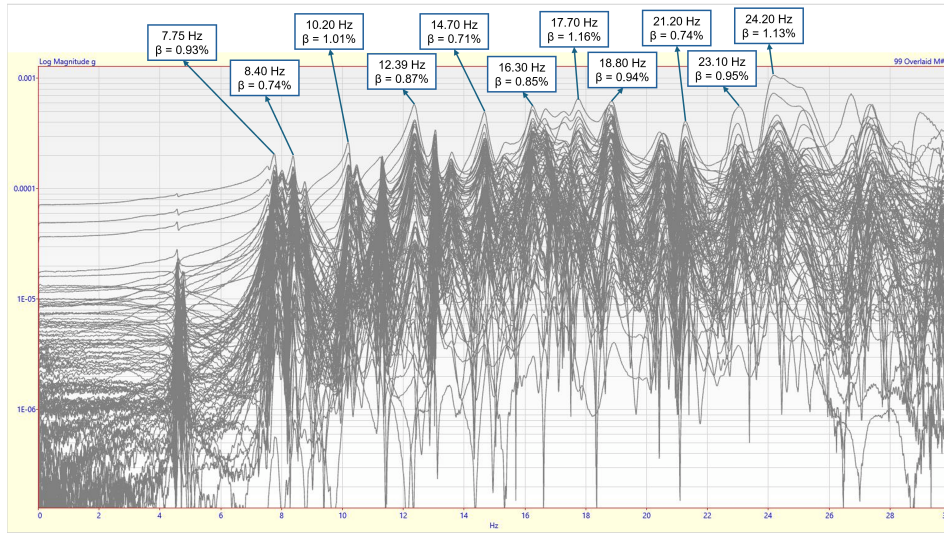


Fig. 4 Frequency response functions (FRFs), natural frequencies and damping ratios of major vibration modes of the light specimen with bare conditions (SM68_RAF), obtained using low-amplitude dynamic shaker excitations.

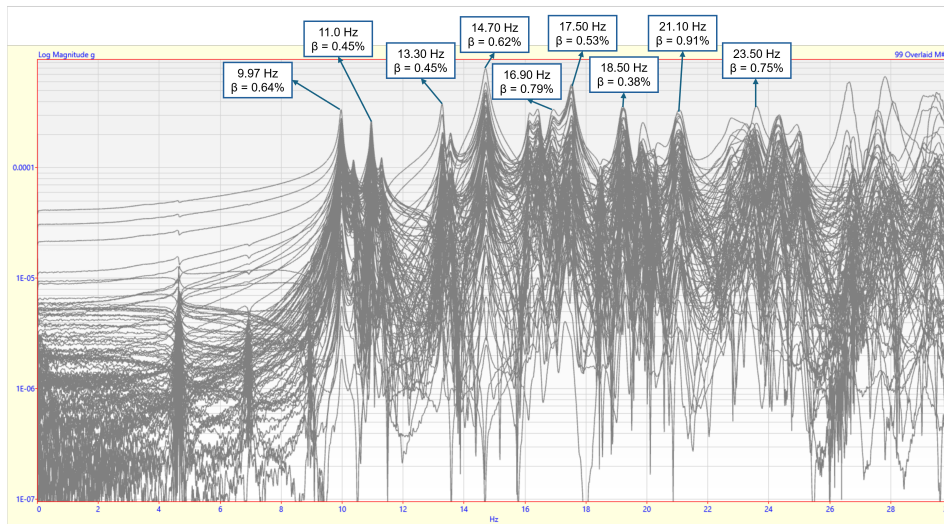


Fig. 5 Frequency response functions (FRFs), natural frequencies and damping ratios of major vibration modes of the light specimen with angle blocking (SM68_RAF_AB_L3), obtained using low-amplitude dynamic shaker excitations.

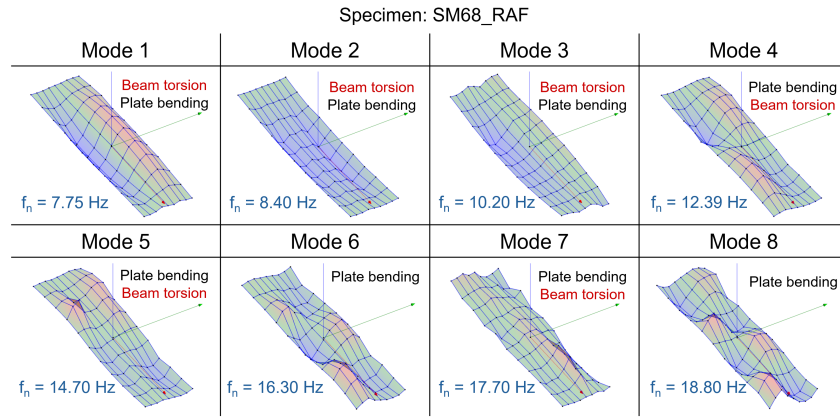


Fig. 6 Mode shapes and natural frequencies of the first eight global modes, obtained from experimental modal testing on the unbraced SM68_RAF specimen.

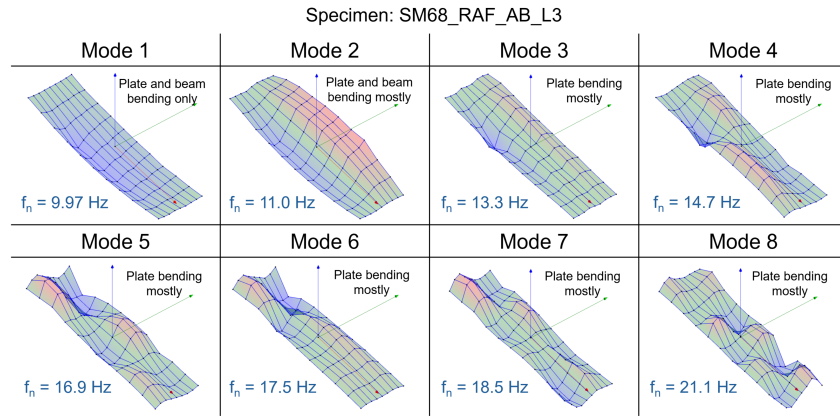


Fig. 7 Mode shapes and natural frequencies of the first eight global modes, obtained from experimental modal testing on the SM68_RAF_AB_L3 specimen with angle blocking.

A comparison of the natural frequencies, mode shapes, and torsional behavior in the modes of SM68_RAF and SM68_RAF_AB_L3 demonstrates the effectiveness of angle blocking in controlling the beam torsion and increasing the stiffness and frequencies of the specimen. The frequency content shifts to the right on the FRF plot, indicating increased stiffness as a result of the angle blocking installation. Specifically, the addition of angle blocking increases the first natural frequency from below 9 Hz to above 9 Hz, thereby rendering the braced specimen a high-frequency floor (HFF) and altering its response to human-induced dynamic loads per design guidance [2]. Furthermore, the second mode of SM68_RAF, which involves plate bending and beam torsion, is eliminated after the addition of the angle blocking. Although the torsional movement of beams is not visible in the mode shapes due to the test setup, separate modal testing was conducted with accelerometers placed perpendicular to the beam webs (perpendicular to the gravity direction) to identify the torsional behavior. Additionally, mode shapes from calibrated finite element models were used to further investigate the torsional behavior of the beams.

The second, heavier specimen exhibits greater stiffness than the lighter one and is classified as a high-frequency floor (HFF), resulting in a less congested frequency content between 0 and 30 Hz. Fig. 8 and Fig. 9 present the merged 99 frequency response functions (FRFs) of the heavy specimen for the unbraced and braced conditions, respectively. Similar to the first specimen, the addition of angle blocking significantly reduces the beam torsion and increases the overall stiffness of the test module. Furthermore, the first mode of SM94_RAF, characterized by beam torsion and plate movement, is nearly eliminated due to the inclusion of angle blocking, as shown in Fig. 10 and Fig. 11. The mode shapes of the first eight global vibration modes are illustrated in Figs. 10 and 11 for the unbraced and braced specimens, respectively.

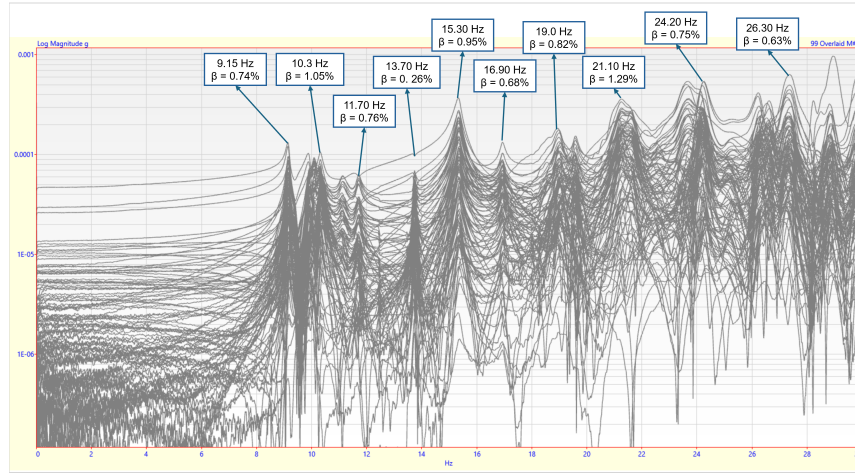


Fig. 8 Frequency response functions (FRFs) with natural frequencies and damping ratios of major vibration modes of the heavy specimen (SM94_RAF), obtained using low-amplitude dynamic shaker excitations.

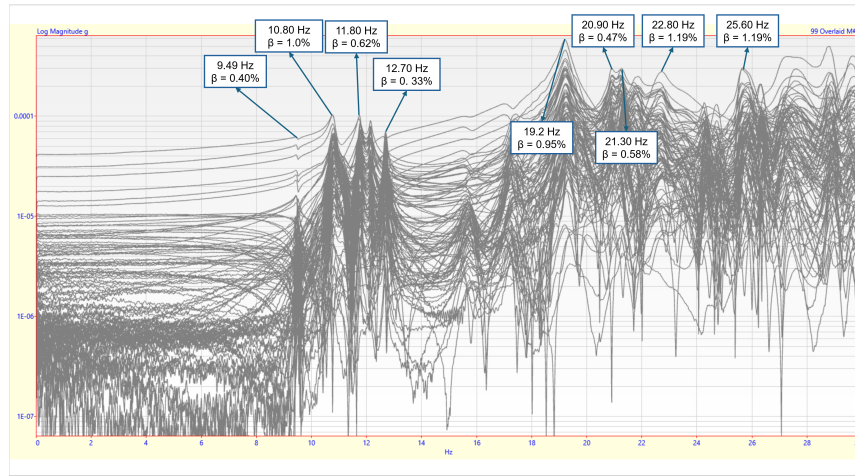


Fig. 9 Frequency response functions (FRFs) with natural frequencies and damping ratios of major vibration modes of the heavy specimen (SM94_RAF_AB_L3), obtained using high-amplitude dynamic shaker excitations.

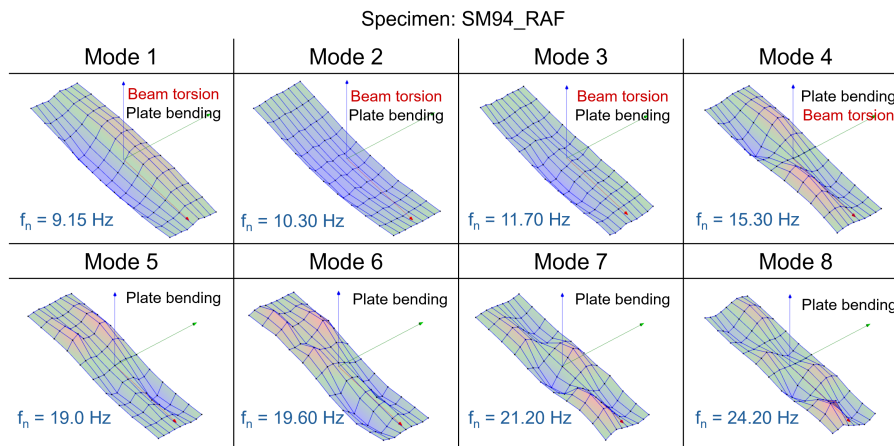


Fig. 10 Mode shapes and natural frequencies of the first eight global modes, obtained from experimental modal testing on the unbraced SM94_RAF specimen.

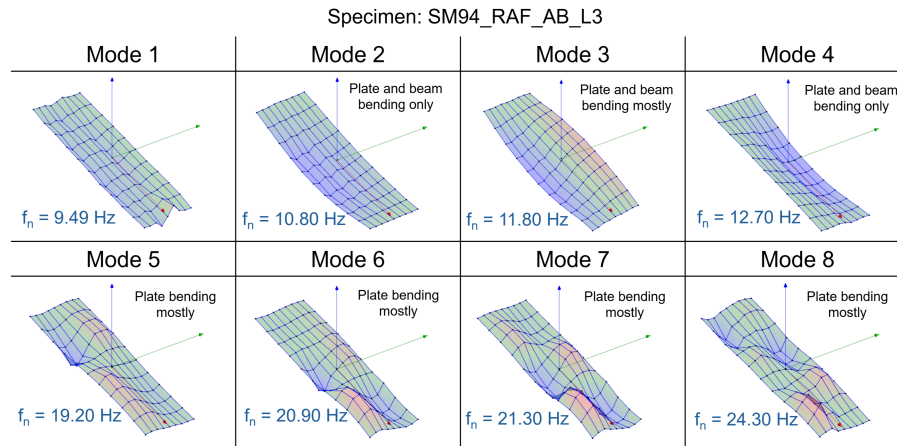


Fig. 11 Mode shapes and natural frequencies of the first eight global modes, obtained from experimental modal testing on the SM94_RAF_AB_L3 specimen with angle blocking.

Conclusions

This study presented a detailed experimental investigation of two all-steel laboratory floor specimens. The effectiveness of the bottom flange angle blocking in improving the dynamic behavior of all-steel laboratory floors was demonstrated. Experimental modal testing and analysis showed that installing angle blocking on the W-beam flanges significantly increases stiffness and limits torsional movement of the beams, thereby enhancing the overall vibration performance of the laboratory floor specimens. These findings contribute to a broader understanding of angle blocking as a practical method for improving the dynamic response of the FastFloor system.

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Chapter 4

Modal Testing and Finite Element Model Updating of a Fast-Floor Specimen: A Modular All-Steel Floor System

Sahabeddin Rifai, Rajshri Chidambaram Muthu Kumar, Onur Avci, Benjamin W. Schafer, Matthew R. Eatherton, W. Samuel Easterling, and Jerome F. Hajjar

Abstract This paper presents the results of modal testing and finite element model updating of an all-steel floor system, designed to increase the speed of construction by eliminating the traditional concrete-on-deck component. A 10 ft $\times$ 40 ft laboratory specimen, comprising three W24 beams attached to a top plate with a raised access floor, was tested to obtain its modal frequencies and mode shapes. The experimental results were used to update a finite element model in SAP2000, which was then validated against the experimental data. The validated model is used to predict the dynamic behavior of a larger 30 ft $\times$ 40 ft system, representing a full bay at an elevated building floor. The study provides valuable insights into the dynamic behavior of steel floor systems and highlights the importance of experimental validation and model updating in ensuring the accuracy of finite element models. The findings contribute to the development of more accurate and efficient design methods for modular steel floor systems.

Keywords Steel floor systems · Modal testing · Finite Element model updating · Dynamic analysis · Modular construction

Introduction

FastFloor is a modular all-steel floor system developed in response to the "Need for Speed" initiative launched by the American Institute of Steel Construction (AISC). This initiative aims to reduce steel construction time by up to 50%. FastFloor achieves this goal by eliminating non-steel components in traditional composite floor systems, thereby increasing the speed of steel construction. The modular units of FastFloor are prefabricated off-site and then transported and erected on steel girders with field connections. Additionally, FastFloor offers several advantages, including reduced weight and a lower carbon footprint [1].

The original design of FastFloor consists of multiple wide-flange steel beams connected to a steel plate using bolts and staggered welding. A typical full-bay FastFloor is designed to be 30 ft $\times$ 40 ft, and the prefabricated full-bay floor can be easily installed and connected to steel girders, which results in significant time and labor savings. This paper presents

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the background and results of dynamic testing on an all-steel floor specimen. The specimen comprises three identical steel beams connected to a steel plate on their top flanges, with a raised access floor (RAF) installed on top of the steel plate. The RAF provides space for various mechanical system components.

Floor systems composed entirely of steel members are characterized by their relatively low self-weight and low inherent damping compared to traditional composite floors. This presents a challenge in achieving acceptable vibration levels in modular steel construction, as beam and girder spans need to be longer in modern building design [1].

This study aims to investigate the dynamic behavior of a scaled-down FastFloor specimen through experimental modal testing and finite element model updating. The experimental results provide valuable insights into the vibration modes and frequencies of the specimen, while the updated finite element model enables the prediction of the dynamic behavior of larger-scale FastFloor systems. The findings of this study contribute to the development of more accurate and efficient design methods for modular steel floor systems, ultimately enhancing the overall performance and sustainability of steel construction.

Test Specimen

The floor specimen tested in the laboratory measures 10 ft in width and 40 ft in length, as shown in Fig. 1-b. This specimen represents a single module of the full-bay FastFloor system, with minor modifications, including a 2 ft extension of the beams beyond the steel plate. The test specimen consists of three W24x68 steel beams and a 3/8 in. thick steel plate on top. Hex Structural Bolts with Shuriken nut keepers were used to connect the plate to the left beam's top flange on one side, while staggered welding was applied to the remaining beam-plate connections.

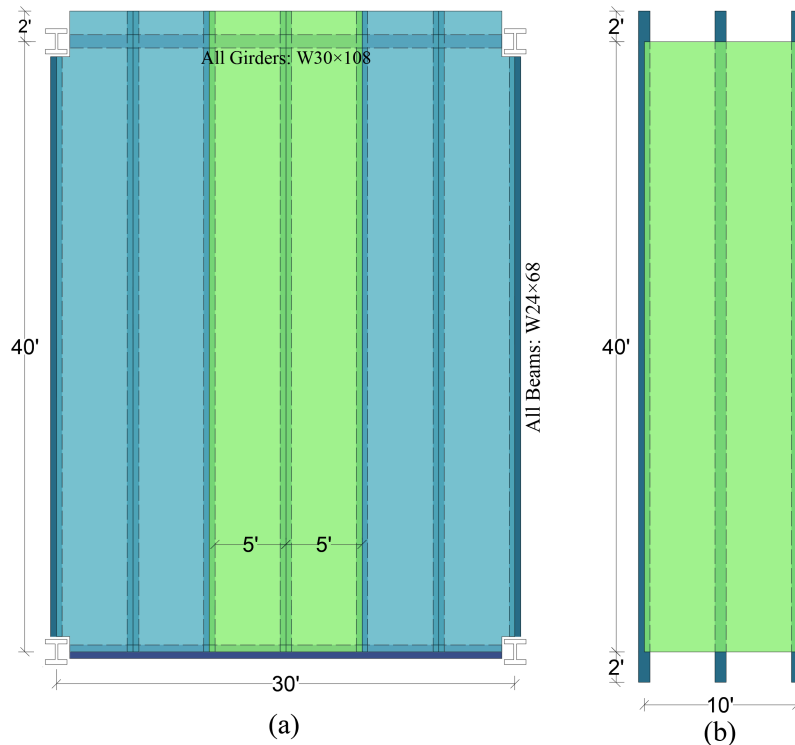


Fig. 1 Layout of the full-bay and single FastFloor modules. (a) A 30 ft $\times$ 40 ft full-bay FastFloor module. (b) The 10 ft $\times$ 40 ft single FastFloor module tested in the laboratory.

To prevent sway deflection in the short direction, web stiffeners with a thickness of 1/2 in. and a width of 4 in. were welded to the beam webs at the support locations. The beam bottom flanges were connected to the bearings using C-clamps to simulate a pin connection. The raised access floor (RAF) was supported by steel pedestals with adjustable heights, which were attached to the steel plate. A total of 125 pedestals were used to support 100 floor panels, each measuring 2 ft $\times$ 2 ft in plan.



Fig. 2 View of the test specimen with W24x68 beams, a 3/8 in. thick plate, and raised access floor (RAF) on top.

Dynamic Modal Testing of the Steel Floor Specimen

To obtain accurate dynamic properties and high-resolution mode shapes, acceleration data were collected from 99 points on the raised access floor (RAF). These points were spaced at 15-inch intervals in the short direction and 48-inch intervals in the long direction. The points were more densely spaced in the short direction because most vibration modes exhibit deflection primarily in this direction. Uniaxial accelerometers were attached to the floor to measure vertical vibrations. Fig. 3 illustrates the locations of the excitation and data acquisition points. A dynamic mass shaker was used to excite the specimen at various frequencies, with both low- and high-amplitude excitations.

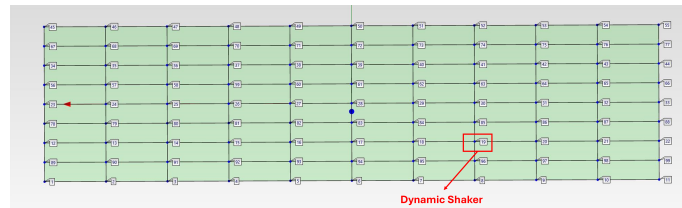


Fig. 3 Accelerometer and dynamic shaker locations on top of the RAF.

The input of the dynamic shaker excitations is measured by the accelerometer placed at point 19 (shown in Fig. 3), placed near the dynamic shaker. Excitations are classified as low-amplitude when the accelerations recorded at point 19 are below 5%g, and as high-amplitude when the accelerations exceed 15%g [2, 3]. Fig. 4 presents 99 overlaid frequency response functions (FRFs) used to identify natural vibration modes, obtained with low-amplitude dynamic shaker excitations. The

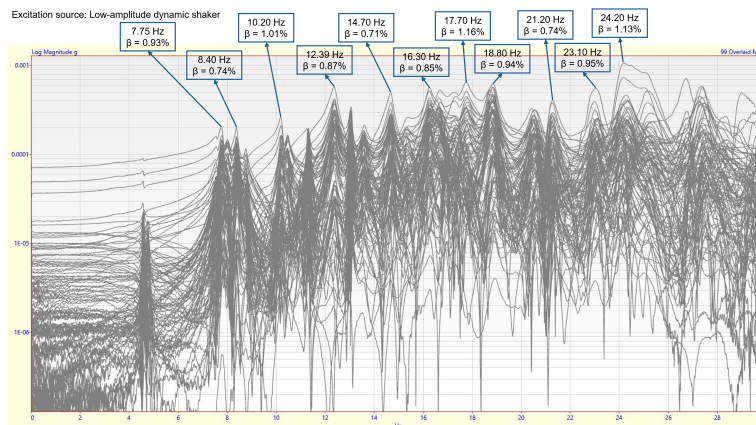


Fig. 4 Frequency response functions (FRFs) with natural frequencies and damping ratios of major vibration modes, obtained using low-amplitude dynamic shaker excitations.

corresponding FRFs for high-amplitude dynamic shaker excitations are shown in Fig. 5. The plots in Fig. 4 and Fig. 5 show the results of experimental modal testing, where acceleration data were processed using MEScope software [4].

The natural frequencies and damping ratios of the global modes are reported in these figures. Local modes are not presented, as they are not utilized in the model updating procedure. The results indicate that the frequency content of the specimen comprises closely spaced multiple vibration modes. Furthermore, the dynamic excitation amplitudes have an impact on the natural frequencies and damping ratios, which is consistent with previous findings [3, 4, 5]. For the purpose of finite element model updating, the natural frequencies obtained from the low-amplitude shaker results were used, as human walking is a low-amplitude activity that is most relevant to the study.

The mode shapes of the first four global vibration modes, obtained from dynamic modal testing and finite element analysis, are illustrated in Fig. 6. For improved visibility, the raised access floor (RAF) components are not shown in the mode shape visualizations. The finite element model in SAP2000 accurately captures the lateral deflection of the beams in all four modes (shown in Fig. 6) and showcases the global dynamic behavior of the specimen in three-dimensional view.

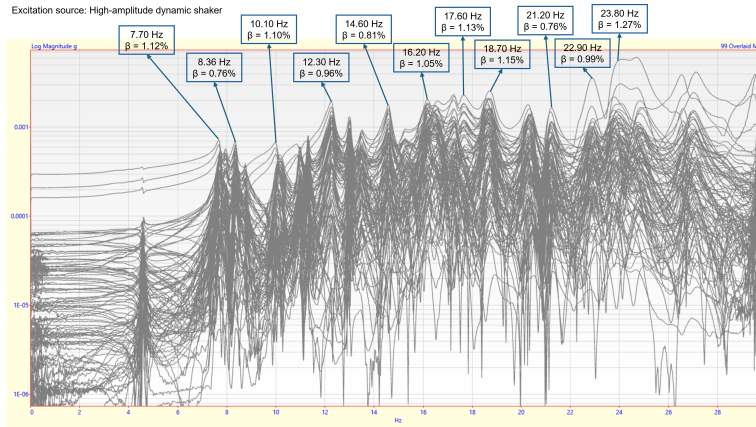


Fig. 5 Frequency response functions (FRFs) with natural frequencies and damping ratios of major vibration modes, obtained using high-amplitude dynamic shaker excitations.

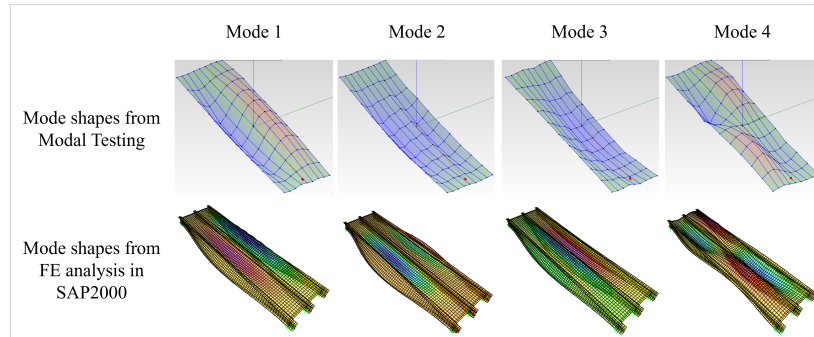


Fig. 6 Mode shapes of the first four global modes, obtained from experimental modal testing and finite element analysis in SAP2000.

Finite Element Model Updating

A well-calibrated finite element model of the 10 ft wide floor specimen is a valuable tool for understanding the dynamic behavior and predicting the response of larger-scale FastFloor systems and full bays, which are impractical to build and test in a laboratory setting. For this reason, the authors focused on updating the finite element model of the tested specimen using SAP2000. Fig. 7 illustrates the 3D floor model in SAP2000. In the initial finite element model, the beams, plate, and RAF panels of the floor specimen were modeled as thin shell elements, while the RAF pedestals were modeled as frame elements.

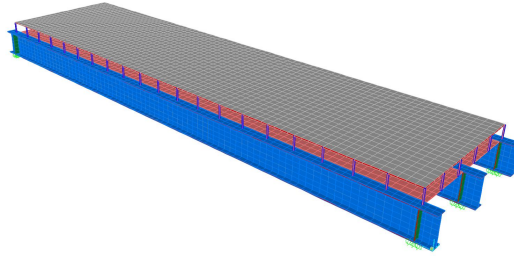


Fig. 7 3D view of the Finite element model of the floor specimen.

The average mean absolute error of the initial finite element model indicates that, on average, the analytical natural frequencies are 30% higher than the experimental natural frequencies, as shown in Table 1. Therefore, model updating was conducted to match the natural frequencies between the experiment and analysis. Since natural frequencies are proportional to the stiffness, the initial finite element model exhibited higher stiffness than the tested specimen. As a result, the authors focused on adjusting model parameters that could reduce the stiffness of the finite element model. The mass of the specimen was kept constant throughout the model updating process, and the mass in SAP2000 was verified using hand calculations.

Table 1 Comparison of experimental and analytical modal frequencies before model updating

Mode	Experimental Frequency (Hz)	Analytical Frequency in SAP2000 (Hz)	Error (%)
1	7.75	10.5	35.5
2	8.40	10.96	30.5
3	11.31	11.29	0.2
4	12.39	18.94	52.9

The main parameters used in the finite element model updating were (1) the continuity of beam-plate connections, (2) the boundary conditions of the specimen, and (3) the modeling of RAF panels. In the initial finite element model, the beam-plate connections were treated as continuous, despite the presence of intermittent welded and bolted connections, with the welded connections considered stiffer than the bolted ones. To accurately reflect the beam-plate connections in the SAP2000 model, the connection joints were first disconnected, then some were reconnected discretely. Different spacings of connected joints were tested depending on the type of connection. In Fig. 8, the discrete beam-plate connections are illustrated where reconnected joints on the bolted side were spaced by 35 in. and on the other three welded connections, spacing of reconnected joints was 24 in. Those spacing values resulted in the lowest frequency errors.

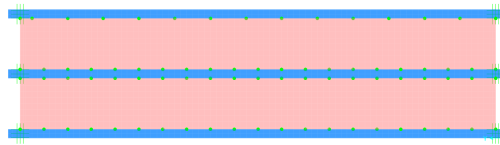


Fig. 8 Modeling of beam-plate connections: The upper connection is bolted, while the remaining three connections are welded [7].

A few adjustments were made to the boundary conditions; however, this parameter did not significantly affect stiffness or frequencies, as only combinations of pin and roller supports were used. The objective of the model updating was to match the frequencies of the global modes with frequencies lower than 16 Hz. The updating process was performed manually by adjusting the three parameters mentioned above while monitoring how changes affected the mode shapes and frequency error values. Model updating attempts were stopped when no further improvement or reduction in the error was observed. Table 2 presents the final updated frequencies from SAP2000 compared to the experimental results. It is noteworthy that the higher the mode, the less it contributes to the overall response. Also, matching the first two frequencies can result in larger discrepancies for higher modes.

Table 2 Experimental and analytical natural frequencies at the end of model updating.

Mode	Experimental Frequency (Hz)	Analytical Frequency in SAP2000 (Hz)	Error (%)
1	7.75	8.18	5.5
2	8.40	8.72	3.8
3	11.31	10.28	9.1
4	12.39	14.72	18.8

Conclusions

This paper presented the results of modal testing and finite element model updating of a 10 ft × 40 ft all-steel floor module. The research team successfully matched the frequency values of the global modes via finite element model updating with reasonable precision. Future work will include experimental and numerical investigations of a full-bay FastFloor laboratory specimen.

Acknowledgments This work was supported by the Charles Pankow Foundation; the American Institute of Steel Construction (AISC); the MKA Foundation; Nucor; Schuff Steel; Herrick Steel; Cives Steel; Zekelman Industries (Atlas Tube/Shuriken); Owen Steel; and Metals Fabrication. In-kind support was provided by Cerami, Sherwin-Williams, Cooper Steel, Clark Construction, and Hensel Phelps. Any opinions, findings, conclusions, or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the sponsors or supporting organizations.

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Chapter 5

IMAC 2025 Modal Expansion of Mixed-Domain Vibration Response Using Neuromorphic Event-based Imaging Data

Gabriella Gallegos, Tessa Lytle, Deborah Fowler, Fernando Moreu, Ryan Schultz, Chandler Smith, Timothy Walsh, and Tariq Khraishi

Abstract Obtaining comprehensive vibration response data is essential for characterizing structural response under vibration and shock environments. Full-field data acquisition methods such as digital image correlation provide valuable insight into structural response, however the computational resources and time required can be prohibitive in applications such as vibration control that require real-time insight. A common low-latency alternative is to expand vibration responses from accelerometer data at limited measurement points to full-field. However, channel count limitations can limit the spatial resolution of physical sensors and the corresponding frequency range of response the expansion procedure can accommodate. The emerging field of neuromorphic imaging presents a low-latency alternative to acquiring full-field vibration measurements, however the quality and confidence in the resulting data can vary based on factors such as displacement magnitude and feature geometry. In this work, the authors examine the combination of neuromorphic imaging and traditional modal-based expansion methods to achieve low-latency estimation of full-field responses with higher accuracy than either technique could independently achieve. A mixed-domain expansion approach is explored to combine displacement data collected using neuromorphic sensing with accelerometer data. The investigation combines neuromorphic imaging data with accelerometer responses by evaluating sensor placement methods that complement both neuromorphic imaging and expansion and implementing the results in a mixed-domain expansion. The result of this research provides real-time response information for vibration control and other applications requiring low-latency dynamic response information.

Keywords Neuromorphic imaging · Expansion · Modal filter

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Introduction

Acquiring comprehensive vibration response data is a critical capability for characterizing structural response under vibration and shock environments. Channel count limitations as well as mass loading effects can limit the spatial resolution of physical sensors such as accelerometers. Full-field measurements can be achieved using technologies such as laser doppler vibrometers (LDV) and digital image correlation (DIC), however both methods are prohibitively expensive and neither of these methods allow for real-time/low latency response insight which is critical for applications such as vibration control or digital twin capabilities. LDVs acquire response data one point at a time using a series of repeated tests. DIC acquires full-field data in a single test, however the extensive post-processing required to extract the relevant dynamic response information means the data cannot be used in real-time.

The emerging field of neuromorphic imaging applies computing techniques inspired by biological nervous systems to process and extract relevant data from image sensors in real-time for low-latency applications. Recent work [1] has used this technology to acquire full-field vibration measurements using digital video cameras. The unique component of neuromorphic cameras (also known as Dynamic Vision Sensors (DVS)) is that for each pixel, when a change in brightness exceeds a certain threshold, an event is triggered at the corresponding pixel location. The event stream is defined by the x and y coordinates, the brightness change indicated by either positive (+1) or negative (-1) polarity, and the time at each coordinate. This concept is illustrated in Figure 1 [18].

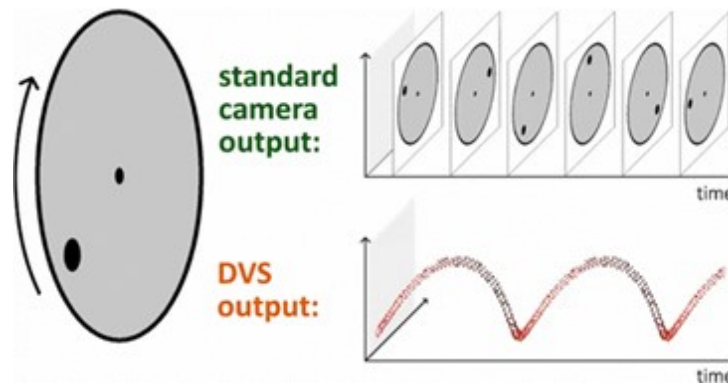


Fig. 1 Dynamic Vision Sensors (DVS) neuromorphic concept.

A popular low-latency alternative to full-field data acquisition is to expand vibration response from accelerometer data at limited measurement points. The most prevalent approach is modal expansion, where the full-field mode shapes derived from a finite element method (FEM) model are used to interpolate between the sensor measurements. Optimal placement of sensors is critical to successful implementation of these expansion procedures. Several approaches exist for optimizing sensor placement for expansion and other modal-filtering applications. These sensor selection methods seek to minimize the number of sensors while maintaining a well-conditioned inversion process. A poorly conditioned inversion will amplify noise and other data contaminants throughout the expansion process, producing poor interpolation results. Regularization methods have been explored as a viable approach to improve expansion results with a poorly-conditioned inversion [2]. However, tuning the regularization parameters is not a straight-forward process. Selecting an optimal degree of freedom (DOF) set is widely regarded as preferable when the test engineer has the ability to do so. Some cases may require both regularization and an optimized set of DOF. This occurs when channel count limitations prevent the ability to include enough DOF to allow for a well-conditioned inversion, or when the experimental data has a large amount of noise. Overall, DOF selection remains a pivotal component to successfully implementing expansion procedures, and various approaches exist to perform the DOF selection process.

Neuromorphic imaging and expansion procedures are both low-latency techniques to achieve full-field response information with different strengths and weaknesses. Expansion methods rely on pre-test analysis to select optimal DOF sets, and a finite element model of the system to generate the required mode shapes. Neuromorphic imaging is an emerging field. While full-field information is acquired, there are varying levels of fidelity and confidence in the results – response locations at points of high displacement and/or at feature edges have shown to be the most coherent, whereas points of low displacement may not result in practical response information. We speculate that combining neuromorphic imaging and model-based expansion could lead to improved low-latency estimation of full-field responses better than either technique could achieve independently. Neuromorphic imaging collects displacement data. Therefore, a mixed-domain expansion approach will need

to be implemented to integrate the neuromorphic displacement data with the accelerometer data. While fundamental expansion work utilizes data collected using a single measurement technique (such as accelerometers), recent work has developed methods to perform expansion using data collected across multiple domains, such as displacement and acceleration or strain and acceleration [3, 4].

The goal of this research is to investigate how to design a modal filter that integrates neuromorphic imaging data with accelerometer responses in a single expansion procedure by (1) evaluating DOF selection methods to place accelerometers in locations that complement the neuromorphic imaging data, and (2) implementing the resulting mixed-domain expansion. This investigation contributes to active areas of research in vibration measurements with neuromorphic imaging and mixed domain expansion procedures, and it will develop a foundation to explore other methods of fusing neuromorphic sensing and expansion techniques for an improved approach to acquire low-latency full-field response data. These methods could ultimately provide necessary real-time response information for vibration control, digital twin implementations, and any other applications requiring low-latency full-field dynamic response information.

Theory

The System Equivalent Reduction Expansion Process (SEREP) method is one of the most widely used modal expansion methods [5]. SEREP is performed in two steps. First, the responses are projected through a modal filter to estimate the modal coordinates (response of each individual mode), using

$$p(t) = U_a^g x_a(t) \quad (1)$$

where p is the matrix of the estimates of modal coordinates, U_a is the matrix of model mode shapes U partitioned to the reduced set of a DOF, and $x_a(t)$ is the matrix of time responses at a DOF. This modal filtering step is extremely powerful and used for a wide range of problems in modal analysis. The second step of the SEREP expansion procedure is the expansion step, where the estimates of modal coordinates are projected back into the physical domain using the full-field model mode shapes using

$$x_n(t) = U_n p(t) \quad (2)$$

where the subscript n is used to refer to the full set of model DOF and $x_n(t)$ is the matrix of experimental responses expanded to full field. SEREP expansion can also be used to expand experimental mode shapes, by using the set of test modes instead of $x_a(t)$. The success of the expansion procedure is dependent on two main challenges: choosing an appropriate set of mode shapes that can fully describe the dynamic response and choosing a set of DOF that can fully describe and differentiate between the mode shapes in the set. These two challenges are intricately linked – the number of mode shapes that can be included is limited by the amount of DOF/channel count available, and the more efficient the DOF set is the more mode shapes can be included for the same number of DOF. This project will focus on the second challenge, DOF selection, although evaluating whether the mode shape set is sufficient is always a necessary part of the process.

There are several popular approaches to DOF selection for modal filtering applications. Effective Independence was used for this work and compared to manually selecting a DOF set.

Effective Independence: Effective Independence [7] is a popular method for DOF selection that uses a computation of the fractional contribution of each sensor location to the linear independence of the mode shapes. At each step of the procedure, the independence distribution vector E_D is defined as the diagonal of the matrix E , using

$$E = U_a [U_a^T U_a]^{-1} U_a^T \quad (3)$$

where U_a is the matrix of mode shapes U partitioned to the reduced set of a DOF. Once computed, the minimum values of E_D are found and the associated DOF is removed from the candidate set. The procedure is repeated until the candidate set reaches the desired size.

Experimental Set Up

This investigation was conducted using a wing-shaped aluminum plate bolted to an aluminum block, with washers placed between the wing and the block. While the structure was relatively simple, complexities were introduced by the bolts and the non-symmetric design of the wing.

A finite element model of the structure was developed using Sierra/SD which can be seen in Figure 2b [19], consisting of an AL6061 block (16"x9"x3") bolted to an AL6061 wing (13"x26"x3/16"). Various boundary conditions were explored, including fixed and a variety of spring supports, along with adjustments to material properties. Ultimately, free-free boundary conditions were applied, with point masses added to simulate the accelerometers. The model consisted of a hexahedral mesh with approximately 770,000 elements and required about 15 to 20 minutes to run. An experimental modal test was then conducted to compare the physical and calculated mode shapes, to ensure the accuracy of the model.

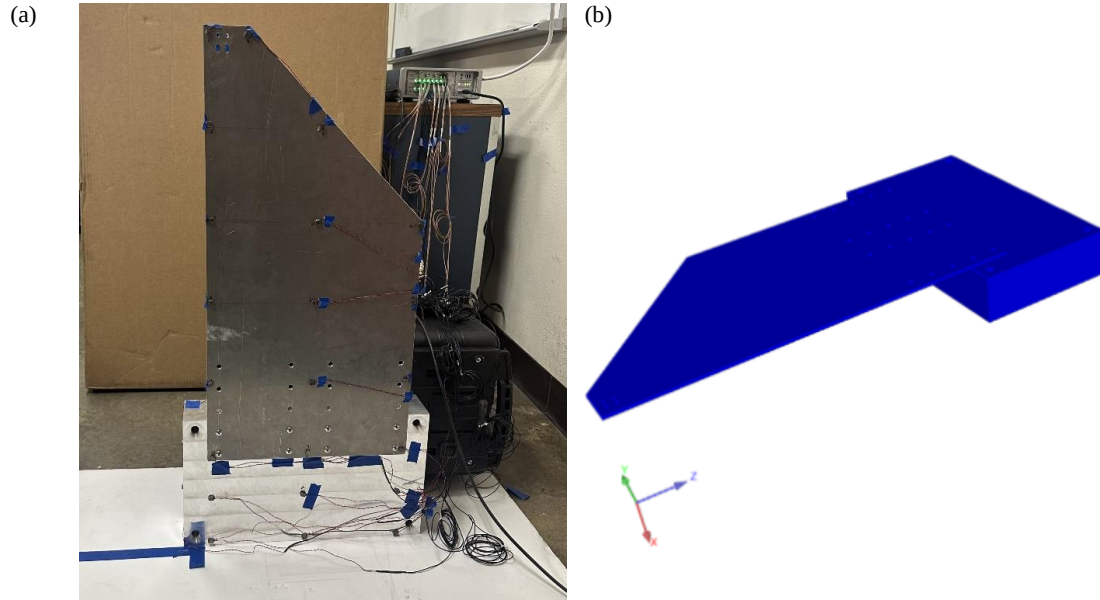


Fig. 2 (a) Experimental setup with 29 DOF. (b) FEM model free-free boundary conditions

An impact hammer test using Simcenter Testlab software [20] was conducted to measure the response at 24 locations, and the results were used to refine the FEM model. The 24 DOF were placed as 16 accelerometers on the wing and 8 accelerometers on the block. Accelerometers were placed on the wing to best capture the mode shapes, and the remaining accelerometers were placed on the block which was expected to have very minimal displacement. The structure was set upright with a thin foam poster board placed between the block and the concrete floor, as shown in Figure 2a. An acquisition time of 5 seconds was used to capture the bandwidth of interest, which was 819 Hz. A white plastic hammer tip was used to excite the desired frequency range of 0 to 600 Hz. This frequency range was chosen to capture the first 10 modes. DOF 3 (as shown in Figure 9) was impacted a total of five times with the hammer.

The FRFs of the 24 accelerometers after an impact at DOF 3 are shown in Figure 3. The coherence was also checked, and throughout the entire frequency range of 0 to 600 Hz, there was no significant decrease, indicating that the collected results were of high quality.

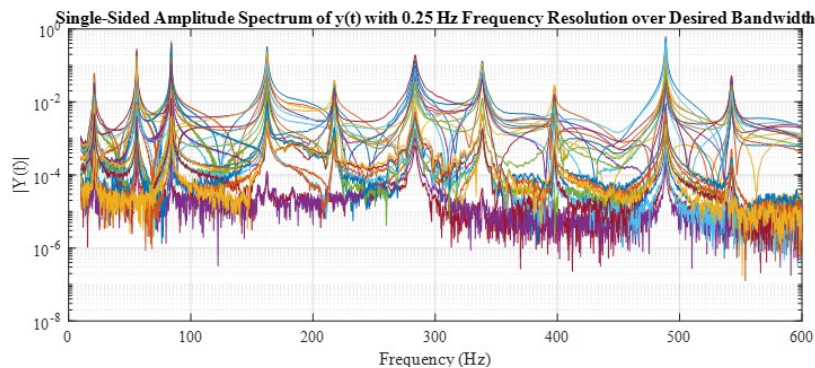


Fig. 3 Impact hammer test FRFs of the 24 accelerometers.

Impact Test Results

After ensuring the quality of the data collected from the impact hammer test, the data was processed using the Polymax curvefitter in Simcenter Testlab. Table 1 presents a comparison of the natural frequencies between the experimental and model natural frequencies.

Table 1 Comparison of Experimental and Model Frequencies for the First 10 Modes.

Mode #	Model (Hz)	Experimental (Hz)	% Error
1	25.5	20.8	22.5%
2	59.2	55.7	6.3%
3	89.0	83.9	6.1%
4	167.8	162.2	3.5%
5	223.9	217.2	2.7%
6	288.9	283.1	2.0%
7	341.4	338.4	0.9%
8	406.7	397.3	2.4%
9	496.3	488.7	1.6%
10	549.8	542.5	1.3%

Table 1 shows the most notable natural frequency discrepancies between lower value modes, particularly Mode 1. These discrepancies are attributed to difficulties in capturing the boundary conditions. A Modal Assurance Criterion (MAC) was used to compare the experimental mode shapes with the model mode shapes [11]. As shown in Figure 4, the MAC demonstrated a high correlation between the experimental and model mode shapes. Mode 1, which had the highest error in natural frequencies, showed a correlation of 0.94, while the remaining nine modes had correlations of 0.99 or higher. Figure 5 shows the comparison between the FEM results and experimental data for Mode 1. The FEM Mode 1 shows more movement along the block compared to the experimental Mode 1 which shows little to no movement along the block. This could indicate that the FEM free-free boundary conditions do not match the experimental boundary conditions of the block sitting upright on the floor.

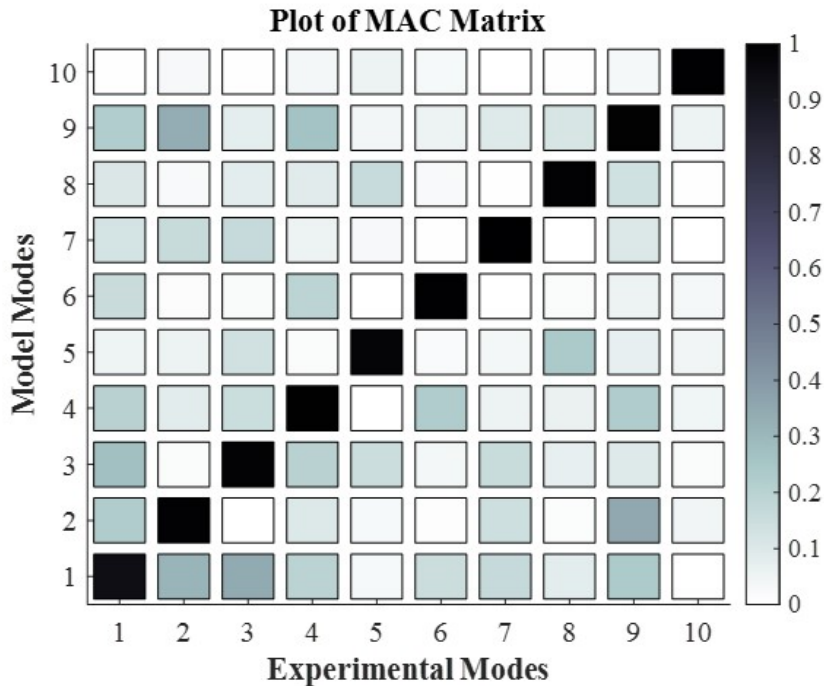


Fig. 4 MAC of experimental and model mode shapes.

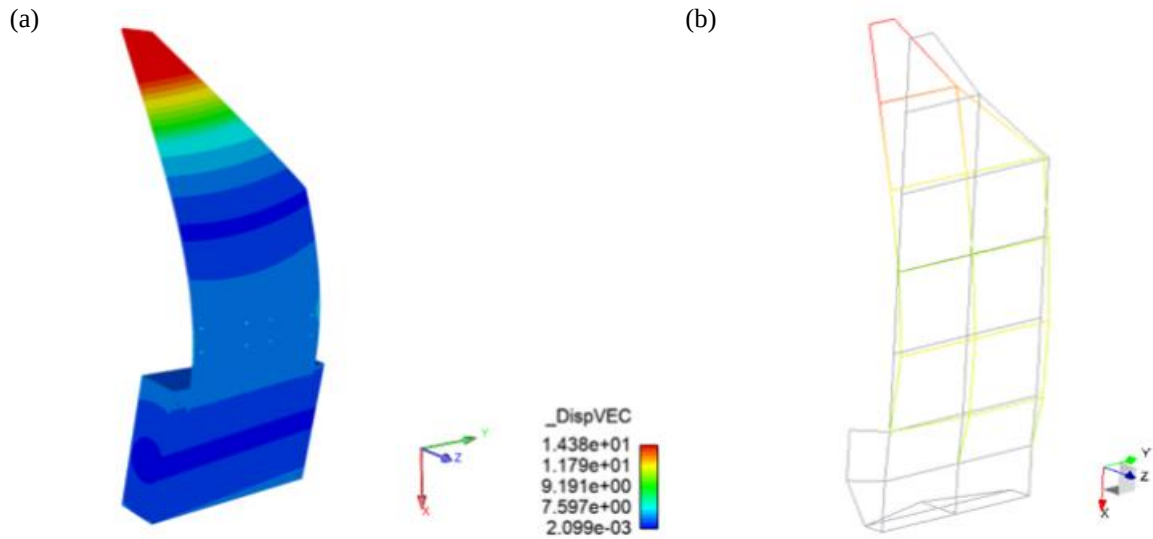


Fig. 5 (a) Experimental setup with 29 DOF. (b) FEM model free-free boundary conditions

Neuromorphic Imaging Testing

Neuromorphic imaging data was collected using a DVXplorer Mini camera with a spatial resolution of 640x480 pixels. The camera offers sub-1ms latency, $200\mu\text{s}$ temporal resolution, and can process up to 450 million events per second. The Dynamic Vision (DV) software used provides options such as event, accumulator, and capture blocks. All data is filtered by a Y-noise filter and a general noise filter before being output as a CSV file. The Y-noise filter removes flickering noise, while the general noise filter stores a map of event timestamps.

The first step was to calibrate the neuromorphic camera by determining the pixel-to-millimeter conversion factor. A 21x21 mm calibration square was taped to the tip of the wing and recorded at distances of two, three, and four inches. With the known spatial resolution of the camera and calibration square, the pixel-to-millimeter conversion factor was computed for the three measured distances. The results were then interpolated for distances from one to seven inches and plotted in Figure 6.

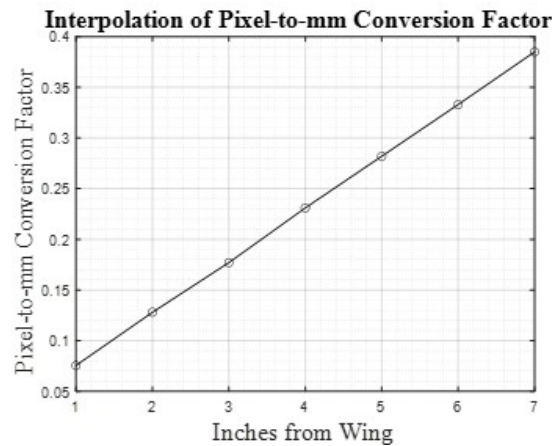


Fig. 6 Pixel-to-mm conversion factor interpolation.

The output of an event stream includes polarity, x and y coordinates, and a timestamp. The collected data is loaded into MATLAB and using the pixel-to-millimeter conversion factor, a specified region of interest is divided into 10x10 frames where events are clustered. The events are grouped into constant-sized clusters, and the average x-pixel values are computed for the specified region of interest. These average x-pixel values over time, combined with the conversion factor, are used to

calculate the displacement and plot the results. The plotted x-pixel values visualize the wing's movement. Figure 7 shows the comparison setup, where the camera was positioned 2 inches from DOF 1, and the LDV was aimed at DOF 1 while DOF 3 was impacted five times.



Fig. 7 Comparison setup with LDV and neuromorphic camera.

To ensure the code accurately captures the displacement, it was further fine-tuned by comparing the calculated displacement with measurements from the Laser Doppler Vibrometer (LDV). Figure 8 shows the comparison of the two methods, where it can be seen that the displacement is being captured.

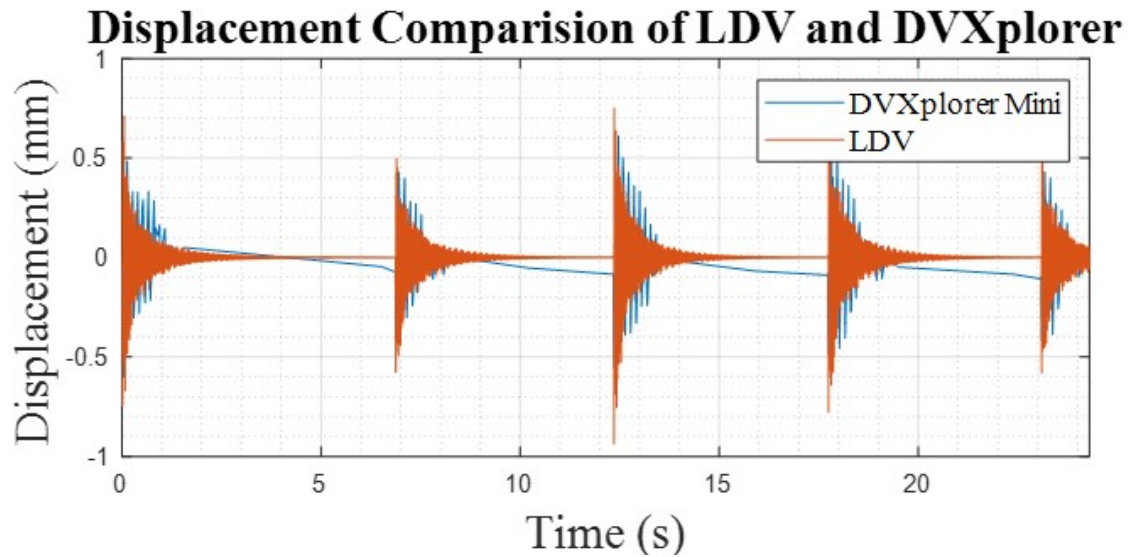


Fig. 8 LDV and DVXplorer Mini displacement comparison.

Neuromorphic imaging data was collected from three DOFs for this experiment: DOF 1, DOF 5, and DOF 7. For each DOF, the camera was placed at a distance of 1 inch from the wing, while an impact was applied at DOF 3. Figure 9a shows the experimental setup for DOF 1, and Figure 9b highlights the DOFs where the camera data was collected, including DOF 3, which was the impact DOF. These DOFs were chosen because they are located on the edges, making them accessible for the camera, and they are areas of high displacement, which allows the camera to record higher quality data.

Using the refined code and the collected data from DOF 1, DOF 5, and DOF 7, the displacement was collected and subsequently post processed. The data were first detrended and resampled into a uniform time vector using interpolation to ensure consistency in sampling. Due to the varying time steps of neuromorphic imaging data, a high sampling rate was used.

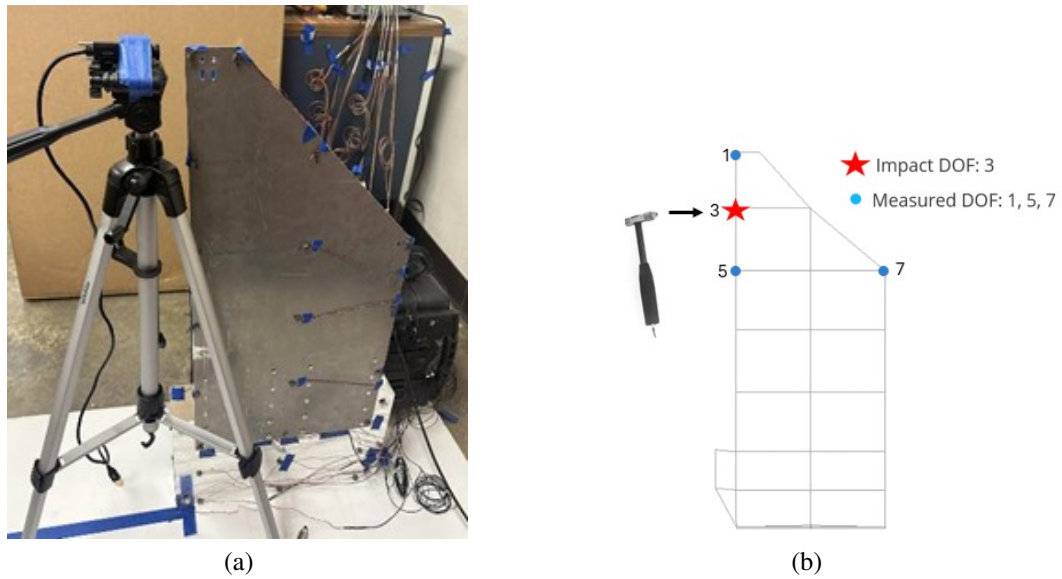


Fig. 9 (a) Neuromorphic imaging data setup for DOF 1. (b) Layout of measured DOF and impact DOF.

A low-pass Butterworth filter was then applied to the resampled data to smooth out high-frequency noise and enhance signal clarity. The filtered displacement data were differentiated twice to obtain the acceleration time history, as shown in Figure 10. Additionally, the filtered displacement data were analyzed in the frequency domain using the Fast Fourier Transform (FFT), as seen in Figure 11. These steps collectively ensure that the data are clean, consistent, and ready for further analysis.

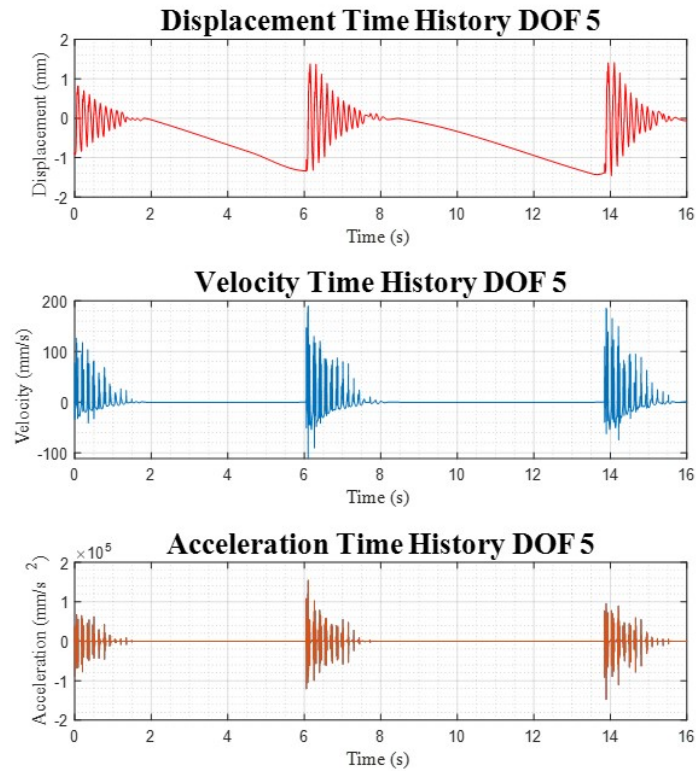


Fig. 10 Displacement, velocity and acceleration time history for DOF 5.

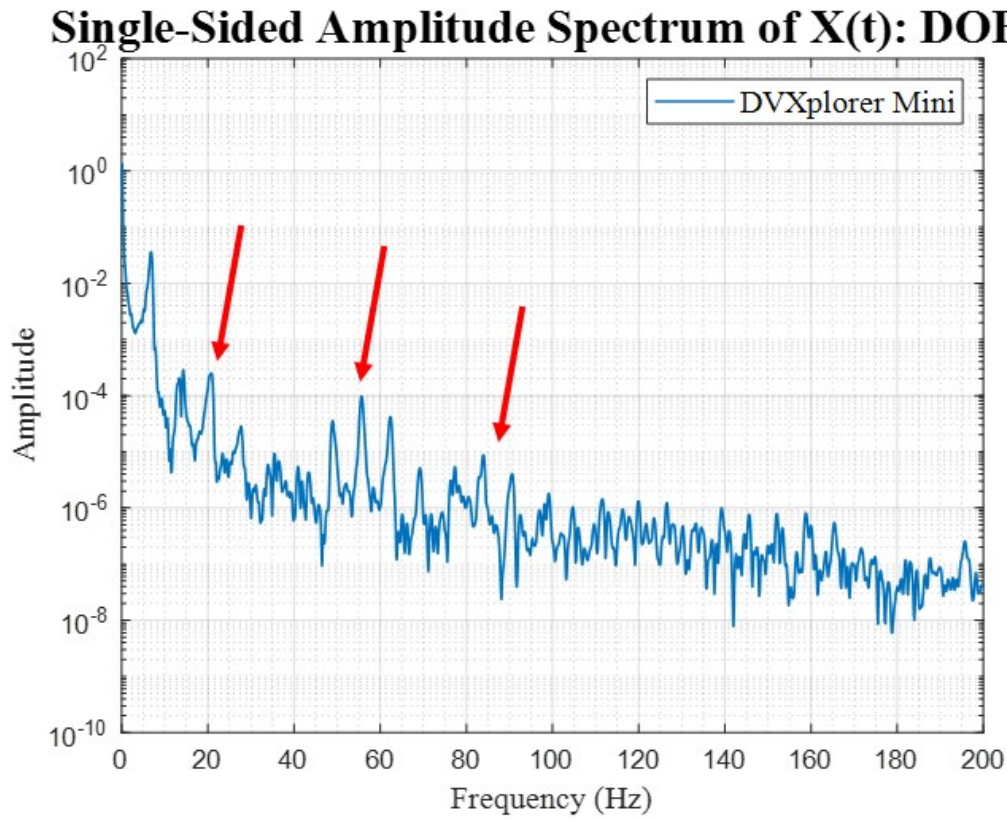


Fig. 11 FFT of DOF 5.

It is noted that the bandwidth of interest drops significantly after 200 Hz, no longer revealing clear peaks of natural frequency locations. For the continuation of the data processing, the focus is on the first three modes, all of which are within the first 100 Hz. Within the FFTs from the collected DOFs, the first three modes do appear as peaks despite the interference from other noise. This can be seen in Figure 11, where the first mode at 20.8 Hz, the second mode at 55.7 Hz, and the third mode at 83.9 Hz can be seen.

Degree Of Freedom Selection

The DOF (degrees of freedom) selection is critical for modal expansion because it determines which structural responses are captured. This selection impacts the accuracy of representing the structure's dynamic behaviors. In general, it is preferable to select the number of DOFs as 1.5 times the number of modes included in your finite element model. This ensures enough DOFs are selected to capture the structural response information of all of the selected modes. In this case, the finite element model included ten modes, so fifteen DOFs were chosen for the expansion. Two methods were explored for selecting the optimal DOF set, Effective Independence (EFI) and Condition Number Optimization.

Condition Number Optimization removed DOF 1, which is a key DOF where neuromorphic imaging data was collected, so this DOF set was not used for the mixed domain expansion. On the other hand, Effective Independence removed DOFs on the block, which did not interfere with the collected neuromorphic imaging data. To further explore the selection, a Manual Selection was also performed. This involved removing DOF 6, located in the middle of the wing, to evaluate how well the expansion would perform in an area with higher displacement, rather than focusing only on the block, which experienced very minimal displacement. The combination of using Effective Independence, as shown in Figure 12a, and manual selection, as shown in Figure 12b, ensured a more comprehensive and targeted comparison of DOF sets.

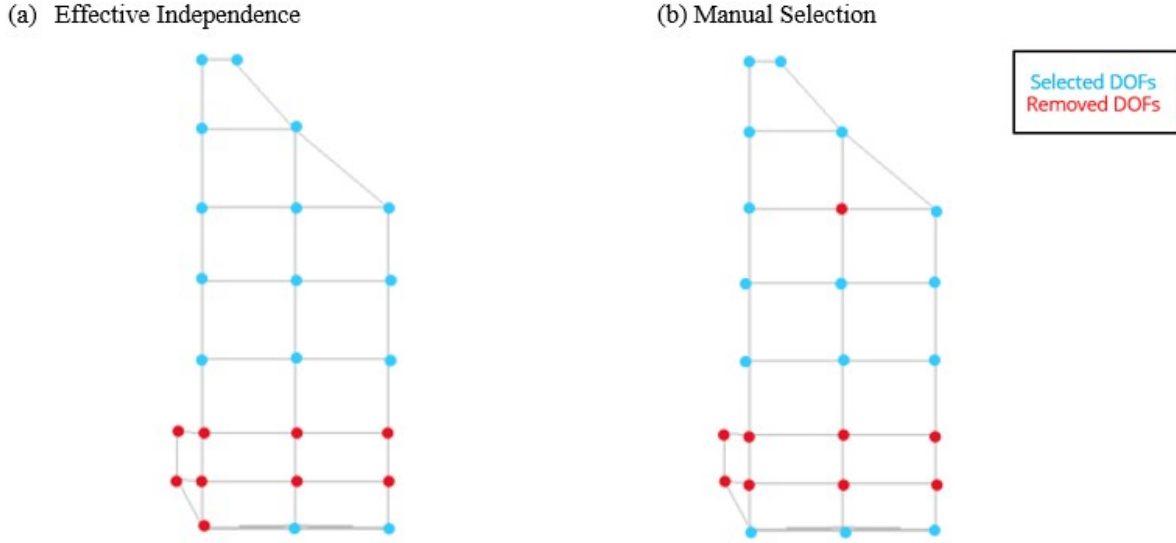


Fig. 12 DOF Selection Methods.

Results & Analysis

The System Equivalent Reduction Expansion Process (SEREP) was the modal expansion method utilized. This method was selected because it is one of the most widely used modal expansion methods. Referring to Equation 2, both single and mixed domain expansions using SEREP were then completed.

Where U_n is a mode shape matrix of all the DOF, U_a^+ is the pseudo-inverse of the mode shape matrix of the selected DOF from the model, and x_a is experimental acceleration response data of the selected DOFs (15 DOF $\times$ n frequency steps). The multiplication of U_n and U_a^+ acts as a modal filter, and x_n represents the estimated acceleration response data for all DOFs.

For single domain expansion, x_a is composed of all the accelerometer data for the selected 15 DOFs.

$$x_a = \begin{bmatrix} \text{Accelerometer DOF 1} \\ \text{Accelerometer DOF 2} \\ \text{Accelerometer DOF 3} \\ \text{Accelerometer DOF 4} \\ \text{Accelerometer DOF 5} \\ \vdots \\ \text{Accelerometer DOF 15} \end{bmatrix}$$

In the mixed domain expansion, the accelerometer data for DOF 1, DOF 5, and DOF 7 were replaced with the post processed neuromorphic imaging data recorded at those DOF. Figure 13 is a reference showing the locations of DOF 1, DOF 5, and DOF 7.

$$x_a = \begin{bmatrix} \text{Neuromorphic DOF 1} \\ \text{Accelerometer DOF 2} \\ \text{Accelerometer DOF 3} \\ \text{Accelerometer DOF 4} \\ \text{Neuromorphic DOF 5} \\ \vdots \\ \text{Accelerometer DOF 15} \end{bmatrix}$$

Using the DOF selected using the Effective Independence method, both a single and mixed domain expansion were completed. Figure 14 shows the comparison between the single domain expansion and the mixed domain expansion, specifically focusing on DOF 15. Figure 15 also references the location of DOF 15 and identifies the DOF set selected using the Effective Independence method. A Root Mean Square Error (RMSE) was then computed for the single and mixed domain expansion

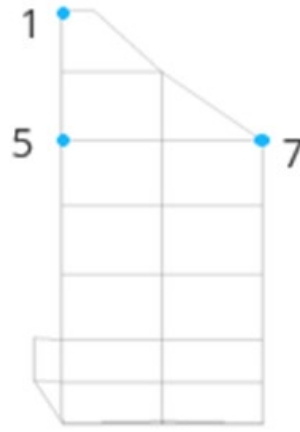


Fig. 13 DOF showing where neuromorphic imaging data was collected for the Mixed Domain Expansion.

estimates. The lower the RMSE, the better the estimation. For the single domain expansion, the RMSE was 0.024, while for the mixed domain expansion, it was 0.022. By computing the RMSE and examining the results, it is evident that the mixed domain expansion provides estimates for the block and represents the measured response data relatively well. This expansion was applied to DOFs which were located on the block. This could be useful in situations where it is difficult to instrument the base of the structure, especially if the base is less accessible.

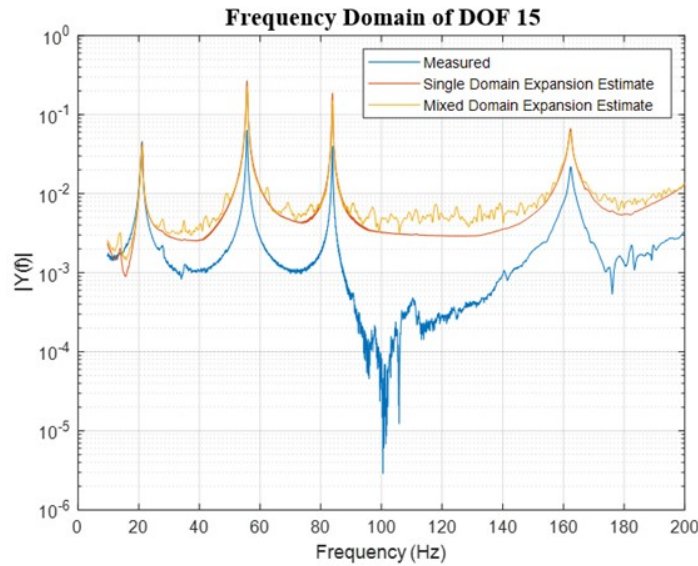


Fig. 14 Comparison of single and mixed domain expansion results for DOF 15 using DOF selected using Effective Independence.

Additionally, both single and mixed domain expansions were computed using DOFs selected from the Manual DOF selection method. Figure 16 shows the results of expanding to DOF 6, which was specifically chosen to assess how well the expansion performs in areas with higher displacement. For the single domain expansion, the RMSE was 0.013, while for the mixed domain expansion, it was 0.010. Although the mixed domain expansion estimate is noisier than the single domain expansion, it is evident that, despite the noise, the overall trend of the mixed domain expansion follows the measured data more closely than the single domain expansion. Figure 17 references the location of DOF 6 and identifies which DOFs were selected using the Manual DOF selection.

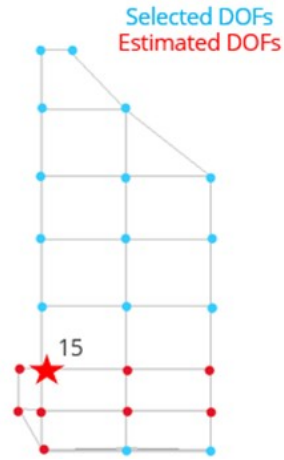


Fig. 15 Reference Image for DOF 15 and DOFs selected using the Effective Independence method.

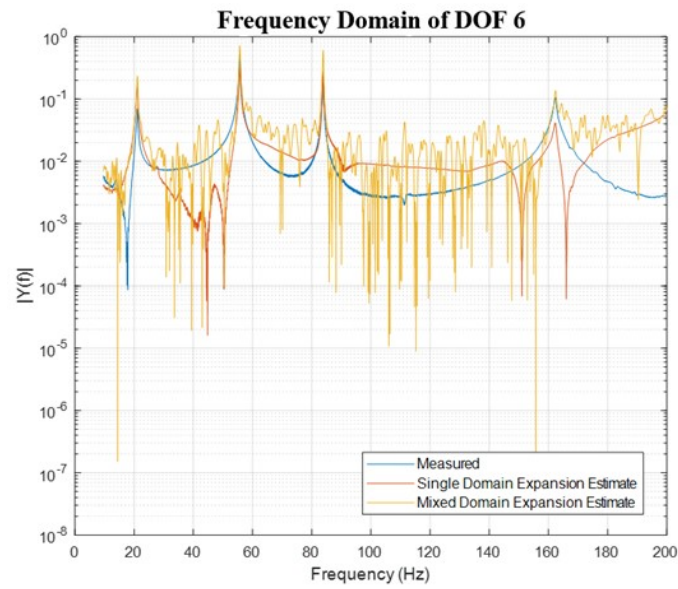


Fig. 16 Comparison of single and mixed domain expansion results for DOF 6 using DOF selected using the Manual DOF selection method.

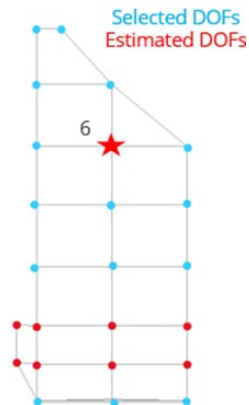


Fig. 17 Reference Image for Manual DOF selection and DOF 6.

Conclusion

From this research, three key conclusions have been drawn. Firstly, a mixed domain expansion can be achieved by combining neuromorphic imaging data with acceleration data. This conclusion is based on our results from applying the combined data to expand to different locations on the wing, yielding an RMSE comparable to that of a single-domain expansion. Second, a foundation for computing this new mixed domain expansion was developed by (1) collecting displacement from a neuromorphic camera, (2) exploring two DOF selection methods, and (3) converting neuromorphic imaging displacement to acceleration to compute a mixed domain SEREP expansion. Third, our results indicate that expanding to places of smaller displacement on the block produced less noisy estimates compared to expansions at points on the wing. This proof of concept lays the foundation for extending this technique, which can be applied to structures that cannot entirely use accelerometers due to, for example, their geometric complexity or their sensitivity to mass loading. It allows structural health monitoring in complicated structures where traditional methods could not be implemented.

These conclusions pave the way for additional areas of research. The first area for improvement is improving the collection and post-processing methods of the neuromorphic imaging data. While mixed-domain expansion is possible, it is limited by the frequency bandwidth capabilities of the camera, the resolution of displacement measurements, and the post-processing of the collected neuromorphic imaging data. Additionally, exploring experimental testing using different excitation methods, such as a shaker, or testing on a more flexible structure could produce better results. The neuromorphic imaging camera was detected as displacements, and a structure with either larger displacements or continuous excitation, rather than a single impact, would be more efficiently captured by the camera.

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Chapter 6

Damage Identification Strategy in Time-Varying Dynamic Systems Combining Cepstral and Image-Based Features

Sina Shid-Moosavi, Costanza Speciale, and Eleonora Maria Tronci

Abstract Damage identification and performance assessment in structural systems are vital for ensuring their safety, durability, and efficiency. Vibration-based pattern recognition strategies have become popular in the past decades to identify anomalies using the vibration response from the monitored structures. However, conventional methodologies often fail to recognize and classify the different types of damage, particularly in dynamic structural environments where patterns change over time. These limitations highlight the need for a more adaptive and comprehensive approach that can adjust to time-evolving conditions and detect subtle variations indicative of damage. This study aims to develop an innovative framework for damage identification by integrating cepstral and time-image-based features to leverage the strength of both sets of indicators. Time image-based features like the Gramian Angular Field capture underlying structural dynamics by encoding temporal correlations among data points as angular values in a polar coordinate system. This technique allows for the application of image-based machine learning methods to time-series analysis by converting the data into a two-dimensional matrix. Additionally, combining cepstral coefficients improves the capability of tracking stationary and nonstationary changes in the frequency content of the systems. By fusing these sets of parameters, this study aims to enhance the accuracy and efficiency of structural damage identification, particularly in systems with time-varying behaviors.

Keywords Damage Identification · Pattern Recognition · Structural Health Monitoring · Cepstral Coefficient · Gramian Angular Fields

Introduction

Vibration-based Structural Health Monitoring (SHM) aims to develop reliable methods for detecting damage by evaluating changes in the vibration responses of civil structures [1, 2]. In this context, structural damage is typically defined as any alteration in a system's material or geometric properties, such as boundary conditions or connectivity, that negatively impacts its performance, both in the present and the future, and ultimately modifies its dynamic response [3]. To identify these changes, SHM often employs Pattern Recognition techniques, which focus on detecting significant variations in parameters that characterize a structure's behavior and health. These parameters, known as Damage Sensitive Features (DSFs), provide a basis for assessing the condition of the monitored system [4].

However, applying these techniques to large-scale, complex, and time-varying systems, such as those found in civil, mechanical, and aerospace engineering, presents numerous challenges. These challenges arise from factors like the scale of the structure, variability in support conditions, sensor noise, limited sensor coverage, and fluctuations in operational loads [5]. The ability to accurately detect damage under such conditions is highly dependent on the selection of effective DSFs, features that must be responsive to structural changes while remaining resilient to external and operational influences and characterized by an easy and user-independent extraction process.

Various parameters have been proposed as DSFs, each with strengths and weaknesses. Modal parameters, like natural frequencies and mode shapes, are widely used due to their connection to structural stiffness and spatial information [6].

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However, they are sensitive to environmental and operational variations, and their extraction is computationally intensive, often relying on user expertise, which limits their use in real-time, dynamic monitoring [7].

This research begins by leveraging cepstral coefficients as damage-sensitive features, which offer a compact and computationally efficient way to capture the spectral content of a system's dynamic response. Initially developed for speech recognition, cepstral coefficients have recently gained traction in structural damage detection [8, 9], thanks to their ability to track both stationary and nonstationary changes in frequency content [10], thereby improving robustness in identifying damage under varying conditions. However, relying solely on cepstral features may not fully capture the complexity of damage patterns in dynamic environments, especially when the system's behavior evolves over time. While cepstral coefficients are effective in capturing the spectral characteristics of dynamic systems, they provide a single-window representation of the system's behavior. This means that although they offer insight into the overall frequency content, they aggregate the response over the window's duration, making it difficult to pinpoint the exact moment when damage occurs within that window. Consequently, this can limit the ability to detect complex, evolving damage patterns, especially when damage manifests as a brief or subtle change within the time window. The inability to precisely localize the damage event within the window motivates the need for additional features that can capture both the temporal evolution of the system and the finer, more complex patterns of damage across time.

To address the limitation of cepstral coefficients' single-window representation, this work introduces a novel framework that combines them with time-image-based features, specifically the Gramian Angular Field (GAF). GAF transforms time-series data into two-dimensional images, capturing temporal correlations by encoding data points as angular values in a polar coordinate system [11]. This allows for a finer resolution of the temporal dynamics, complementing the cepstral coefficients' ability to track stationary and nonstationary frequency content changes. By merging these two techniques, this approach enhances the robustness and accuracy of damage identification, particularly in dynamic environments where damage patterns evolve over time and require precise localization [10, 12].

The novelty of this framework lies in its ability to merge two complementary feature types: cepstral coefficients provide valuable insight into the system's spectral properties, while GAF captures the temporal evolution of vibration signals in a format that can be analyzed using advanced machine learning techniques. This dual-feature approach enables a more comprehensive analysis of the system's behavior, especially in time-varying conditions where traditional methods often fail to detect subtle, evolving damage patterns.

Case Study: Aventa AV-7 Dataset

This research utilizes data from the Aventa AV-7 [13], a 7 kW wind turbine operated by ETH Zurich (Figure 1), which has been monitored to evaluate SHM methods as the turbine approaches the end of its design life. Since 2020, sensors have



Fig. 1 Aventa AV-7 [13].

been deployed throughout the turbine to facilitate system identification, modal analysis, and failure detection studies, with data collected under four specific operational scenarios: normal operation, pitch system failure, aerodynamic imbalance, and rotor icing. For each condition, datasets include time series measurements recorded over specified dates between 2022 and 2023.

The Aventa AV-7 turbine operates with a variable-speed, variable-pitch control, beginning energy generation at a 2 m/s wind speed and ceasing at 14 m/s for a rated power of 7 kW. It has a three-blade rotor with a 12.9 m diameter, positioned 18 m above ground on a concrete and steel tower. A belt-driven generator and a frequency converter enable variable-speed operation.

The turbine is outfitted with eleven accelerometers distributed along the tower, nacelle, main bearing, and generator, as well as two full-bridge strain gauges at the tower base for measuring axial and bending strains. All data is recorded at 200 Hz, while SCADA (Supervisory Control and Data Acquisition) data, such as wind speed, rotor speed, and power output, is recorded at 7 Hz, with temperature and humidity recorded at 1 Hz. The accelerometers are paired and located at specific tower sections to capture the x - and y -axis accelerations aligned with wind direction, allowing comprehensive monitoring of the turbine's structural response.

The SCADA system provides insights into power, wind speed, rotor speed, and yaw angle, which are critical for analyzing turbine performance, especially under "normal operation" conditions. The power curve, derived from these SCADA readings, illustrates the relationship between wind speed and power output, with distinct operational regions (cut-in, rated, and cut-off speeds) [14].

The dataset further captures three specific fault conditions, providing valuable data for condition monitoring methods.

In February 2022, a failure occurred in the flexible coupling of the pitch drive system, which is critical for adjusting blade angles to optimize performance under changing wind conditions. SCADA data captured deviations in rotor speed, power output, and yaw position compared to normal operation. Before the complete failure, the pitch system became stuck, reducing the turbine's output to 1-2 kW while the rotor speed dropped to around 40 rpm. An emergency stop was triggered to prevent further damage, as indicated by sharp changes in SCADA measurements at approximately 270 seconds. Acceleration data from sensors across the tower and nacelle show a significant structural response to the failure, highlighting its impact on the turbine's dynamics.

In December 2022, rotor icing caused a near-complete halt in turbine operations, reducing rotor speed to almost zero and stopping power generation. This highlights the importance of early detection in cold climates where ice accumulation can significantly reduce aerodynamic efficiency and increase vibration.

Then, data collected after roughness tape was applied to one blade in December 2022 revealed imbalances that affected rotor speed and induced mechanical stress on key turbine components, underscoring the need to identify imbalances early to prevent failures.

This research focuses on the pitch drive failure, as it represents a sudden and critical rupture in the turbine's system. This abrupt failure is particularly challenging for SHM due to the need for immediate detection and intervention to prevent further damage. By concentrating on this sudden event, the study aims to improve SHM techniques' ability to quickly identify and respond to rapid, unforeseen failures, which are often more difficult to manage than gradual damage.

Analysis

The proposed methodology consists of a two-step feature extraction and clustering approach designed to identify structural damage in the turbine system. First, cepstral coefficients are extracted from the system's acceleration time history, providing a compact and computationally efficient representation of the frequency content, which is sensitive to changes in dynamic response caused by damage. Next, image-based features are derived using Gramian Angular Fields, which transform time-series data into two-dimensional images, preserving temporal correlations.

These two types of features are then combined into a unified feature set by applying Principal Component Analysis to reduce dimensionality while retaining the most significant components. Finally, the reduced feature set is analyzed through hierarchical clustering, which groups the data based on similarities in spectral and temporal patterns. This process identifies distinct clusters corresponding to different health conditions, allowing for the detection and classification of damage across various regions of the turbine's operational data.

Cepstral Coefficients

Cepstral coefficients are obtained by applying the Inverse Discrete Fourier Transform (IDFT) to the logarithm of the squared magnitude of a signal's Discrete Fourier Transform (DFT). In the context of SHM, the available signals are typically the system's displacement or acceleration time histories. Accordingly, a set of D cepstral coefficients is defined as follows:

$$CC[q] = \mathcal{F}^{-1} \{ \log(|\mathcal{F}\{x[n]\}|^2) \}, \quad q = 0, \dots, D-1 \quad (1)$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the DFT and IDFT, respectively, and $x[n]$ ($n = 0, \dots, N_a - 1$) represents the discrete acceleration time history of the system under investigation. The selection criterion for the number of cepstral coefficients (D) and its impact on classification performance is detailed in [15].

Examining the formulation of cepstral coefficients reveals their benefits for system representation. The extraction process is both compact and computationally efficient, making it suitable for real-time monitoring and large-scale applications. Additionally, it requires minimal user input, as the only decision involves selecting the number of coefficients (D), reducing the level of expertise needed.

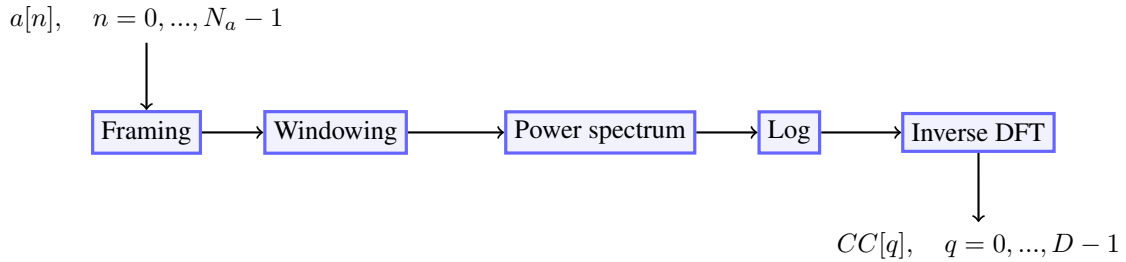


Fig. 2 Cepstral coefficients extraction procedure.

The extraction process follows five key steps, as depicted in Figure 2. Beginning with the system's acceleration time history $x[n]$, the signal is divided into equal-length frames during the framing phase. Each frame is multiplied by a Hamming window to mitigate edge effects (windowing phase). Afterward, D cepstral coefficients are computed for each frame by calculating the power spectrum and applying the logarithm, following Equation 1. Typically, the first coefficient is omitted due to its sensitivity to external forces and sensor noise.

Breaking down the extraction process reveals an additional advantage of cepstral coefficients: their ability to be extracted from small portions of the signal independently. This allows for the sequential analysis of small frames, enabling the description of the system's evolution over time, even in non-stationary signals or complex dynamics. This feature enhances the applicability of cepstral coefficients for analyzing systems characterized by time-varying dynamics.

When structural damage occurs, it alters the system's properties, which leads to changes in its dynamic response. Cepstral coefficients can capture these changes by analyzing the spectral content of the acceleration signal. However, relating specific changes in structural properties to variations in the cepstral coefficients can be challenging, particularly for complex systems.

The analytical formulation presented in [15] clarifies this connection. For an N -degree-of-freedom system subjected to a generic input excitation, the q -th cepstral coefficient ($q > 0$) for the acceleration at the i -th degree of freedom is given by:

$$CC_i[q] = \theta[q] + \gamma_i[q] \quad (2)$$

where,

$$\begin{cases} \theta[q] = \frac{1}{q} \left[\sum_{l=1}^N 2e^{-\xi_l \omega_l \Delta T} \cos \omega_{d,l} \Delta T q - 1 \right] \\ \gamma_i[q] = \frac{1}{q} \left[- \sum_{l=1}^{2N-1} Z_l^{(i)q} \right] \end{cases} \quad (3)$$

In these equations, ΔT represents the sampling time, ω and ω_d are the undamped and damped natural frequencies, ξ is the damping ratio, and $Z_l^{(i)q}$ are the roots of a polynomial equation that incorporates both the system's structural properties and the input excitation. While this formulation is straightforward for single-degree-of-freedom systems, it becomes more complex for multi-degree systems, as demonstrated in [15].

Nonetheless, this formulation clarifies that the cepstral coefficients have two main components: $\theta[q]$, which is linked directly to the system's natural frequencies and damping ratios, and $\gamma_i[q]$, which also depends on factors such as input excitation and sensor location.

Image-Based Features

This section extracts image-based features from the acceleration window frames used to extract the cepstral coefficients. The first step consists of encoding the time series as images using the Gramian Angular Field [16], representing time series data in a polar coordinate system instead of the conventional Cartesian coordinates. This approach provides a unique method for capturing the temporal patterns of the data, as each value is mapped to an angular coordinate, while time is represented by the radial coordinate [17, 18].

Given a time series $x[n]$ ($n = 0, \dots, N_a - 1$), the first step is to rescale the values so that they fall within a fixed range, typically $[-1, 1]$ or $[0, 1]$:

$$\tilde{x}_{-1}^i = \frac{(x_i - \max(X)) + (x_i - \min(X))}{\max(X) - \min(X)} - 1, \quad \text{or} \quad \tilde{x}_0^i = \frac{x_i - \min(X)}{\max(X) - \min(X)} \quad (4)$$

Next, the rescaled time series is transformed into polar coordinates ($\tilde{X}$), where the time values are mapped as radial components and the rescaled values as angular components:

$$\begin{cases} \phi = \arccos(\tilde{x}_i), & -1 \leq \tilde{x}_i \leq 1, \tilde{x}_i \in \tilde{X} \\ r = \frac{t_i}{N_a}, & t_i \in N_a \end{cases} \quad (5)$$

where t_i represents the time stamp, and N_a is a constant factor that regularizes the polar coordinate space. This transformation offers a novel way to interpret time series data by warping values across a range of angles, with the polar coordinate representation preserving the absolute temporal order of the data.

Once transformed, we can compute temporal correlations between different time points by exploiting trigonometric functions. The Gramian Summation Angular Field (GASF) and Gramian Difference Angular Field (GADF) are defined as follows:

$$\text{GASF} = [\cos(\phi_i + \phi_j)] = \tilde{X}' \cdot \tilde{X} - \sqrt{I - \tilde{X}'^2} \cdot \sqrt{I - \tilde{X}^2}^\top \quad (6)$$

$$\text{GADF} = [\sin(\phi_i - \phi_j)] = \tilde{X} \cdot \sqrt{I - \tilde{X}'^2} - \tilde{X}' \cdot \sqrt{I - \tilde{X}^2}^\top \quad (7)$$

where I is the unit row vector $[1, 1, \dots, 1]$. After converting the time series into polar coordinates, each time step is treated as a one-dimensional metric space, where the inner product between two points x and y is defined as follows:

$$\langle x, y \rangle = x \cdot y - \sqrt{1 - x^2} \cdot \sqrt{1 - y^2}, \quad \text{or} \quad \langle x, y \rangle = \sqrt{1 - x^2} \cdot y - \sqrt{1 - y^2} \cdot x \quad (8)$$

This process results in two types of Gramian Angular Fields, GASF and GADF, which can be viewed as quasi-Gramian matrices.

$$G = \begin{bmatrix} \langle x_1, x_1 \rangle & \dots & \langle x_1, x_n \rangle \\ \vdots & \ddots & \vdots \\ \langle x_n, x_1 \rangle & \dots & \langle x_n, x_n \rangle \end{bmatrix} \quad (9)$$

The GAFs offer several advantages. First, they maintain temporal dependencies as time progresses from the top-left to the bottom-right of the matrix. Each matrix element $G_{i,j}$ represents the relative correlation between two-time points, determined by the summation or difference of angular values for the corresponding time interval k . The main diagonal $G_{i,i}$, which corresponds to the case when $k = 0$, retains the original value or angular information. This diagonal can be used to reconstruct the original time series from high-level features extracted by deep learning models [16].

After generating these Gramian matrices, they are converted into images for further processing.

The next step involves extracting features from these images using DenseNet [19], a deep convolutional neural network architecture with a unique connectivity pattern. In DenseNet, each layer is connected to all preceding layers within a dense block, allowing each layer to access the feature maps of all previous layers. This dense connectivity facilitates feature reuse and supports efficient gradient flow, leading to a compact model less prone to overfitting [20]. Additionally, DenseNet enables direct supervision of each layer through shortcut connections, enhancing the model's ability to capture detailed pixel-level information without requiring extensive preprocessing [20]. The feature extraction process in DenseNet is particularly suited for capturing the complex patterns within these time series-based images.

Features Fusion and Clustering

This study utilized a combination of cepstral coefficients and image-based features to capture both the spectral and temporal dynamics of the system's vibration signals. Cepstral coefficients provide a concise representation of the frequency content in the signals, while GADF and GASF features convert time-series data into two-dimensional images, preserving temporal relationships between data points. This dual-feature approach allows for a more comprehensive analysis, as it integrates the frequency-domain insights of cepstral coefficients with the temporal correlations encoded by the image-based features.

Once these features were extracted for each time window, Principal Component Analysis (PCA) [21] was applied to reduce the dimensionality of the combined feature set. PCA efficiently selects the most significant components, reducing the complexity of the data while retaining the essential features that contribute most to variance within the dataset. The unified feature set obtained from this dimensionality reduction step is thus optimized for further analysis, making the dataset manageable while still capturing the critical information necessary for detecting and classifying damage.

The next step in the methodology involved applying unsupervised clustering to the reduced feature set in order to identify distinct damage conditions without prior labeling. Clustering was used to group the data based on shared characteristics. This clustering approach is crucial for understanding how the system responds to different types of damage, particularly in complex and evolving environments [22].

Hierarchical clustering was chosen as the primary technique for this task, as it is well-suited for exploratory data analysis and allows for the visualization of relationships between data points in the form of a dendrogram. The hierarchical clustering algorithm works by initially treating each data point as its own individual cluster. At each iteration, the two most similar clusters are merged, progressively reducing the number of clusters until all points are grouped into a single large cluster. This iterative process produces a tree-like structure, which can be "cut" at different levels to reveal distinct clusters that represent different damage states [23].

The process of hierarchical clustering involves three key steps: (1) calculating the similarity between each pair of objects, (2) linking the objects into a hierarchical tree structure based on these similarities, and (3) determining an appropriate level to "cut" the dendrogram to define distinct clusters. In this analysis, Euclidean distance was used as the measure of similarity between clusters, ensuring that the clustering process captured both the spectral and temporal characteristics of the data [22].

A notable advantage of hierarchical clustering is its deterministic nature and its ability to provide a clear method for selecting the final number of clusters based on the hierarchical tree structure. However, a significant drawback of this approach is its computational intensity, especially when dealing with large datasets, since the similarity between every pair of objects must be calculated [24]. Despite this, hierarchical clustering was chosen due to its ability to reveal the underlying structure of the data and its effectiveness in grouping complex, high-dimensional data into meaningful clusters that correspond to different damage conditions.

Results

This study was conducted to analyze the pitch drive failure case, which is a mechanism that can be considered developing in three regions, as shown in Figure 3. Initially, the turbine operated under normal conditions, classified as the undamaged case, where rotor speed, wind speed, and power output were stable. A significant decrease in rotor speed and power was observed in damaged region 1, marking the onset of the pitch system failure. Notably, the wind speed remained steady, highlighting that mechanical failure rather than external factors caused the performance degradation. Following this, the pitch system became stuck in an incorrect position, leading to severely reduced power generation (down to nearly 0 kW) despite the rotor continuing to rotate at a lower RPM, identified as damaged region 2. Again, wind speed remained consistent, emphasizing that the turbine's inability to generate power was due to internal failure. Ultimately, the system performed an emergency stop to prevent further damage, labeled as damaged region 3.

The rupture in the pitch drive system limits the turbine's ability to produce adequate power relative to wind speed, significantly affecting the structure's dynamic response. The pitch drive typically adjusts blade angles for optimal performance, but its failure leads to a mismatch between mechanical forces and operational demands. This misalignment alters load distribution and reduces the effective stiffness, particularly in the drivetrain and support structures. As stiffness decreases, the natural frequencies drop, making the structure less resistant to dynamic loads. In the damaged regions, the cepstral analysis shows irregular frequency content, with lower-frequency components dominating, reflecting the structural degradation and stiffness loss caused by the pitch failure.

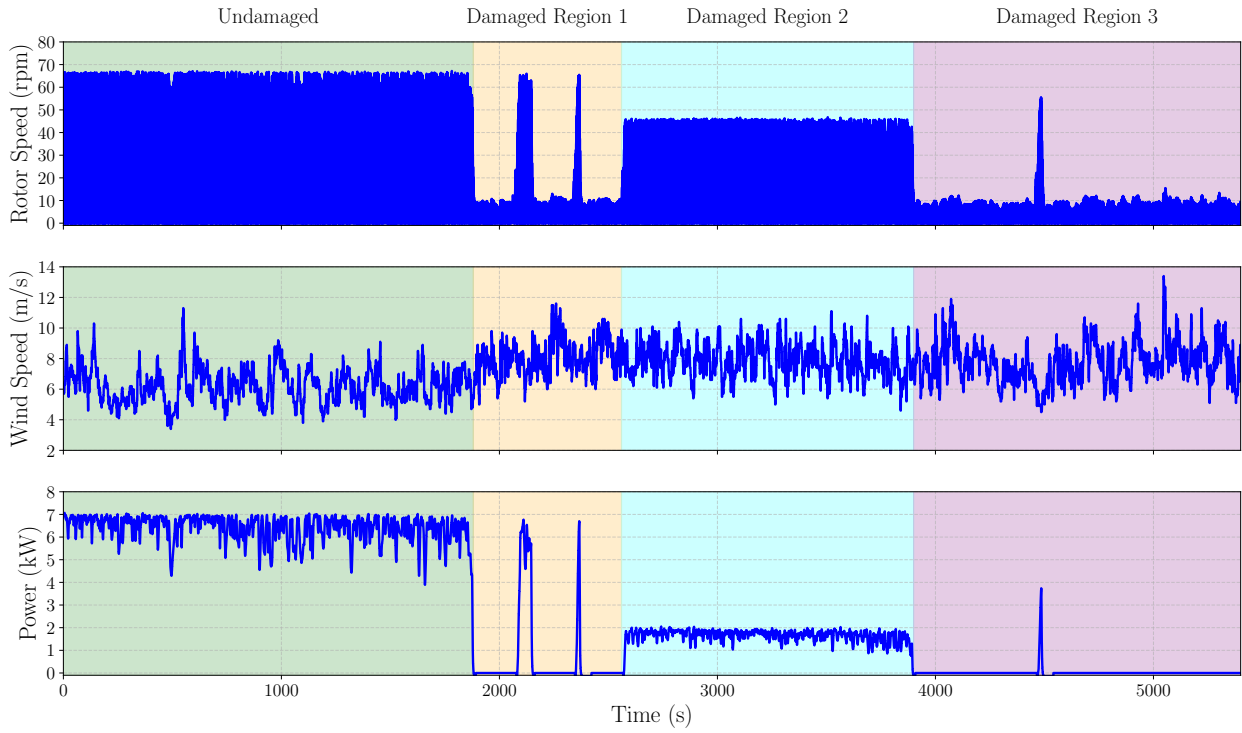


Fig. 3 Progression of turbine operational states from normal to pitch failure across identified damage regions.

Figure 4 illustrates the variation of cepstral coefficients across the turbine's operational states, focusing on how the pitch drive failure affects the system's dynamic response. After discarding the first cepstral coefficient, the remaining coefficients provide insights into the structural changes occurring due to the failure. The lower-order coefficients (second, third, etc.) show noticeable deviations as the damage progresses, particularly in damaged region 1, where the smoothness seen in the undamaged state begins to break down. This reflects significant structural changes, as these coefficients are sensitive to shifts in the system's primary spectral content, such as changes in natural frequencies or damping. As the failure worsens in damaged region 2, the higher-order coefficients become increasingly irregular, capturing finer details of the dynamic response and reflecting more complex, nonlinear effects caused by the pitch failure. The growing irregularity across both lower- and higher-order coefficients highlights the progressive degradation of the system, with lower-order coefficients capturing broad structural changes and higher-order coefficients revealing more detailed variations in the frequency content. Together, these changes provide a comprehensive picture of the system's instability as the pitch drive failure progresses.

The GADF and GASF images across various regions in Figure 5 illustrate how pitch drive failure influences temporal dynamics correlations. The fusion of GADF and GASF images facilitates the capture of both global patterns and localized fluctuations, as GADF accentuates time-series anomalies, while GASF emphasizes collective behavioral trends within the data [11]. The images for undamaged regions reveal higher rotor speeds, consistent with stable operational behavior. Conversely, in damaged regions 1 and 3, where rotor speeds are lower, distinct deviations in image patterns can be observed, indicative of impaired functionality. Damaged region 2 displays characteristics suggestive of malfunction, though with distinct features differing from regions 1 and 3, highlighting specific anomalies in rotor speed that further underscore varying degrees of pitch drive impairment.

After extracting cepstral coefficients and image-based features, the two sets of features were fused as previously described. Initially, the individual features were subjected to PCA to reduce dimensionality, with only the major components retained to form a combined feature set. This unified feature set was then used as input to the hierarchical clustering algorithm. The resulting dendrogram, displayed in Figure 6, illustrates the hierarchical grouping of samples, where each node represents the merging of clusters based on their similarity. Along the x -axis, the distance between clusters is shown, which reflects the level of similarity across samples within each group.

Based on the dendrogram, three clusters were selected to categorize the dataset. The merged features were thus organized according to these three clusters, as illustrated in Figure 7. Here, rotor speed and acceleration data are color-coded based on their cluster assignments, clearly distinguishing damaged states from undamaged ones. Notably, damage regions 1 and 3 are

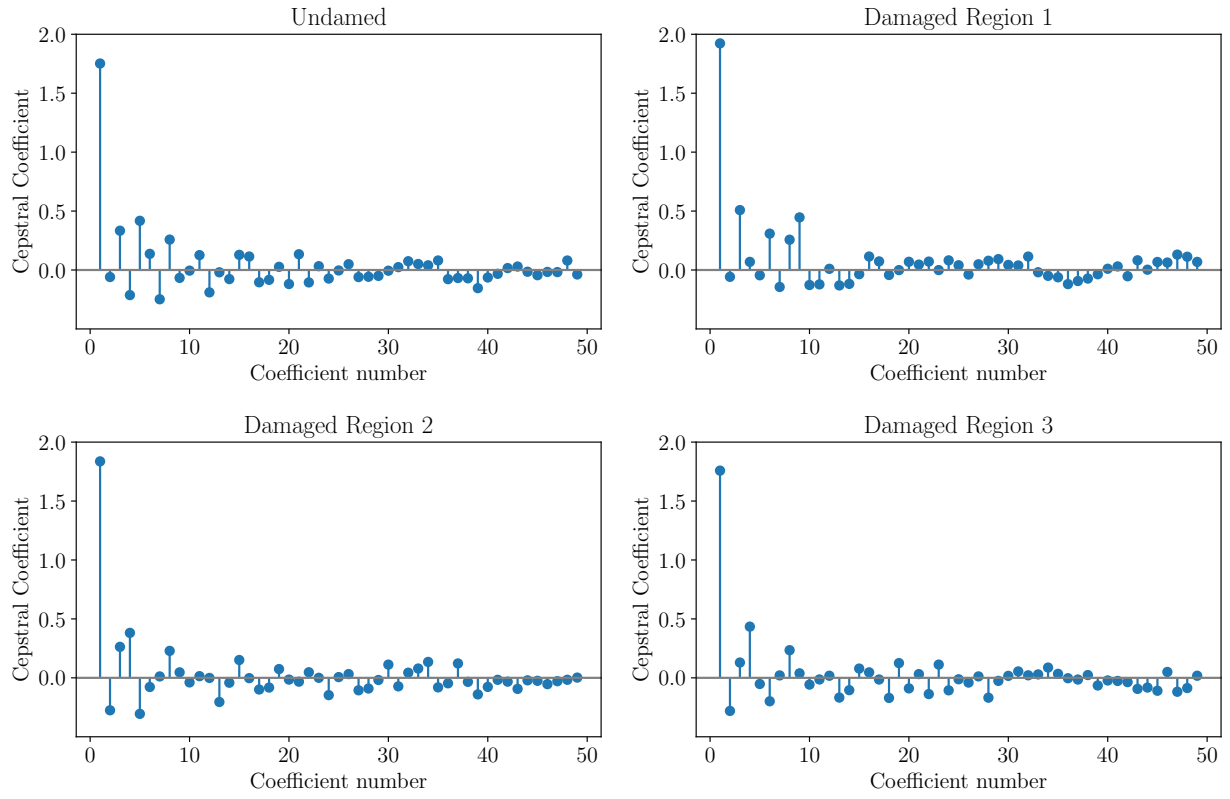


Fig. 4 Sample cepstral coefficient variations across regions of pitch drive failure.

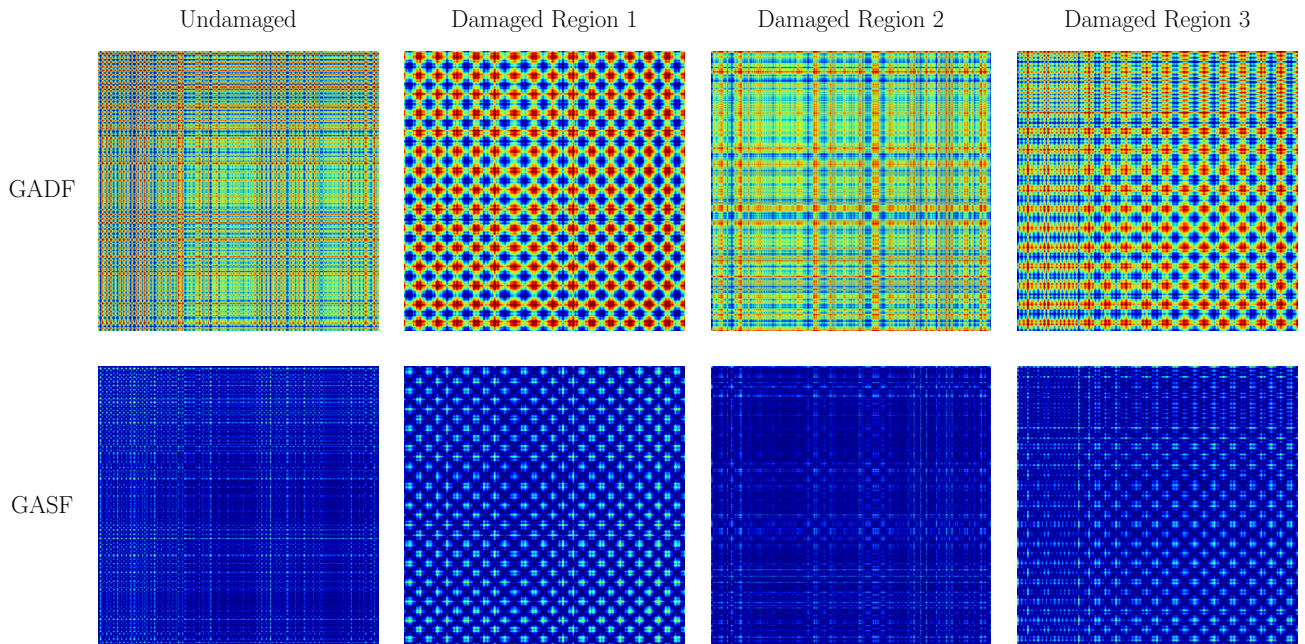


Fig. 5 Sample GAF image variations across regions with pitch drive failure.

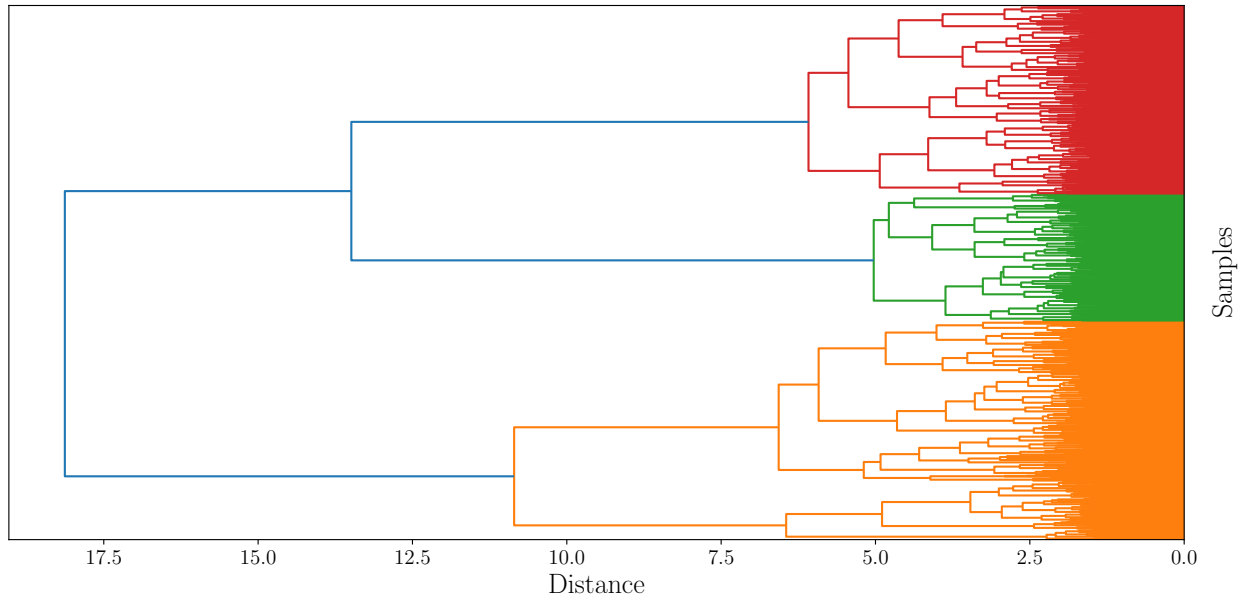


Fig. 6 Dendrogram showing hierarchical clustering of pitch drive failure time-series samples over time.

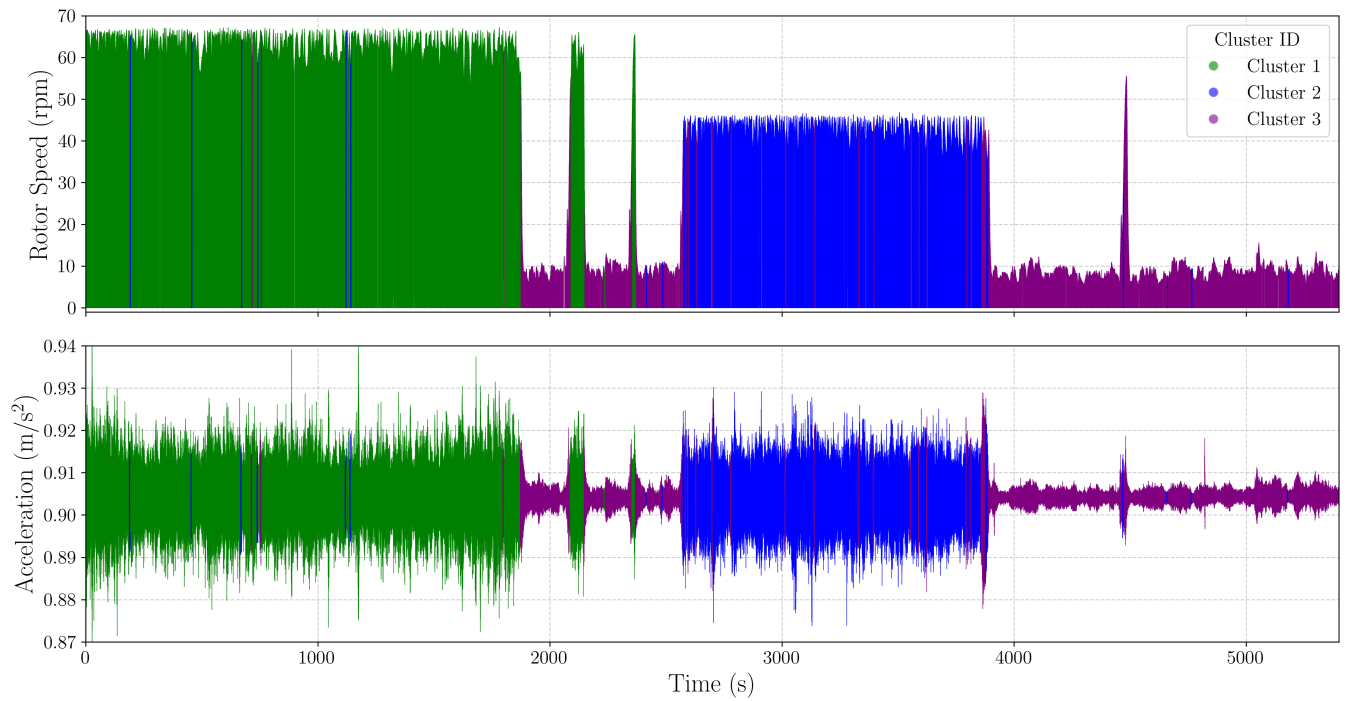


Fig. 7 Clustered rotor speed and acceleration, distinguishing damaged from undamaged states.

grouped into the same cluster, indicating shared dynamic characteristics between these regions. However, region 2, exhibiting a higher rotor speed than regions 1 and 3, is distinctly clustered, underscoring the influence of different operational dynamics on damage classification.

Conclusion

In conclusion, this study presents an integrated framework for damage identification in dynamic structural systems by combining cepstral and time-image-based features. The application of cepstral coefficients, originally from the field of speech recognition, enables effective tracking of frequency content changes, providing a robust foundation for identifying stationary and nonstationary DSFs. However, due to limitations in capturing fine-grained temporal dynamics, these coefficients were augmented with GAF transformations, which allow time series data to be visualized and analyzed as images. This dual-feature approach not only enhances damage detection accuracy but also allows for the precise localization of evolving structural damage.

The methodology was tested on the Aventa AV-7 wind turbine dataset, particularly focusing on the pitch drive failure scenario, which demonstrated significant degradation of the system's frequency response. This approach successfully classified distinct damage states through hierarchical clustering, validating the framework's capacity to capture complex, time-evolving structural behaviors. These results indicate that a more adaptive and comprehensive solution for SHM applications in dynamic environments is achievable by merging cepstral and image-based analysis.

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Chapter 7

Experimental Comparison of Feedback Control Methods for Active Vibration Damping with Piezoelectric Sensor/Actuator Pairs

Amirali Sadeqi, Jan Høgsberg, Niels L. Pedersen, and Ayech Benjeddou

Abstract This paper presents a comparison of the most common feedback control methods for collocated piezoelectric transducers, including integral force feedback (IFF), direct velocity feedback (DVF), proportional-derivative feedback (PDF), and positive position feedback (PPF). The methods are assessed experimentally with a laboratory setup comprising a set of piezoelectric transducers mounted symmetrically on a cantilever beam as collocated sensor/actuator pairs. The control laws are developed based on voltage-based expressions to be directly tuned and implemented in the experiments with low-cost real-time control boards. The active damping of the feedback control methods is quantified, and the comparison between them underlines the superiority of the presented PPF control in attainable damping. The findings from the experiments can be used to suggest rules of thumb for digital implementation and tuning of collocated feedback controls using piezoelectric sensor/actuator pairs.

Keywords Active vibration damping · feedback control · piezoelectric transducers · collocated sensor/actuator · IFF · DVF · PPF · PDF

Introduction

Passive vibration control systems are simple regarding integration and operation but provide small forces and may not achieve desired damping targets in all applications. For instance, viscoelastic materials are appropriate for high frequencies, the performance of shunt damping depends on the electromechanical coupling, and the dielectric elastomers require high voltages and provide a high impedance and low actuation [1–5]. On the other hand, active vibration control systems allow for damping optimization according to the system's dynamic properties and modal characteristics. One of the common approaches is collocated active control, which requires the sensor and actuator to be mounted at the same point in the structure. Such a pair can provide strong damping by relating their input/output through a control law such as positive position feedback (PPF), proportional-derivative feedback (PDF), integral force feedback (IFF), and direct velocity feedback (DVF). Regarding the two latter, the phase difference between the local force feedback due to the integral or derivative of the sensor measurements provides an unconditionally stable and decentralized control system. It has been demonstrated that the IFF, DVF, and PPF can be robust and significantly reduce vibration amplitudes if the control parameters are tuned and optimized [1, 2, 6–8]. Meanwhile, studies have reported the use of proportional-derivative control in nonlinear applications such as vibration suppression of rotating blades and micro-contact systems [9–11].

Piezoelectric materials have simultaneous actuation and sensing abilities and have been extensively used for active control and vibration damping. They are typically low-cost and lightweight with low power consumption, but require high voltages to generate significant forces and deformations in order to effectively contribute to control, especially for massive structures. They also have the flexibility to be customized in various sizes and shapes for energy harvesting, vibration

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absorbers, and biomechanical applications [4, 12]. It has been suggested that an active element consisting of a collocated piezoelectric actuator-sensor can behave like a muscle [1, 13]. So, as applied to a space truss, one can substitute some structural elements with the active elements to achieve a large enough extension to control the structure.

The IFF damping and performance were compared later with passive resistive and inductive shunting by adding the electromechanical coupling factor as the second control index, [14]. Apart from the drawbacks in the modal fraction index to quantity damping, IFF was reported to be more effective in achieving critical damping compared to classical DVF. In contrast, DVF is more efficient if the piezoelectric transducers have a small mechanical impedance and are softer and smaller than the structure [2]. The PD controller, in general, adds the derivative of the process error to the proportional control output. However, PD feedback (PDF) with a collocated sensor-actuator can be considered an extension of DVF. The simplicity, guaranteed stability, compatibility with physics-based models, and susceptibility to improvement have made both IFF and DVF appealing control laws for various applications. Yet, they have some limitations, such as low damping for high-order modes and practical instability (or loss of compliance at low frequency) due to high-pass filtering of the control signal [15].

On the other hand, PPF, compared to IFF and DVF, is more sensitive to the sampling frequency and tuning control parameters to remain stable. However, it can achieve higher damping in real-time campaigns and be easily implemented in analog circuits to suppress multiple vibration modes, given that they are known beforehand. One can apply optimization methods such as genetic algorithms to achieve multiple control gains [16]. Fractional-order exponents have also been applied as an alternative to tuning PPF control parameters with integer exponents. The studies have demonstrated the efficiency of fractional-order PPF for requiring less actuation voltage [17] and higher robustness in a broader band of effective phase compensation [18]. Yet, the effectiveness, efficacy, and suitability of IFF, DVF, and PPF, as well as their extensions and modified versions, may vary from one case to another.

The underlying project supporting the present work, seeks the possibility of a control system to suppress forearm tremors, which are rhythmic involuntary muscle contractions with noticeable shaking in the upper limb. Various pharmacological and non-pharmacological treatments have been introduced to overcome tremors and ease the patients' posture at rest or action [19]. Non-pharmacological and non-invasive aids, such as robotic assists and wearable orthoses, are commonly used for tremor suppression. Patients must carry such instruments all the time, which can restrict or discomfort the movement [20]. This work investigates the primary challenges of designing and implementing a semi-invasive active control system attached to the forearm bones. To that end, in the following, a cantilever beam with a piezoelectric sensor-actuator pair is experimentally studied. The voltage-based electro-mechanical equations of the classic control methods, IFF, DVF, PDF, and PPF, are derived, and their performance is assessed and compared.

Feedback Control by Sensor/Actuator Pairs

Figure 1 illustrates the schematic of the vibration control system to be investigated here. It comprises a cantilever beam with a collocated piezoelectric sensor/actuator pair. The beam is excited by the external force in the transverse y -direction, while the control by the piezoelectric pair introduces opposite bending moments at the two points of attachment (i and j).

Considering only the axial contribution of the piezoelectric transducers, the sensor deformation Δu_s and actuation force f_a can be related to the measured sensor voltage V_s and the prescribed actuation voltage V_a , respectively, through the electromechanical coupling parameter θ , and the blocked capacitance in constant strain C_p , by the two expressions [13, 21]:

$$\Delta u_s = -\frac{C_p}{\theta} V_s \quad (1)$$

$$f_a = \theta V_a \quad (2)$$

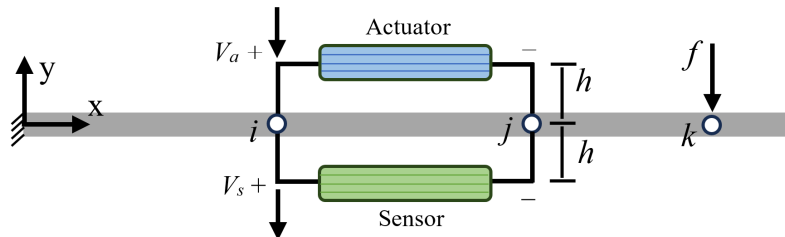


Fig. 1 Schematic of a cantilever beam with a collocated piezoelectric actuator/sensor pair. The offset of each piezoelectric transducer is h to the beam's neutral axis.

Here, θ is obtained as $\theta = d_{31}A_pE_p/t_p$ where A_p , t_p , E_p , and d_{31} are the piezoelectric transducer's area of cross-section, thickness, elastic modulus, and strain coefficient, respectively. Also, by ignoring the transducer's inertia and damping, the relation between the sensor's force and deformation becomes:

$$f_s = -k_s \Delta u_s \quad (3)$$

where k_s is the stiffness of the piezoelectric sensor, which is assumed to be identical to the stiffness of the corresponding piezoelectric actuator, i.e. $k_s = k_a$.

Integral force feedback (IFF)

When using a force sensor to measure the sensor force f_s , inspired by the classical integral force feedback method from [13], the desired actuation force f_a is prescribed by the present IFF control via the direct integral relation:

$$f_a = g' \int f_s dt \quad (4)$$

in which g' is the control gain that scales the required control force for the actuator relative to the measured force from the sensor. In the present application, the above force relation must be converted to a voltage-based control law. By substituting Δu_s from Eq. (1) into Eq. (3), followed by substitution of f_s from Eq. (3) and f_a from Eq. (2), the force-relation in Eq. (4) can be expressed in the desired voltage-form as:

$$V_a = g \int V_s dt \quad (5)$$

defining the normalized gain $g = g'k_sC_p/\theta^2$ for this voltage-based IFF, where the sensor voltage is integrated in real time to determine the actuator voltage upon scaling with the gain g . Hence, instead of a force sensor, one can directly use a piezoelectric voltage and Eq. (5), which provides maximum damping if g is tuned to be optimal in experimental testing. It is noteworthy that depending on the wiring conditions of the sensor and actuator, the current directions change, and g might have a positive or negative sign. According to Fig. 1, the sensor and actuator in this experiment have the same wiring direction, whereby $g > 0$.

Direct velocity feedback (DVF)

The desired actuation force in the DVF control sense can be related to the axial velocity $\Delta \dot{u}_s$ as:

$$f_a = -g' \Delta \dot{u}_s \quad (6)$$

where g' is the control gain when using a velocity sensor. Substituting f_a from Eq. (2) and the derivative of Δu_s from Eq. (1) into Eq. (6), and defining $g = g'C_p/\theta^2$, the required voltage to reach a desired actuator velocity based on DVF follows from:

$$V_a = g \dot{V}_s \quad (7)$$

Hence, when measuring the transducer's voltage rate $\dot{V}_s$ instead of the velocity $\Delta \dot{u}_s$ in (6), the control gain, $g > 0$, is the single parameter to be tuned optimally to achieve maximum damping.

Proportional-derivative feedback (PDF)

Including inertial and stiffness terms in Eq. (6), one can extend the actuation force into the general control law:

$$f_a = -g'_1 \Delta \ddot{u}_s - g'_2 \Delta \dot{u}_s - g'_3 \Delta u_s \quad (8)$$

Similar to Eq. (7), a voltage-based equation can be obtained by eliminating Δu_s and f_a from Eqs. (1) and (2), respectively. To simplify the otherwise general control equation (8), the apparent inertia term is cancelled by $g'_1 = 0$. Hereby, the desired proportional-derivative feedback (PDF) control is obtained as:

$$V_a = g_1 \dot{V}_s + g_2 V_s \quad (9)$$

where $g_1 > 0$ is the derivative gain and $g_2 > 0$ is the proportional gain to be tuned to the optimal values, resulting in maximum damping.

Positive position feedback (PPF)

The PPF control, in general, applies a second-order low-pass filter, in the following referred to as 2^{nd} -order PPF. Disregarding the actuator's inertia, the simpler 1^{st} -order PPF can also be efficiently applied to feedback control using a collocated actuator/sensor. The voltage-based control equations for both are developed in the following.

2^{nd} -order PPF

The governing equation of motion of the actuator can be expressed in terms of the displacement δ , damping ratio ζ_a and angular cut-off frequency ω_a [22, 23]:

$$\ddot{\delta} + 2\zeta_a\omega_a\dot{\delta} + \omega_a^2\delta = \omega_a^2\Delta u_s \quad (10)$$

where the actuation force and displacement can be scaled with a gain g' as $f_a = -g'\delta$.

Substituting Δu_s from Eq. (1) and $\delta = -f_a/g'$ into Eq. (10), also eliminating f_a using Eq. (2), the PPF control can be re-written into the desired voltage-based format as:

$$\ddot{V}_a + 2\zeta_a\omega_a\dot{V}_a + \omega_a^2V_a = g\omega_a^2V_s \quad (11)$$

where $g = g'C_p/\theta^2$ is the control gain. This PPF control achieves maximum damping, once the control parameters ω_a , ζ_a and g are optimally tuned. It is noteworthy that one may find Eq. (10) in the literature, e.g., [8, 17, 18] stated in terms of modal coordinates.

1^{st} -order PPF

Deriving optimal tuning relations for the above 2^{nd} -order PPF may follow various objectives, i.e., maximising damping or minimising vibrations by a resonant control. Thus, reducing the second-order format to a simpler first-order control may be convenient and ease in tuning. In Eq. (11), the apparent inertia term can be cancelled by letting $\omega_a \rightarrow \infty$. Hereby, the simpler first-order PPF control can be expressed as:

$$\dot{V}_a + \frac{1}{\tau}V_a = \frac{g}{\tau}V_s \quad (12)$$

when introducing the time-scale $\tau = 2\zeta_a/\omega_a$ and the non-dimensional control gain $g = g'C_p/\theta^2$. In this 1^{st} -order PPF, the apparent gain $g > 0$ and time-scale $\tau > 0$ must be tuned to maximum damping during the experiments.

It is noted that for the present PPF control, a vanishing control effort can be achieved by either $g = 0$ or $\tau \rightarrow \infty$. As argued in the following section, the desired complex pole properties in root-locus diagrams are obtained by choosing the time-scale τ as the variable control parameter, whereby the short-circuit limit is recovered when $\tau \rightarrow \infty$.

The implementation of equations introduced for IFF and DVF is straightforward, as the actuation voltage V_a can be obtained using the direct integral or derivative of the sensor's voltage measurements V_s . Meanwhile, obtaining the actuation voltage from the ordinary differential equations (11) or (12) is more time-consuming as it requires a numerical integration scheme such as the Euler method [24]. However, as demonstrated later in the experimental analysis, the PPF control outperforms the direct methods, due to its ability to apply a negative stiffness component [25].

Optimal Tuning and Maximum Damping

For the piezoelectric feedback control strategies outlined in the previous section, the theoretical tuning of the control parameters relies on maximizing the modal damping ratio for the targeted vibration mode, which, in the present case, is the fundamental first bending mode of the cantilever.

For all feedback control strategies, the governing closed-loop poles will follow semi-circular trajectories in the complex frequency plane for an increasing damper parameter. An example is illustrated in Fig. 4 for the 1^{st} -order PPF control in Eq. (12). The semi-circular path is for this 1^{st} -order PPF obtained by varying the time-scale τ for a fixed control coefficient g , with $\omega = \omega_0$ representing the natural frequency of the closed-loop system when $\tau \ll 1/\omega_1$, while the opposite limit $\omega = \omega_\infty$ is reached when $\tau \gg 1/\omega_1$. It follows directly from Eq. (12) that $V_a = 0$ when $\tau \rightarrow \infty$, whereby ω_∞ corresponds to the natural frequency of the cantilever when the actuator piezo is short-circuited.

It follows directly from the construction of the semi-circular root-locus trajectory in Fig. 4, that the damping ratio associated with the top of the half-circle is approximately

$$\zeta_{max} = \frac{\text{Im}[\omega]}{|\omega|} \simeq \frac{\frac{1}{2}(\omega_\infty - \omega_0)}{\frac{1}{2}(\omega_\infty + \omega_0)} = \frac{\omega_\infty - \omega_0}{\omega_\infty + \omega_0} \quad (13)$$

where $\omega_{opt} = \frac{1}{2}(\omega_\infty + \omega_0) \simeq |\omega|$ approximates the argument of the complex natural frequency.

The present section presents a so-called Modal Control Coupling Factor (MCCF) and expressions for the optimal tuning of the simplest control methods introduced in the previous section. The concept behind the MCCF and the tuning methods has been presented recently in [26], while further details will be derived in [27].

DVF and IFF

For the DVF control in Eq. (7), when $g = 0$, the actuation voltage $V_a = 0$ and the control force $f_a = \theta V_a$ vanishes as well, whereby the short-circuit condition is retained. In the opposite limit $g \rightarrow \infty$, the closed-loop system – in theory – becomes undamped at a larger natural frequency $\omega_\infty > \omega_0$. In practice, this limit might be difficult to reach, as the actuator voltage V_a in Eq. (7) scales directly with g . Instead, the limiting frequency ω_∞ can, for the DVF control, be determined numerically, assuming that the piezoelectric actuator link becomes fully rigid. The tuning of the feedback control depends on the properties of the piezoelectric actuator, via the apparent electric stiffness θ^2/C_p , the modal properties of the structure and a modal control coupling factor (MCCF), defined as

$$K_1^2 = \frac{\omega_\infty^2 - \omega_0^2}{\omega_0^2} \quad (14)$$

where subscript 1 denotes the fundamental mode as the control target. The optimal gain scales directly with the MCCF in the following way [27],

$$g_{opt} = K_1^2 \frac{C_p k_1}{\theta^2 \omega_1} \quad (15)$$

where k_1 is the modal stiffness for the mode shape normalized to unit deflection across the actuator terminals.

For the IFF control, the only change relative to DVF is the phase delay of the control, whereby the optimal gain g in Eq. (5) will scale inversely with the natural frequency. Thus, for IFF, the optimal control will be

$$g_{opt} = K_1^2 \frac{C_p k_1 \omega_1}{\theta^2} \quad (16)$$

It is worth noting that the optimal gain g for both DVF and IFF scale with the MCCF K_1^2 and with the modal stiffness k_1 , while it is inversely proportional to the electric stiffness θ^2/C_p of the piezoelectric transducer. For the DVF control, the natural frequency ω_1 in the denominator of (15) accounts for the time-differentiation in (7). Thus, for the IFF control gain in (16), ω_1 must appear oppositely in the numerator because of the integration performed in the control equation (5). The present results show that while the attainable damping ratio in Eq. (13) is evaluated directly by the limiting natural frequencies, the optimal tuning expressions require knowledge of both piezoelectric and modal system parameters.

1st-order PPF

For the 1st-order PPF control in Eq. (12), either the timescale τ or the gain g can be chosen as the tuning variable. However, to obtain the desired semi-circular root locus trajectory in Fig. 4, the timescale τ must be used as the control variable, while g is chosen within the stability limit associated with PPF control.

As mentioned previously, the fundamental short-circuit limit is retained by $\omega = \omega_\infty$ when $\tau \rightarrow \infty$. In the opposite limit ($\tau \rightarrow 0$) it follows from Eq. (12) that $V_a \rightarrow gV_s$, is experienced by the cantilever beam as a negative stiffness that reduces the natural frequency, i.e. $\omega_0 < \omega_\infty$.

For the 1st-order PPF, the MCCF is formally defined as for the DVF in Eq. (14). However, because ω_∞ corresponds to the short-circuit natural frequency of the beam, the MCCF is conveniently defined relative to this short-circuit frequency as:

$$K_1^2 = \frac{\omega_\infty^2 - \omega_0^2}{\omega_\infty^2} \quad (17)$$

The difference between the two definitions in Eq. (14) and Eq. (17) is small for typical levels of control authority. The optimal tuning of the control timescale can then be approximated as (see [27]):

$$\tau_{opt}\omega_1 = \frac{g}{K_1^2} \frac{\theta^2}{C_p k_1} \quad (18)$$

For very large timescales τ , the 1st-order PPF must recover the IFF in Eq. (5) with g in IFF being equal to g/τ for the 1st-order PPF. This equivalence is verified by comparison of the two tuning formulae in Eq. (16) and Eq. (18), respectively.

Tuning in practice

It is seen from the three tuning expressions in Eq. (15) for DVF, in Eq. (16) for IFF and in Eq. (18) for the 1st-order PPF control, that the tuning scales with the MCCF K_1^2 , while it also depends on the modal properties, in the present case the modal stiffness k_1 and natural frequency ω_1 , and the electric stiffening parameter θ^2/C_p . Thus, the tuning will most likely require a dedicated numerical model (twin), from which these modal parameters can be derived. As this is beyond the scope of the present analysis, further studies are needed on how to conduct a robust and accurate optimal tuning of collocated feedback control methods for piezoelectric vibration damping.

The above expressions for optimal tuning clearly show that the MCCF (K_1^2) is a crucial governing parameter, which may be determined experimentally by either Eq. (14) or Eq. (17), if the two limiting frequencies ω_0 and ω_∞ can be determined. For IFF in Eq. (5) and DVF in Eq. (7), the issue is associated with the determination of ω_∞ , which requires the gain to be very large. For the filtered control in 1st-order PPF, large control efforts are instead associated with the timescale $\tau \rightarrow 0$ (which gives ω_0).

In the following experimental campaign, the optimal control parameters for the various control methods are determined by ad-hoc parameter variation for a sinusoidal loading of the cantilever. To determine the associated optimal natural frequency, impact tests are then performed to derive ω_{opt} . As illustrated in Fig. 4, the optimal frequency can be assumed to represent the mean value of the two limiting frequencies: $\omega_{opt} = \frac{1}{2}(\omega_0 + \omega_\infty)$. In the following, either ω_0 (for IFF and DVF) or ω_∞ (for 1st-order PPF) can be determined as the short-circuit frequency, while the other limiting frequencies can be eliminated in the expressions for K_1^2 by the estimated ω_{opt} . The determined MCCF K_1^2 can then be used to determine the damping ratio in (13), which may be re-formulated as

$$\zeta_{max} \simeq \frac{\omega_\infty - \omega_0}{\omega_\infty + \omega_0} = \frac{\omega_\infty^2 - \omega_0^2}{(\omega_\infty + \omega_0)^2} \simeq \frac{1}{4} K_1^2 \quad (19)$$

when introducing $\omega_\infty + \omega_0 \simeq 2\omega_0$ or $2\omega_\infty$. This estimated optimal damping ratio can then be compared with the decay rates obtained during impact tests. The latter expression for the damping ratio in (19) is identical to that derived in [28] for piezoelectric vibration damping using a pure resistive shunt.

For the control methods not explicitly tuned in the present section, the calibration procedures are too extensive for the present analysis or have several (sometimes detrimental) objectives. Thus, these are not addressed directly, and only tuned manually by the ad-hoc calibration applied in the following experimental section.

Experimental Setup

The objective of the present experimental analysis is to examine the feedback control methods introduced in the previous section through the experimental vibration of a cantilever beam. Figure 2 exhibits a photo of the setup and instruments from the top and front views. It consisted of an aluminium cantilever beam ($300 \times 30 \times 1.6$ mm) with two collocated piezoelectric pairs, i.e., four patches ($40 \times 13 \times 0.5$ mm). The four piezoelectric patches are identical and made of Lead Zirconate Titanate (PZT) 5A4E piezoceramic material (Piezo Systems, Inc) with the strain piezoelectric coupling constant, $d_{31} = -190 \times 10^{-12}$ m/V and other characteristics presented in Table 1.

Table 1 Characteristics of the applied transducers, D220-A4-303XE, 5A4E PZT, Piezo Systems, Inc.

Weight (gr)	Stiffness (N/m)	Capacitance (nF)	Rated voltage (V)	Resonant frequency (kHz)	Free deflection (μ m)	Blocked force (N)
2.7	7×10^6	52	± 90	24.5	± 3.6	± 26

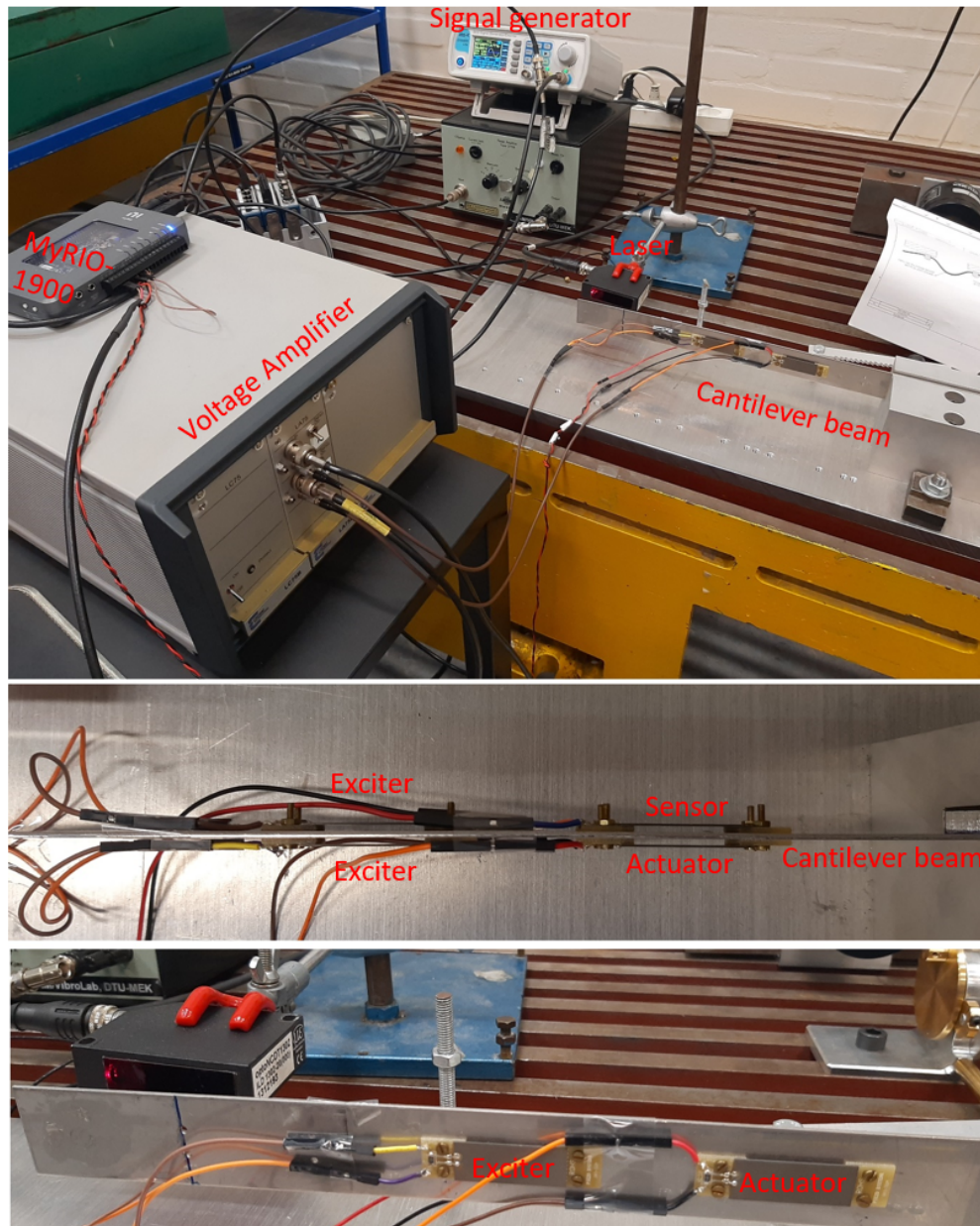


Fig. 2 Experimental test setup of a cantilever beam with piezoelectric patches and data logging system.

The patches were connected to the beam with a gap distance of $h = 2$ mm. The first pair of patches close to the free end (125 mm from the tip) was used to excite the beam. The next two patches close to the support (50 mm from the support) were used as a sensor/actuator pair. The data logging system consisted of a portable device, National Instruments MyRIO 1900, to process the input/output (sensor/actuator) signals in Lab VIEW installed on a computer, Intel Core i7-3.8GHz-RAM 16GB. A laser sensor (Micro-Epsilon, optoNCDT-1302) also measured the displacement at the free end of the beam only to monitor the performance of the control, i.e. its measurements were not used in the control loop.

Two types of experiments were performed based on impact and resonance excitations. First, the system's frequencies and damping ratios are identified when the (PPF) control is off and on and the beam is subjected to impacts. In that case the exciter piezoelectric pair was short-circuited and they had only elastic contributions. In order to sustain the vibration and increase the signal-to-noise ratio when the control is turned on and causing significant damping, multiple impacts via a small hammer were applied around the free end of the beam.

In the second experiment, the performance of the control methods is assessed when the structure is excited by a sine wave with the first resonance frequency of 13.8 Hz. The sine wave was generated via a signal generator and applied to the first

piezoelectric pair, see Figure 2, with the ± 90 V range realized through an intermediate voltage amplifier. Next, the sensor measures the vibration-induced voltage V_s and sends it as a feedback input to MyRIO 1900 and the control environment developed in Lab VIEW. Applying IFF, DVF, PDF, or PPF to the input voltage signal yields the prescribed output voltage signal to be sent from MyRIO1900 to the voltage amplifier and then to the actuator. The tests were performed in equal conditions and a time step of 10^{-4} s for all the control methods in 60 s, while the control was off in the first 30 s.

Modal identification via impact test

The displacement of the beam subjected to multiple impacts was measured via the laser sensor at the free end. Figure 3 shows the displacement time history and power spectral density (PSD) when no control is applied. Employing the displacement time history data in a stochastic subspace identification (SSI) algorithm yields the frequencies 13.80 Hz and 83.79 Hz and the damping ratios 0.61% and 1.06% for the first two vibration modes, presented in Table 2.

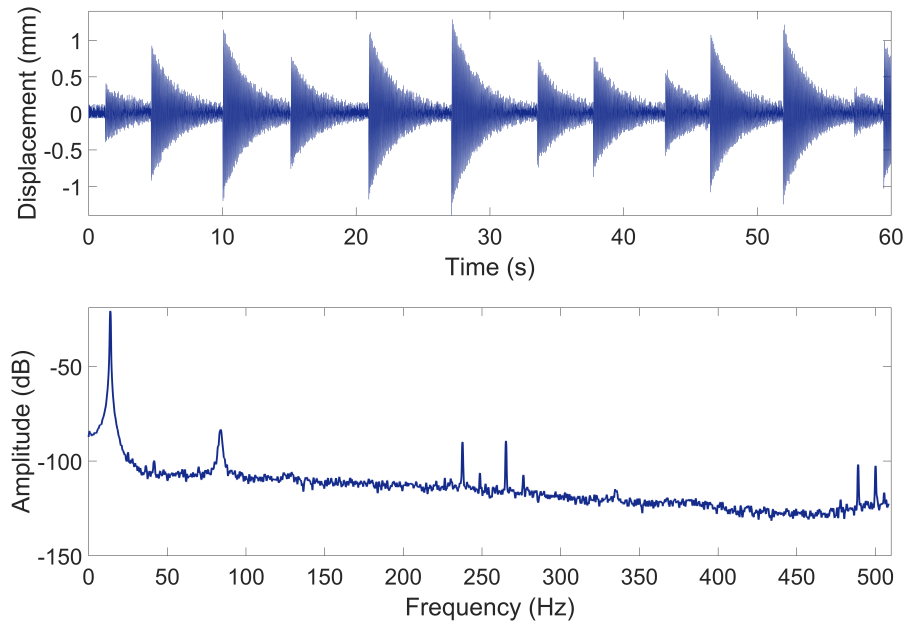


Fig. 3 Displacement time history (top) and PSD (bottom) when the control is off and the beam is subjected to multiple impacts

When applying the control, for example the 1st-order PPF, the poles of the closed-loop system for a given gain g vary with τ , as shown in Figure 4. Hence, for the given g , there is an optimal τ that results in the maximum imaginary part of the eigenvalue.

So, the optimal damping ratio, ζ_{opt} , and the MCCF, K_1^2 , can be obtained from Eqs. (13) and (17), respectively. In practice, ω_{opt} is achieved by searching for the optimal τ in a finite range. Also, ω_∞ can be achieved by setting τ to a large value. Meanwhile, ω_0 is difficult to reach in practice as setting τ to zero can prescribe infinite voltage to the actuator beyond its operational range. However, from the circular trajectory in Figure 4 one can deduce $\omega_\infty - \omega_{opt} = \omega_{opt} - \omega_0$, and rewrite

Table 2 System's frequencies and damping ratios for the first two modes with and without PPF control

	No Control	$\tau = 100(\text{ s}) \approx \text{Infinite}$	$\tau = 10^{-3}(\text{ s}) \approx \text{Optimal}$	$\tau = 10^{-4}(\text{ s})$
$\omega_1/2\pi(\text{ Hz})$	13.80	13.78	13.22	13.25
$\zeta_1(\%)$	0.61	0.58	3.16	0.81
$\omega_2/2\pi(\text{ Hz})$	83.79	83.77	83.40	83.40
$\zeta_2(\%)$	1.06	1.75	0.44	2.01

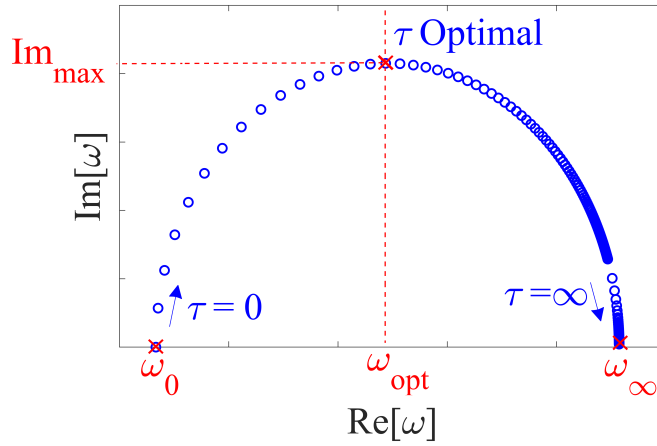


Fig. 4 Real part vs imaginary part of the closed-loop state-space system's eigenvalues (ω) for various τ

Eq. (13) as

$$\zeta_{opt} = \frac{\omega_{\infty}}{\omega_{opt}} - 1 \quad (20)$$

In the second experiment, the system's frequencies and damping ratios are identified in three tests when the beam was subjected to multiple impacts, and the 1st-order PPF was applied with the optimal gain of $g = 3.3$: first, $\tau = 100$ s was chosen as infinity, second, $\tau = 10^{-3}$ s which is found to be the optimal value for damping the first mode, and third, $\tau = 10^{-4}$ s which is the smallest value greater than zero leading to a high actuation voltage. Figure 5 displays the time history of displacement of the free end of the beam in each test and the prescribed actuation voltage. It is seen that the magnitude of the voltage changes reversely with τ so that the displacement diagram for $\tau = 100$ is similar to that in Figure 3.

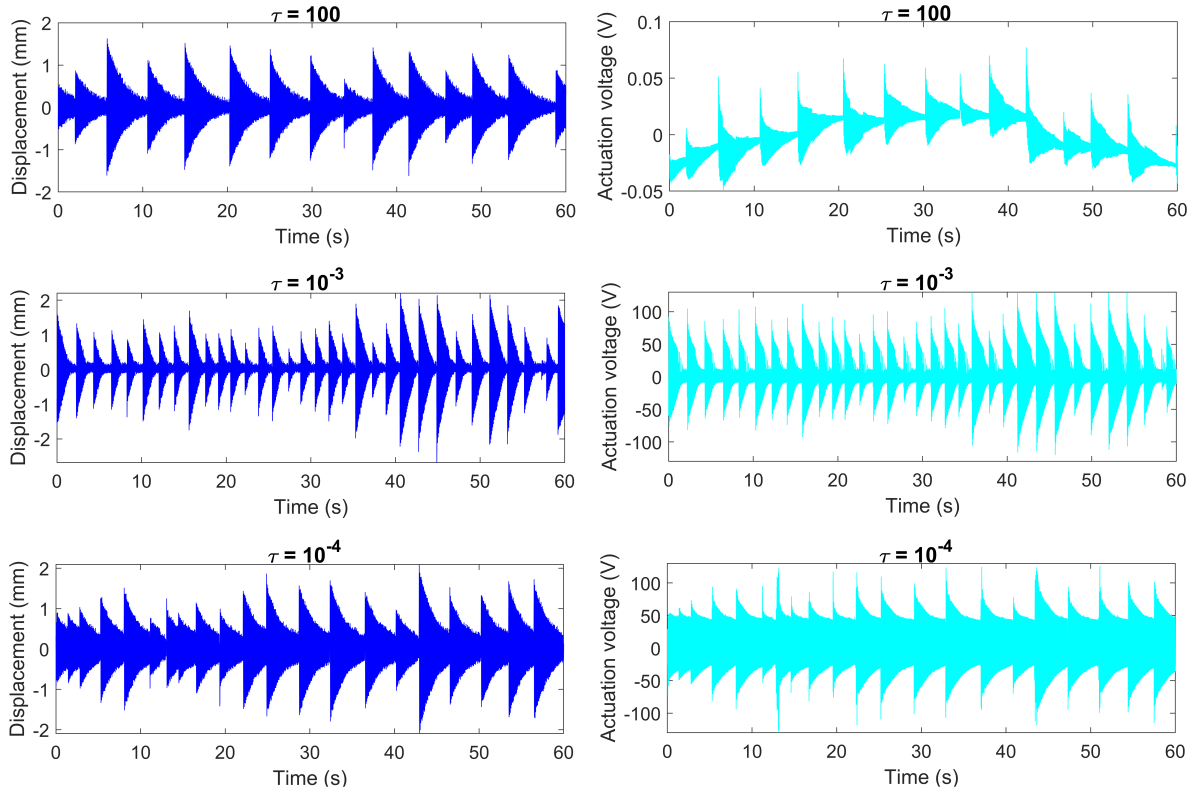


Fig. 5 Displacement and actuation voltage when 1st-order PPF applied and beam subjected to multiple impacts

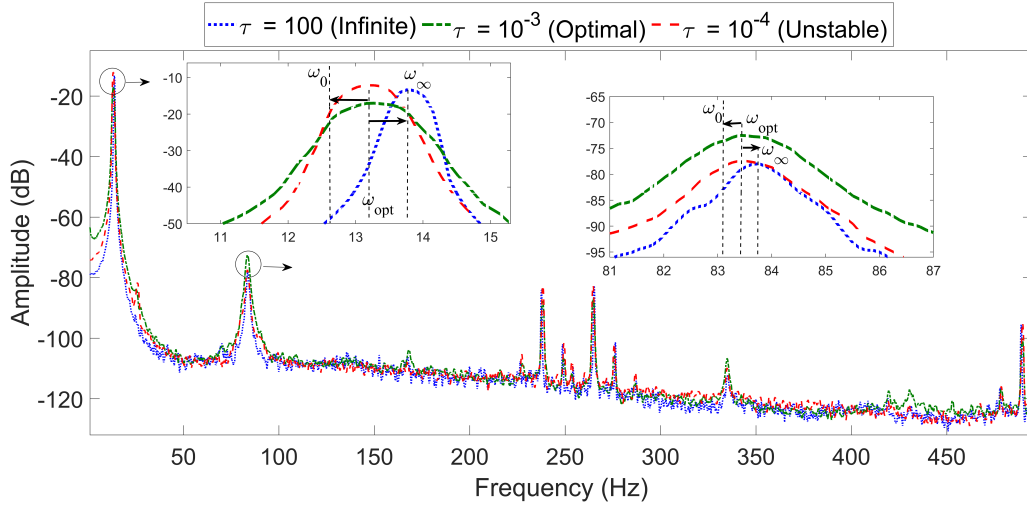


Fig. 6 PSD of the displacement when 1st-order PPF applied and beam subjected to multiple impacts

Figure 6 illustrates the PSD of the displacements. The first two peaks are plotted in the zoomed-insets, and the identified parameters via SSI are tabulated in Table 2. Notably, $\tau = 10^{-4}$ s was the smallest value prescribing actuation voltage in the allowable range. Therefore, there is no actual PSD curve in Figure 6 to represent ω_0 as it corresponds to $\tau = 0$, leading to infinite voltage. However, regarding the circular trajectory in Figure 4, ω_0 is hypothetically demonstrated in Figure 6 to be equidistance with ω_∞ with respect to ω_{opt} .

According to Table 2, the damping ratios (and frequencies) for the case with $\tau = 10^{-3}$ s and 10^{-4} s are almost the same. However, the latter prescribes a higher voltage to the actuator, causing instability and higher amplitude than the optimal case. Furthermore, the optimal damping ratio for the first mode obtained from Eq. (20) is 4.71%, which is about seven times greater than the case with no control (i.e., 0.61%) but different from 3.16% obtained via the identification. The reason is the time-variant nature of the damped response with PPF control in free vibration due to impact. Therefore, the stability of the identified parameters highly depends on the selected state-space model order in SSI. On the other hand, for the second mode, the damping decreases for the optimal τ . Hence, $\tau = 10^{-3}$ s is only valid for the first mode, and every mode requires its optimal gain to achieve maximum damping.

Comparison via resonance test

The performance of the introduced control methods for damping the first resonant mode is assessed herein. The optimal gains presented in Table 3 were applied to Eq. (7) for DVF, Eq. (9) for PDF, Eq. (11) for 2nd-order PPF, and Eq. (12) for 1st-order PPF. The control gain g in the latter was locked to unity since it was found to be less influencing the damping than ω_a and ζ_a . Figures 7 - a to d, show the recorded signals when the IFF control was turned off for 30 s and then was turned on for the next 30 s with an optimal gain. The zoomed-inset figures show the first two seconds after the control activation. Figure 7 - a displays the time history of the applied excitation load (sine wave) via the piezoelectric exciter pair with the resonance frequency of 13.8 Hz. The zero flat lines in Figures 7 - c and d represent the sensor and actuator signals, respectively, when the control is off for 30 s. After activating the control and applying the integral to the sensor's voltage, as introduced in Eq. (5), the optimal gain can be searched for maximum amplitude reduction to prescribe the required actuation voltage. As seen in Figures 7 - c and d, both diagrams drop and become stable in less than 2 s.

Table 3 Optimal gains employed in different control methods

IFF	$g = 2.1 \times 10^3$ (1/s)
DVF	$g = 2.2 \times 10^{-4}$ (s)
PDF	$g_1 = 1.8 \times 10^{-4}$ (s), $g_2 = 2$
PPF-1 st Order	$g = 3.3$, $\tau = 1.5 \times 10^{-3}$ (s)
PPF-2 nd Order	$g = 1$, $\omega_a = 86.5$ (rad/s), $\zeta_a = 0.045$

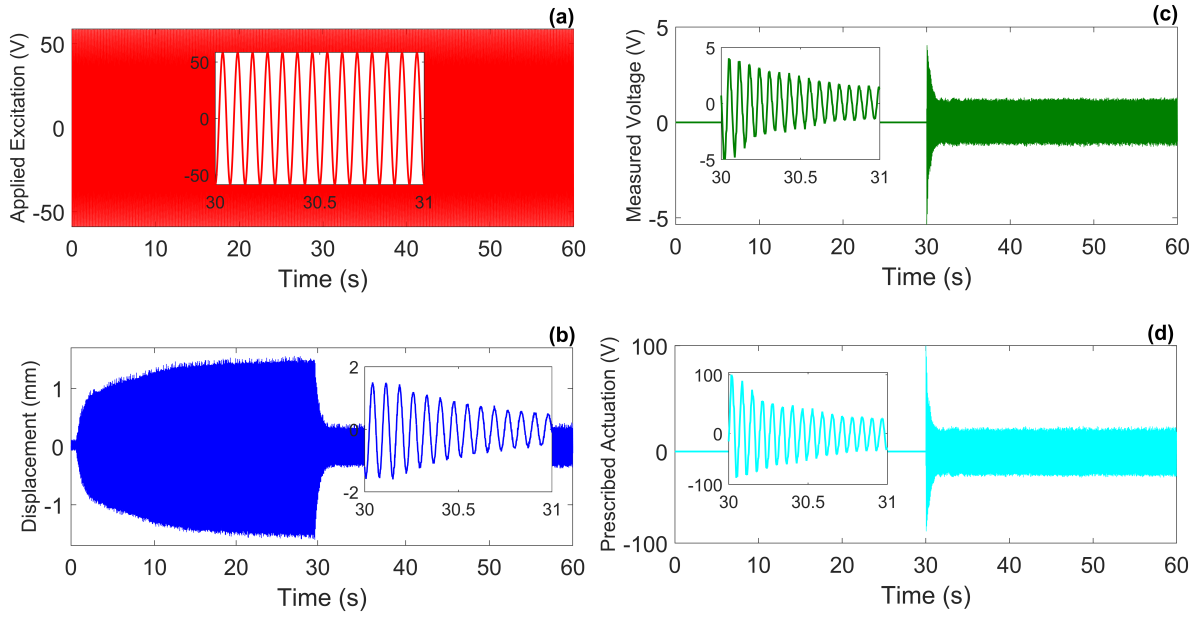


Fig. 7 Time histories of a) applied excitation, b) displacement measured via laser, c) deformation-induced voltage measured via the piezoelectric sensor, d) voltage prescribed to the actuator

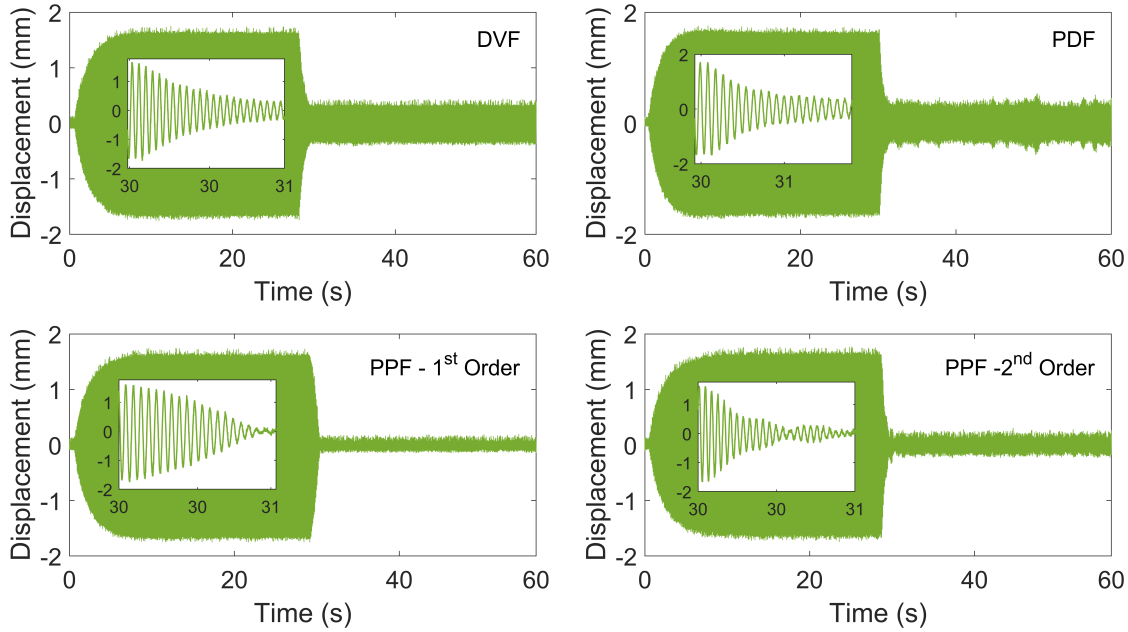


Fig. 8 Displacement time histories measured via laser sensor at the free end of the beam for various controls

A similar trend is observed in Figure 7 - b, representing the displacement of the beam's free end measured via the laser sensor. The vibration amplitude is increasing while the control is off for 30s. Then, it drops after applying IFF with the optimal gain $g = 2.1 \times 10^3 \text{s}^{-1}$, obtained manually beforehand. Likewise, the next diagrams in Figure 8 exhibit the displacement time histories when applying other controls with optimal parameters. The PPF control demonstrates an overall superior performance compared to IFF and DVF in terms of amplitude reduction. The reduction is quantified and compared through the envelope of displacement settling time presented in Figure 9. Each diagram connects the average of the upper and lower peaks in the displacement time histories in 2.5 s after activating the control (30-32.5 s) but displayed herein from time 0. The 2nd-order PPF and PDF, almost reach 50% amplitude equally fast with a settling time of 0.25 s, slightly faster than DVF and IFF, and about three times faster than 1st-order PPF. However, all controls reach 0.15% of their max amplitude

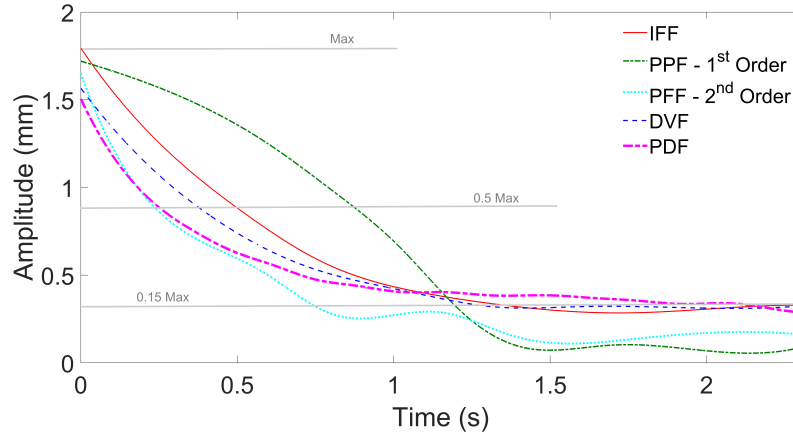


Fig. 9 Envelope of displacement time histories during settling time

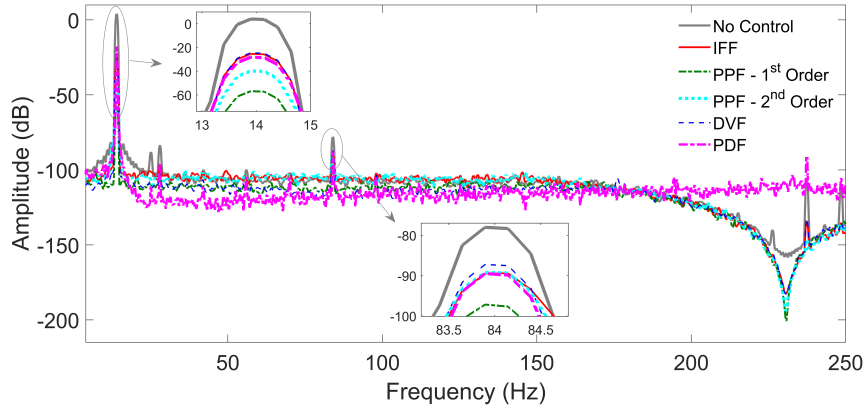


Fig. 10 PSD of displacement and zoomed first two resonant peaks with and without control

almost simultaneously after 1.2 s. The 1st-order PPF provides more significant damping and a lower but more unstable plateau than IFF and DVF, implying an impulse trend of the reduced vibration.

The effectiveness of the controls on the resonance amplitude is also studied in the frequency domain. Figure 10 depicts the PSD of the displacement time histories previously shown in Figure 8. The zoomed insets display the peaks at the first two resonance modes. The grey solid line curve corresponds to the 30 s of displacement when the control is off, whereas the other curves correspond with the next 30 s.

The amplitude of the first mode drops from 4 dB to -24.5 dB with IFF, -25 dB with DVF, and -28 dB with PDF. Meanwhile, the 1st-order and 2nd-order PPF lead to further reduction with -57 dB and -40 dB, respectively. Compared to DVF, the improvement due to having a proportional term in PDF is not remarkable meanwhile subharmonics appear and increase in the latter.

Notably, the actuation voltage for IFF and DVF was obtained using a direct integral and derivative of the vibration-induced voltage measured by the piezoelectric sensor. In contrast, the forward Euler method used for PPF requires further computations than IFF and DVF. In particular, the 2nd-order PPF again requires further computations than the 1st-order PPF, causing a higher plateau in 2nd-order PPF due to data accumulation in the buffer of the processor, whereas it is more efficient at the beginning. Hence, achieving PPF's maximum damping in real-time depends on data logging and signal processing speeds. MyRIO 1900 facilitates input/output processing at a high speed, yet using the FPGA (Field-programmable gate array) feature, one can make the data logging and control process independent from other computations running in parallel in the computer. Despite the excellence of PPF in providing damping in real-time and high-speed processes, the optimal parameters are more complex to find as the control stability can easily be disturbed. Especially when the gain g is not tuned, the control parameters ω_a and ζ_a might be tuned with unrealistic values, such as tuning ζ_a with large values.

Conclusion

This research developed the essential voltage-based relationships for some classic feedback control methods, namely, IFF, DVF, PDF, and PPF, and experimentally assessed their performance. The optimal control gains in IFF, PDF, and DVF were obtained by directly tuning the prescribed actuation voltage. In contrast, the system's resonance frequency was employed in second-order PPF. The superiority of PPF in generating higher damping in the given test setup was found to be considerable compared to IFF, PDF, and DVF, but it is more sensitive to the control parameters and test conditions. Furthermore, the damping generated by PPF depends on data logging and signal processing speeds. It was demonstrated that the second-order PPF leads to a shorter settling time, thus faster vibration reduction. On the other hand, the proportional term in the PDF does not significantly improve its performance compared to DVF. Further investigations regarding the damping mechanism of the discussed control methods in numerical simulations will be conducted in future research.

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Chapter 8

Initial Validation of a Novel Output-to-Output Frequency Response Function Mapping Method for Impact Localization in Dispersive Media

Andrew T. Gothard and Steven R. Anton

Abstract In this study, a novel impact localization mapping algorithm is proposed to solve the source localization problem in solid media, namely the output-to-output frequency response function (OO-FRF) mapping method. Previous work has shown that in media where mechanical waves travel with a constant velocity the time difference of arrival (TDOA) method can be used to solve the source localization problem. In solid media, the TDOA method is an insufficient solution due to mechanical waves being distorted by dispersion, wave reflections, and frequency-dependent damping. The ΔT mapping method was developed as a data driven Modified-TDOA (M-TDOA) method, which is able to account for path dependent velocities in complex structures by creating TDOA mapping functions across the surface of a structure; however, the ΔT mapping method does not account for the dispersive effects when different frequency ranges are excited by impacts. In order to improve upon the previous work, this work proposes the OO-FRF mapping method. The OO-FRF mapping method is a frequency response function (FRF) localization method and does not rely on TDOA values but instead uses the relative frequency response between pairs of sensors to localize impacts. By using OO-FRFs, this method naturally captures frequency dependent relationships across the structure without needing to know the input force on the system. The OO-FRF method is validated using a numerical study with a 1D finite element beam and compared against the ΔT mapping method. The results show that the localization error of the ΔT mapping method and OO-FRF mapping method are 0.0957 *m* and 0.0029 *m*, respectively. Based on these results, the OO-FRF mapping method shows clear improvement over the ΔT mapping method, and the results demonstrate the potential for these methods to be used to localize impacts in complex dispersive structures.

Keywords Impact localization · Output-to-output FRF · Mapping function · Dispersive media

Introduction

The general source localization problem can be defined as the determination of the location of an event using the characteristics of the signals emitted by that event. The source localization problem has been widely studied for wireless communication systems where a mobile station emits a signal, such as radio waves, and is located using access points [1, 2]. Source localization in solid media is also an important area of research that has been studied for a variety of structural health monitoring (SHM) and damage localization applications in composite and metallic structures, such as planes, naval vessels, civil infrastructure, and others [3–6]. Two of the major subsets of the source localization problem in solid media are impact localization and acoustic emission (AE) source localization. The AE emission source localization problem is typically focused on the localization of cracks or other damage within a structure. Impact localization in solid media has been largely studied to identify the location of potentially damaging impacts; however, more recently with the development of vibration based smart building technology, impact localization has also been used to monitor human building interactions (HBIs) for occupant tracking, gait analysis, and fall detection to improve occupant safety and comfort [7–9]. Typically, the source localization problem is solved using the time difference of arrival method (TDOA), which estimates the source location based on the speed of the wave and the difference in the arrival times of the wave at multiple sensors. The TDOA method works well for

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wireless communication systems due to the fact that the emitted signals travel at a constant speed and receive little interference from the environment. However, for source localization in solid media, the emitted mechanical waves are affected by dispersion, frequency dependent damping, and reflections from the boundaries of the structure. Due to these complexities, the velocity of the mechanical waves is not constant throughout the media, which can cause large localization errors when using the TDOA method even for simple structures.

Previous work has been done to improve source localization in solid media using modified TDOA (M-TDOA) methods. While many types of algorithms exist, the majority of the M-TDOA methods fall into the following major categories: velocity modeling methods [10–12], dispersion compensation methods [13, 14], machine learning methods [15, 16], and ΔT mapping methods [17–19]. Velocity modeling methods typically relax the assumption of a single propagation velocity by fitting a velocity model to a set of training data, which is then used by a TDOA algorithm to localize an impact. Another type of velocity modeling method uses a TDOA optimization scheme to predict the most likely velocity and impact location from a set of possible velocities. By relaxing the constant velocity assumption, the velocity modeling methods reduce localization errors due to dispersion. The dispersion compensation methods typically use a continuous wavelet transform (CWT) to perform a time-frequency analysis on the signals received at each sensor to remove the effects of dispersion. Since wavelets act as bandpass filters with a dominant center frequency, the CWT can be used to extract particular center frequencies from a signal. The center frequencies are not perfectly isolated due to the tradeoff between temporal and frequency resolutions inherent to the CWT; however, the effect of dispersion on the mechanical waves is reduced. A TDOA algorithm can then be used to localize an impact based on the arrival times of the isolated center frequency. While the velocity modeling and CWT dispersion compensation methods reduce error due to dispersion and are an improvement over regular TDOA methods for simple plate like structures, these methods do not account for the effects of path dependent velocities that are often present in complex structures. Other groups have proposed improved TDOA methods, which use machine learning models. The machine learning methods often use neural networks (NNs) or convolutional neural networks (CNNs) and a set of training impacts in order to learn complex patterns in the data and then predict the location of test impacts. While machine learning methods have been successful in localizing impacts in complex structures, they have the disadvantage of being black box models, which means it is difficult to determine if the model is learning physically relevant features that will be applicable to new impacts. Additionally, machine learning algorithms often require large amounts of training data that sufficiently covers the type of impacts that could be seen in the structure, which is difficult to define. Finally, ΔT mapping methods are another type of M-TDOA method that have been developed to handle impact localization in complex structures. ΔT mapping methods use a data driven approach where a grid of calibration impacts is taken across the surface of a structure and TDOA values are calculated for each grid location and for each sensor pair. TDOA maps (called ΔT maps) are generated by interpolating between impact locations. Impacts are localized by minimizing the difference between the TDOA value of the test impact and the ΔT maps. By using mapping functions for a structure, the ΔT mapping methods compensate for path dependent velocities for a particular type of impact and demonstrate the usefulness of mapping functions for localization in complex structures. While the ΔT mapping method allows for calibration across complex structures, it makes the assumption that the ΔT maps are independent of the type of impact on a structure. In reality, different impacts can excite various frequency ranges in a structure and the dispersive effects of the wave propagation can cause variabilities and inaccuracies in the ΔT maps leading to localization errors.

In this work, the output-to-output frequency response function (OO-FRF) mapping method is proposed as a dispersion compensated mapping method. The OO-FRF mapping method is a modified version of the spatially sparse FRF impact localization method developed by Alajlouni et al., which uses input-to-output calibration FRFs to localize impacts [20]. The OO-FRF mapping method does not rely on TDOA calculations but instead calibrates a surface using OO-FRF maps. The calibration OO-FRFs take the ratio between the frequency domain representations of the outputs of two sensors at a particular location. By mapping the ratio between frequency domain representations across a structure, the proposed method naturally accounts for frequency dependent relationships and path dependent velocities in the structure. Another benefit of the OO-FRFs is that the input force is not required to generate the calibration OO-FRF maps. Since the input force is not needed, any object that excites the desired frequency range in the structure can be used for calibration. This may be useful for calibrating vibration based smart building systems for HBIs using natural impacts like footsteps. The OO-FRF mapping method was inspired by the work done by Davis et al. who showed that OO-FRFs could be used to localize impacts, but the proposed method improves upon the work done by this group by incorporating interpolation and a minimization scheme that allows for impact prediction between calibration locations [21]. The OO-FRF mapping method is validated through a numerical study of a 1D homogenous beam using a finite element model (FEM) and compared to the ΔT mapping method.

Methods

Output-to-Output FRF Mapping Method:

The OO-FRF mapping method is a data driven method developed to localize impacts in complex mechanical structures using a set of dynamic sensors and a calibration dataset. The proposed algorithm has two main stages: a calibration stage and a localization stage.

In the calibration stage, impacts are recorded across a structure and OO-FRFs are formed for each impact location. The OO-FRFs are formed by taking the ratio between the frequency domain representations of the outputs of pairs of sensors. As an example, for a pair of sensors s_1 and s_2 at calibration impact location k , the OO-FRF is formed using the following expression,

$$\vec{R}_{s_1 s_2, k}(f) = \frac{\vec{H}_{s_1, k}(f)}{\vec{H}_{s_2, k}(f)} = \frac{\vec{A}_{s_1, k}(f)}{\vec{F}_k(f)} \frac{\vec{F}_k(f)}{\vec{A}_{s_2, k}(f)} = \frac{\vec{A}_{s_1, k}(f)}{\vec{A}_{s_2, k}(f)} \quad (1)$$

where $\vec{R}_{s_1 s_2, k}(f)$ is the OO-FRF between sensors s_i and s_j , respectively, $\vec{H}_{s_1, k}(f)$ and $\vec{H}_{s_2, k}(f)$ are the acceleration FRFs between s_1 and s_2 and point k , respectively, $\vec{A}_{s_1, k}(f)$ and $\vec{A}_{s_2, k}(f)$ are the FFT of the acceleration response at s_1 and s_2 , respectively, and $\vec{F}_k(f)$ is the FFT of the input force at k . Note that the $\vec{F}_k(f)$ term cancels so that $\vec{R}_{s_1 s_2, k}(f)$ is dependent only on $\vec{A}_{s_1, k}(f)$ and $\vec{A}_{s_2, k}(f)$. As previously mentioned, the benefit of using OO-FRFs is that you do not need to know the input force to perform calibrations and can perform the calibration with any object or even a person walking. It should be noted that because input-to-output FRFs are not used the force of the impact cannot be reconstructed. Once the calibration OO-FRFs are formed, cubic spline interpolation is used to increase the spatial resolution between calibration locations.

Next, in the localization stage, test impacts are localized by finding the location that minimizes the difference between the calibration OO-FRFs and the ratio between the frequency domain representation of the acceleration response between pairs of accelerometers. For a set of N sensors $S = \{s_1, \dots, s_N\}$ and a set of M interpolated calibration points $K = \{1, \dots, M\}$ the minimization is described using the following equation,

$$k_{impact} = \min_{k \in K} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^N a_{s_i s_j, k} \left\| \vec{R}_{s_i s_j, k} - \vec{A}_{s_i} \oslash \vec{A}_{s_j} \right\| \right) \quad (2)$$

where k_{impact} is the predicted impact location, $\|\cdot\|$ represents the L_2 norm, $a_{s_i s_j, k} = \frac{1}{\|\vec{R}_{ij, k}\|}$ is a normalization factor, $\vec{R}_{s_i s_j, k}$ is the calibration OO-FRF between sensors s_i and s_j , respectively, at location k , $\vec{A}_{s_i}$ and $\vec{A}_{s_j}$ are the FFT of the sensor outputs of s_i and s_j , respectively, and $\oslash$ is the Hadamard division operator which represents element wise division. Sensor pairs are not repeated for reciprocal relationships since this would not add information to the localization problem. It should be noted that the minimization scheme should only include the range of frequencies excited in the structure during the calibration stage.

1D FE Beam Model:

While the OO-FRF mapping method presented is designed for complex structures, it is first necessary to validate the method on a simple structure. Therefore, in this work, the OO-FRF mapping method is validated on a 1D homogenous finite element (FE) aluminum 3003-H14 beam model with fixed-fixed boundary conditions. The results are also compared to the ΔT mapping method. The parameters of the beam model can be found in Table 1. Aluminum 3003-H14 is used as the material due

Table 1 1D FE beam model parameters.

Parameter	Parameter Value
Material	Aluminum 3003-H14
Modulus	68.9 GPa
Poisson's Ratio	0.33
Density	2730 kg/m ³
Length	0.4572 m
Width	0.0254 m
Thickness	0.00635 m

to the availability of the material for future experimental investigation. The FE model is developed using the commercially available Ansys Mechanical software package and is validated against analytical natural frequencies and mode shapes from the literature [22]. The model uses quadratic Timoshenko beam elements of size 0.00635 m . It should be noted that manual mesh refinement is performed to determine that the results converged at this mesh size. Proportional damping is assumed in the model and is estimated by using realistic mass coefficient (α) and stiffness coefficient (β) values from the literature for a lightly damped aluminum structure [8]. The α and β damping values are selected as 1 and $7 * 10^{-8}$, respectively. The sampling rate for the model is selected to be 25 kHz to provide enough temporal resolution to accurately capture the difference in arrival times for the ΔT mapping method.

Localization Algorithm Evaluation:

Once the model is developed, sensor locations, calibration impact locations, and test impact locations are selected, which can be seen in Figure 1. Two sensor locations are selected 0.0254 m from each end of the beam. The locations are chosen to be at each end of the beam due to the fact that for 1D TDOA localization only impacts between two sensors can be distinguished if a constant propagation velocity is assumed, which is required for the ΔT mapping method. Next, 10 evenly spaced calibration points are placed along the beam between the two sensors. The test locations are chosen to be at $0.2L$, $0.3L$, $0.4L$, and $0.5L$, where L is the length of the beam. It should be noted that $0.1L$ is excluded because this is outside of the calibration locations. Impacts at each calibration and test location are simulated using a narrow Gaussian force distribution, which has been used in previous literature to represent impulses [23, 8]. The Gaussian distribution can be described using Equation (3),

$$f_g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \quad (3)$$

where $f_g(t)$ is the Gaussian impulse, σ is the standard deviation parameter of the distribution, and μ is the mean parameter of the distribution. σ and μ control the width and center of the distribution, respectively. In order to demonstrate that the ΔT mapping method is affected by dispersion from different frequency ranges being excited and that the OO-FRF mapping method is able to handle impacts that excite different frequency ranges, two types of impacts are used at each calibration and test location. For Impact Type 1 and Impact Type 2, the σ parameter is selected as 0.001 and 0.003, respectively, and μ is selected to place the impulse in the center of the impulse window. These impact functions are selected because Impact Type 1 excites a much broader frequency range than Impact Type 2, which can be seen from the FFT of the impact types shown in Figure 2. Average calibration maps are formed for both the ΔT and OO-FRF mapping methods by taking the average of the calibration maps for each impact type across the structure. Then the average calibration maps are interpolated to increase spatial resolution. It should be noted that the OO-FRF mapping method does not require averaging across multiple impact types since it accounts for frequency dependent relationships; however, the ΔT mapping method requires averaging across multiple impact types since the ΔT calibration maps are dependent on the type of impact. Therefore, in order to fairly compare the ΔT and OO-FRF mapping methods, average calibration maps are used for localization in both algorithms. Each localization method is then used to predict the location of the four test impacts for each impact type and localization errors are calculated by finding the difference between the predicted locations and the actual test locations. Average localization errors are computed for the ΔT and OO-FRF methods by taking the average of the localization errors for both types of test impacts for each method. It should be noted that the interpolation resolution of the cubic spline interpolation for the OO-FRF mapping method is selected to be $\Delta x = 0.004\text{ m}$. Additionally, the frequencies used for the localization calculation are limited to 200 Hz and below since this is where the majority of the frequency content exists for the beam. ΔT maps are also generated using the same calibration dataset and interpolation resolution. Arrival times are calculated using the AIC as performed in previous literature [18].

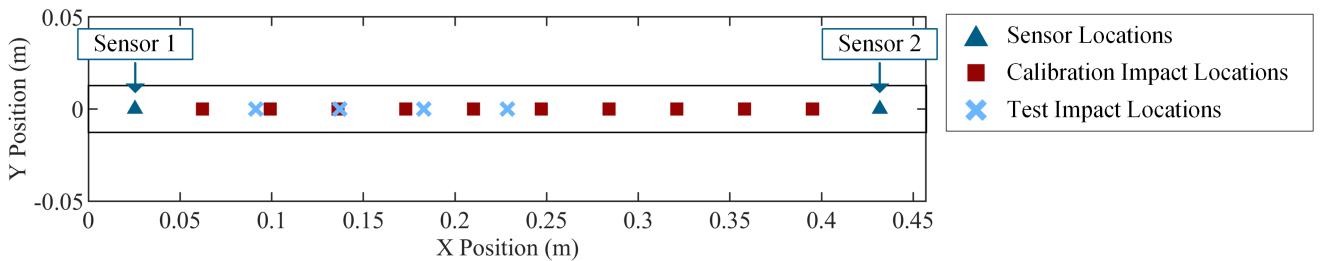


Fig. 1

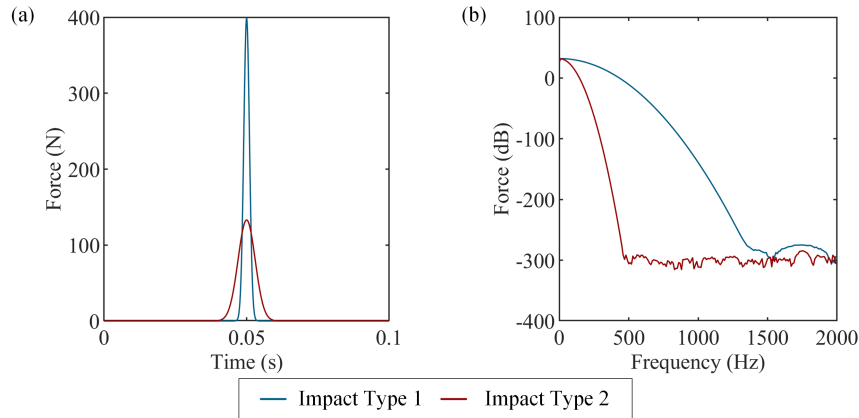


Fig. 2

Results

The localization accuracy of the ΔT and OO-FRF mapping methods are calculated and compared. The average localization error for the ΔT and OO-FRF mapping methods are 0.0957 m and 0.0029 m, respectively. Based on these results the OO-FRF mapping method shows significant improvement to the localization error of the ΔT mapping method. This demonstrates that, even when using averaged calibration maps, the ΔT mapping method struggles to localize different impact types; however, the OO-FRF mapping method is able to compensate for the dispersive effects of different impact types and localize the different impact types accurately.

Conclusions

In this work, the output-to-output frequency response function (OO-FRF) mapping method is proposed as a novel source localization algorithm in dispersive solid media. The OO-FRF mapping method naturally compensates for dispersive effects that occur when different frequency ranges are excited in a structure by using OO-FRF maps to calibrate a structure. The OO-FRF maps use the ratio between the frequency domain representations of pairs of sensors to form calibration maps across a structure. The OO-FRFs do not require TDOA calculations and naturally capture frequency dependent relationships of the structure. Additionally, the OO-FRF mapping method does not require a known input force to calibrate the system. The proposed method is validated and compared to the ΔT mapping method using a simulated 1D FE beam model. The results show that the proposed mapping method has a better localization accuracy than the ΔT mapping method. Future work will include experimental validation of the OO-FRF mapping method on both a simple beam structure and structures with increased complexity.

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Chapter 9

Reduced Order Model Efficiency for Structural Health Monitoring using Modal Data: A Case Study of Milad Tower

Akbar Esfandiari, Maryam Vahedi, Saeed Behboodi, and Masoud Sanayei

Abstract There are many methods for parameter estimation and damage detection of ordinary structures. However, for high-rise and massive structures where extraction of higher mode shapes and higher-frequency structural responses, which are more sensitive to structural variation, are challenging, the available indices such as MAC or COMAC may not work. Hence, this research presents a modified form of COMAC indices through weighting for structural health monitoring of Milad Tower, as a case study of a high-rise structure. A reduced-order model of the Milad Tower is used instead of the full finite element model. The capabilities of the different parameter estimation methods for model updating and damage assessment are investigated. The effectiveness of the proposed method is evaluated using numerically simulated data that demonstrates acceptable levels of accuracy and reliability in detecting, locating, and quantifying structural damages. Additionally, unavoidable challenges such as incomplete measurements of structural responses, modeling errors, and measurement errors are investigated. Also, the measured structural responses are used to assess the behavior of the proposed indices. The reduced-order model's efficiency and the proposed method's practical applicability using the new indices are demonstrated. The proposed weighted form of the COMAC index is more sensitive to changes in structural parameters. In addition, a model updating algorithm is proposed for damage detection using the new reduced-order model based on the consistent mass matrix. The proposed method was successfully applied to the Milad Tower using modal data. It resulted in a more accurate reduced-order model for damage detection.

Keywords Towers · Reduced finite element model · Modeling error · Modeling efficiency · Computational efficiency · Modal data

Introduction

In structural model updating for health monitoring procedures, utilizing a full-scale three-dimensional finite element model of a high-rise structure provides a massive amount of data, which may result in errors in data collection. In addition, the data analysis of large-scale structures using the full-scale model could be prohibitive in terms of modeling time and computation time. In this regard, developing a lower-dimensional model that accurately captures the characteristics of the original model in the frequency range of interest would be beneficial. Generating a reduced Finite Element model is based on selecting the desired number of Degrees of Freedom (DOFs), which results in reduced-order mass, damping, and stiffness matrices [1].

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Many researchers have addressed the issue of high-rise building monitoring, based on which it can be concluded that dealing with large data and finite element modeling of large-scale real-life structures poses challenges when used in numerical simulations [2–7]. These challenges are magnified in inverse problems and model updating algorithms. Additional specialized scripts are needed to communicate with commercial software for structural health monitoring and interactive communication between commercial software and research software [8]. This way, SHM and model updating of a large structure using different commercial software is time-consuming.

Model order reduction techniques aim to create a simplified model, commonly called a reduced-order model (ROM), based on a large and complex full-scale model (FSM) using commercial software. The approximation errors must be negligible compared to the full-scale FE model in terms of the frequency range of interest and the dynamic properties and characteristics of the complete model. Furthermore, it must be computationally efficient, reliable, and low-cost.

The Guyan condensation method can reduce finite element models using static condensation [9]. Hence, it does not preserve the inertial forces applied to the model. Dynamic condensation methods have been developed to remove the weakness of the Guyan method [10,11]. O’Callahan et al. [11] proposed the system equivalent reduction expansion process (SEREP). This method is introduced as the most accurate method in many studies. Marinone et al. [12] proposed an improved method by combining the SEREP and Guyan methods. Regardless of the existing and challenging problems of the proposed data expansion and model reduction methods, applying these methods to large civil infrastructure is impractical. For structural model updating and conceptual analysis of large structures, it is preferred that the entries of the stiffness matrix are meaningful and that connectivity between the DOFs is preserved. Such a model can be used for sensitivity analysis and investigation of the influence of different parameters on structural responses. The methods above may not preserve the connectivity of elements for model reduction. Furthermore, the transformation matrix for model reduction shall be updated at each iteration.

Large models can be simplified at the substructure level based on the available condensation techniques [13]. This approach preserves the connectivity between the master DOFs. Many researchers frequently use the Guyan static condensation and dynamic condensation methods. Comprehensive studies on these methods were conducted by Allen et al. [14] and Qu [15]. However, these methods require unavailable details of stiffness and mass matrices as inputs, limiting their application to large structures modeled by commercial software.

Wu and Li [16] updated a simple finite element model of the 310 m tall Nanjing TV Tower based on ambient vibration measurements. The FE model consisted of 17 beam elements and lumped masses. The tapered beam elements used by Cheng [17] were replaced by equivalent piece-wise uniform prismatic beams, and lumped masses were assumed.

The simplified models of large civil structures are based on fundamental principles and definitions of structural behavior. Yi et al. [18] extracted the lumped mass matrix and entries of the stiffness matrix of the Dalian World Trade Center’s building. The simplified model was used to study the optimal sensor placement on the building. Ni et al. [19] proposed a reduced model for structural health monitoring of the Canton Tower. They assigned six active DOFs at each defined level. The stiffness matrix was extracted sequentially by applying unit loads at active DOFs one at a time. The mass matrix was extracted as a lumped mass. Only the reduced model’s stiffness parameters were tuned to remove the observed discrepancy between the full-scale and the reduced model. The ROM was used in other studies focused on structural health monitoring and structural identification of the Canton Tower [20, 21]. The difference between the modal properties of the reduced and full-scale models was increased due to inaccurate mass modeling.

Xiong et al. [22] proposed a reduced model for the Shanghai Tower, a 125-story, 632-meter building. The stiffness matrix was extracted by applying unit displacement at the defined DOFs at the interfaces of individual zones, and further steps extracted the lumped mass matrix. The mass and stiffness matrix of the ROM was calibrated to match the extracted model data by the full-scale model.

The lumped mass matrix is a source of modeling error that adversely affects the model characteristics and, consequently, model updating accuracy. Coarse meshing further increases induced modeling errors by a lumped mass matrix. It must be emphasized that reducing modeling errors is an essential task for structural model updating.

In this research, a novel method is proposed to extract the consistent mass matrix of a large tower. The algorithm is based on the extracted data from the structure’s full-scale finite element model. Then, the proposed reduced-order model based on the reduced consistent mass matrix model is verified by comparing the dynamic characteristics of the tower’s ROM versus the FSM. A new form of the Coordinate Modal Assurance Criterion (COMAC) index is proposed by appropriate weighting (normalization) of the mode shapes. The proposed weighted form of the COMAC index is more sensitive to changes in structural parameters. The mass matrix of the obtained ROM is then updated to be utilized for damage detection. In addition, a model updating algorithm is proposed for damage detection using the new reduced-order model based on the consistent mass matrix. The proposed method was successfully applied to the Milad Tower using modal data. It resulted in a more accurate reduced-order model for damage detection.

Research Methodology

a. General overview of the Milad Tower

Milad Tower, also known as Tehran Tower, is a multi-purpose tower in Tehran, Iran. It consists of five main parts: the foundation, the lobby, the main body of the tower (shaft), the head structure (basket), and an antenna mast with a length of 120 meters. Tower construction started in 1997, and it opened on October 7, 2008.

The main shaft, rising through the middle of the main lobby, transmits loads from the upper parts of the tower, such as the head and antenna, to the foundation. It is a concrete structure that extends from the ground level to 315 meters.

The head structure of the tower is a 12-story building that includes a variety of utilities, including telecommunication floors, shops, a restaurant, and an observation deck. The head structure weight is about 2,000 tons and is attached to the tower's main body from the level of 247 to 308 meters. The minimum diameter of the head structure is 25 meters, and the maximum diameter is 60 meters. The last part of the Milad Tower is the antenna structure made of pure steel with a length of 120 meters and an approximate weight of 350 tons. The total weight of the Milad Tower, including the shaft, head, and antenna, is about 150,000 tons. A schematic of the tower's elevations and plan sections of the concrete core is illustrated in Figure 1.

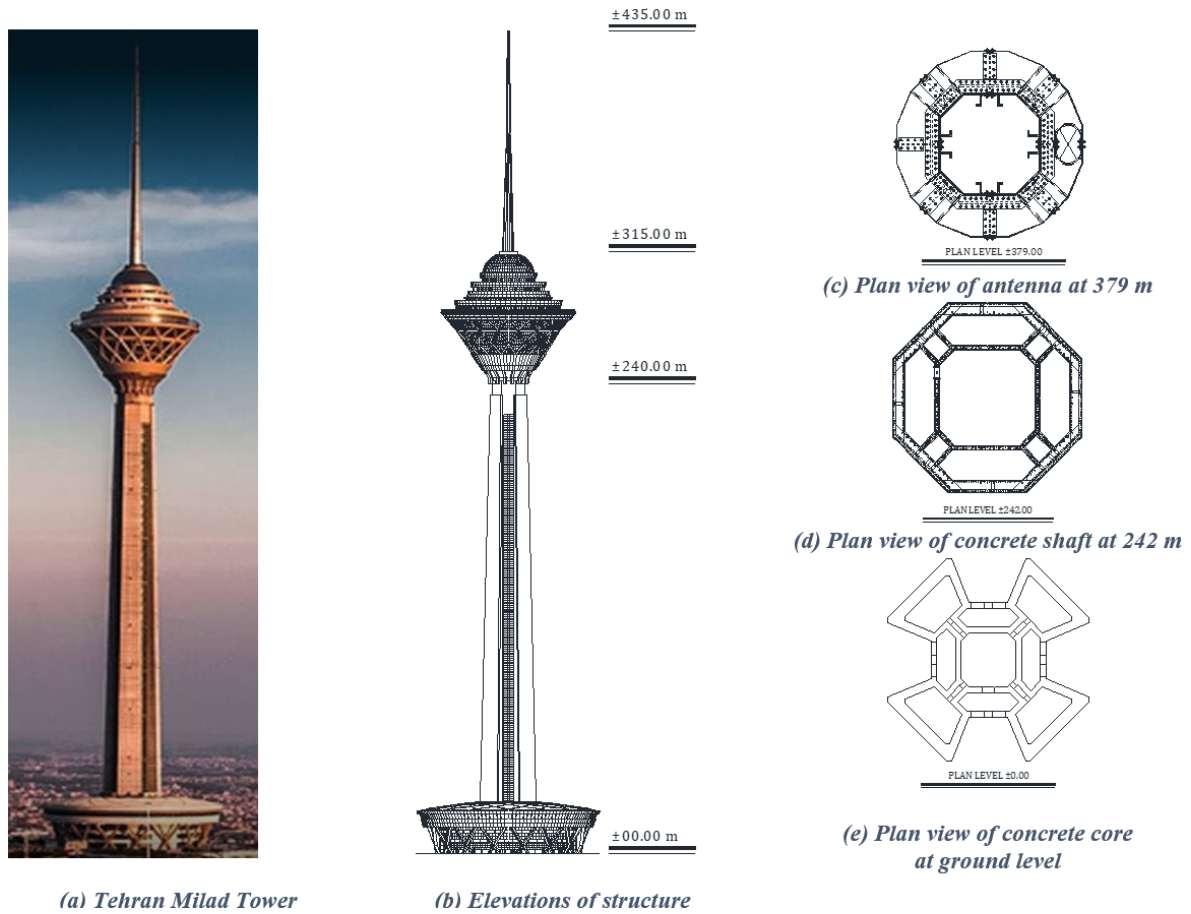


Fig. 1 Elevations of Milad Tower and plan sections of concrete core at various levels.

b. Full-scale model of the Milad Tower

The full-scale 3D finite element model of the Milad Tower was primarily developed as a baseline model based on design drawings of the Tower. This FSM, which consists of 4,495 frame elements, 18,489 shell elements, 17,850 nodes, and 106,524 degrees of freedom, was created in the SAP2000 commercial finite element analysis software. The main part of the Milad Tower structure is the concrete shaft modeled with shell elements. The head structure is a 12-story steel structure modeled

as a space frame with additional floor elements. The loads are transferred from floors to beams and columns to the shaft. The natural and mode shapes of Milad Tower are shown in Figure 2.

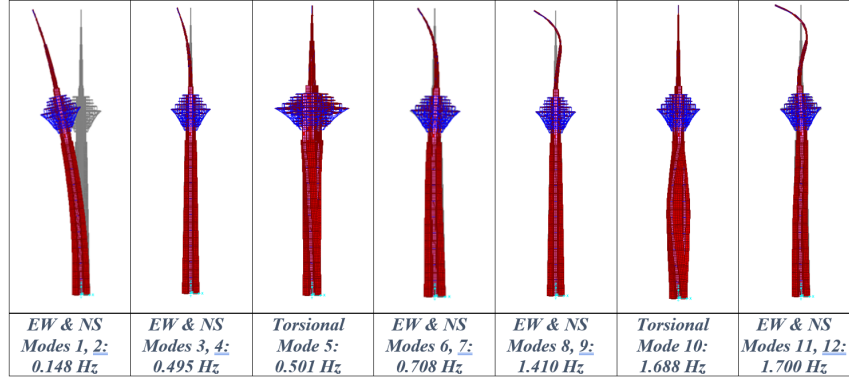


Fig. 2 Milad Tower modes of vibrations obtained from finite element FSM.

The natural frequencies of the first 12 modes were below 1.7 Hz. Modes 1 and 2 are the tower/antenna repeated bending modes about the principal axes. Whereas modes 3, 4, and 6, 7, as well as 8, 9, and 11, 12 represent four pairs of repeated modes. Modes 5 and 10 are the torsional modes of the tower.

c. 3D Reduced-Order Model

The proposed method for estimating the reduced consistent mass matrix is a general method that can be applied to different types of structures. However, the method is discussed in detail based on the geometry and mechanical properties of the Milad Tower to develop the stiffnesses and mass super-elements for each segment of the structure. To create the ROM, the FSM is discretized and reduced into 34 super-elements along the shaft's centroidal axis for modeling axial, shear, and bending behaviors. The elements' end node selection was dependent on the floor slab levels in the shaft and the desired sensor locations for future structural health monitoring.

The ROM's first 20 elements are related to the shaft (height of 0 to 248.5 m), element 21 corresponds to the head structure (height of 248.5 to 302.4 m), and the last 13 elements represent the antenna structure (height of 302.4 to 435.55 m). Therefore, the overall reduced-order model includes 34 elements with 204 degrees of freedom.

Extraction of the stiffness matrix: For each element of the ROM, a unit displacement must be applied to each defined 12 degrees of freedom one at a time while the rest of the DOFs are constrained. The magnitude of the force required at each active DOF represents the column of the stiffness matrix corresponding to the active DOF. However, using any commercial software, this procedure could be performed by applying a unit force at the active DOF, constraining all other master DOFs, and obtaining the resisting forces. Then, the obtained values of the resisting forces shall be magnified to achieve a unit displacement. A schematic of the applied unit displacement is presented in Figure 3. Repeating the above procedure for each of the 34 super-elements will result in stiffness matrices at the element level that are assembled to form the global stiffness matrix of the ROM.

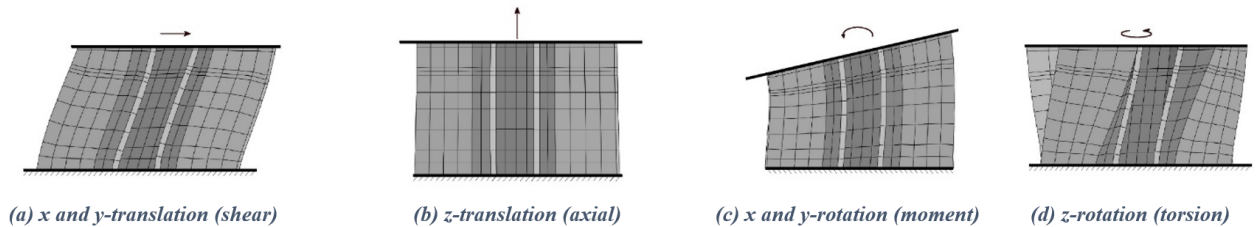


Fig. 3 Unit translation or rotation (force or moment) imposed at different master DOFs of a super-element.

Extraction of the reduced lumped mass matrix (RLMM): While extracting the stiffness matrix of a ROM based on commercial software is straightforward, extracting the mass matrix is challenging. Hence lumped mass matrices are often used

by reduced-order algorithms. In the RLMM, the total mass of each element is divided directly equally between the translational degrees of freedom for each direction at the adjacent nodes. The rotational entries of the lumped mass matrix are diagonal and defined as Eq. (1):

$$m_r = \sum_{i=1}^s l_i^2 m_{ii} \quad (1)$$

where m_{ii} is the translational mass at the i th node and l_i is the distance between node i and the reference (master) node of the reduced model. This formulation depends on the structure's overall geometry and does not present the dominant behavior of the structural elements. A lumped mass matrix introduces modeling errors that can corrupt the model updating process. Additionally, the mass modeling error by the lumped mass method becomes large when using a coarse mesh.

Extraction of the reduced consistent mass matrix (RCMM): The following steps are followed to extract the consistent mass matrix for each super-element.

Step1) Mass at translational and rotational DOFs: A typical super-element of a segment of the full-scale model is shown in Figure 4. A master DOF of this super-element is considered to be active, while all other degrees of freedom are constrained. The master DOF can be translational or rotational to study the lateral modes of vibration.

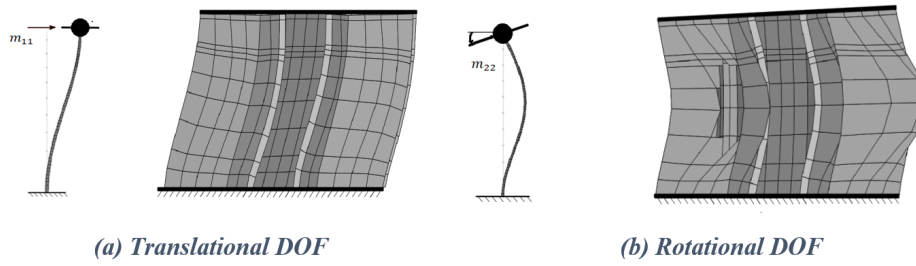


Fig. 4 A single active DOFs for extraction of a consistent mass matrix of a super-element.

In such cases, the equation of motion of the ROM will be equal to the equation of motion of a single degree of freedom model, and the eigenvalue equation shown in Eq. (2) is used to find the corresponding mass:

$$k_{ii} - \omega_i^2 m_{ii} = 0 \quad (2)$$

where k_{ii} is the stiffness at the i th node and ω_i is the i th circular natural frequency. The corresponding stiffness entry k_{ii} was previously estimated during the stiffness extraction process. By using the calculated natural frequency using each segment of the full-scale model, the corresponding diagonal entries of the ROM mass matrix can be calculated by Eq. (3):

$$m_{ii} = \frac{k_{ii}}{\omega_i^2} \quad (3)$$

This step must be repeated for translational and rotational DOFs of each super-element of the FSM for calculation of the ROM for each element.

Step2) Mass coupling of a translational and a rotational DOF: While transverse and rotational deformations are uncoupled from both axial and torsional deformations in a bi-symmetrical section, the transverse and rotational DOFs are coupled together. To extract the off-diagonal entries of the ROM mass matrix in a 3D space, both coupled DOFs must be active simultaneously while the other 10 DOFs are constrained. A schematic representation of such a case is given in Figure 5 for the extraction of off-diagonal entries of the consistent mass matrix.

The circular natural frequencies of the two DOF models are expressed by the corresponding eigenvalue equation in Eq. (4):

$$\det \left(\begin{bmatrix} k_{ii} & k_{ij} \\ k_{ij} & k_{jj} \end{bmatrix} - \omega_{ij}^2 \begin{bmatrix} m_{ii} & m_{ij} \\ m_{ij} & m_{jj} \end{bmatrix} \right) = 0 \quad (4)$$

where i and j are active DOFs of a segment of the structure and ω_{ij} is the circular natural frequency of the constrained segment that is evaluated using the FSM. Note that the corresponding stiffness matrix is already estimated in the previous steps. Hence, the only unknown term is the off-diagonal mass entry m_{ij} due to the coupling of the horizontal and rotational DOFs is calculated using Eq. (4). This procedure must be repeated for all off-diagonal entries of the mass matrix ($m_{2,4}, m_{1,5}, m_{5,7}, m_{4,8}, m_{8,10}, m_{7,11}, m_{1,11}, m_{2,10}$).

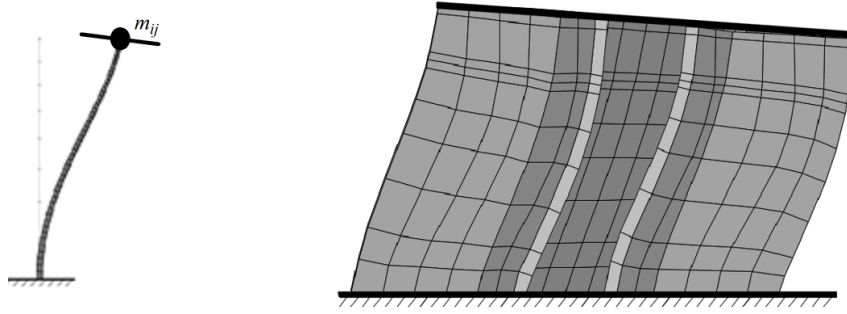


Fig. 5 Simultaneous activation of two coupled DOFs (transverse & rotational).

According to the structural stability concepts, two transverse DOFs of a single segment cannot be defined as active simultaneously. Hence, to extract such an off-diagonal mass entry, two segments of the model must be considered simultaneously for computing Eq. (4).

Torsional stiffness: Since the estimation of the torsional modes is frequently imprecise and model updating should be avoided, an alternative technique is proposed for calculating the corresponding torsional entry in the stiffness matrix. For this purpose, the fundamental definition of torsional stiffness has been implemented. A torsional moment T applied at the free end of the model causes a twisting deformation θ . Then the magnitude of the torsional stiffness can be evaluated by Eq. (5):

$$K_t = \frac{T}{\theta} \quad (5)$$

The above technique can be applied at all defined levels to obtain the stiffness matrix entries corresponding to all torsional DOFs of each segment.

Milad Tower Reduced-Order Model Verifications

The ROM must maintain not only the static behavior but also the dynamic behavior of the original FSM. For this purpose, the dynamic characteristics of the ROM are compared with the FSM, including natural frequencies and mode shapes. The natural frequencies of the ROM modal analysis for the first ten lateral modes in the x-direction are presented in Table 2. The lateral mode shapes of the x and y directions are identical due to symmetry. The modal mass ratios were more than 90 percent using these mode shapes. A comparison of the natural frequencies in Table 1 demonstrates the accuracy of the predicted natural frequencies by the ROM based on the reduced consistent mass matrix model compared to the FSM. Additionally, the reduced lumped mass matrix (RLMM) and the reduced consistent mass matrix (RCMM) models are compared to the FSM. The relative difference of RLMM and RCMM w.r.t. FSM is shown in Table 1. Clearly, the RCMM-based model has outperformed the RLMM-based model based on the calculated natural frequencies.

Table 1 Comparison of natural frequencies of first 10 lateral modes

Mode No.	1	2	3	4	5	6	7	8	9	10
FSM (Hz)	0.148	0.495	0.709	1.41	1.695	2.844	3.128	4.346	4.701	4.767
RLMM (Hz)	0.146	0.514	0.655	1.392	1.519	2.555	2.675	3.683	4.662	5.100
RCMM (Hz)	0.150	0.507	0.742	1.456	1.782	2.828	3.21	4.112	4.726	5.405
% Difference of RLMM w.r.t. FSM	-0.013	0.039	-0.076	-0.013	-0.104	-0.102	-0.145	-0.153	-0.008	0.070
% Difference of RCMM w.r.t. FSM	0.014	0.024	0.047	0.033	0.051	-0.006	0.026	-0.054	0.005	0.134

Additionally, to further assess the accuracy of the mode shapes obtained from the FSM and ROM, the first ten lateral mode shapes are shown in Figure 6. It must be mentioned that the mode shapes demonstrated as solid lines are related to the

updated model of RCMM, and the detailed process of updating is stated in the next section. The mode shapes are normalized for comparison based on the corresponding vector size. Since the lateral modes in the x and y-directions are similar, only the x-direction is presented in Figure 6. As shown, the predicted mode shapes by the ROM are very accurate and represent the real behavior of the model using a few elements, considering that the random loads (e.g., wind and earthquake) mainly excite the lower modes. The mode shapes of the updated models based on the RCMM, labeled “Updated RCMM”, will be discussed later.

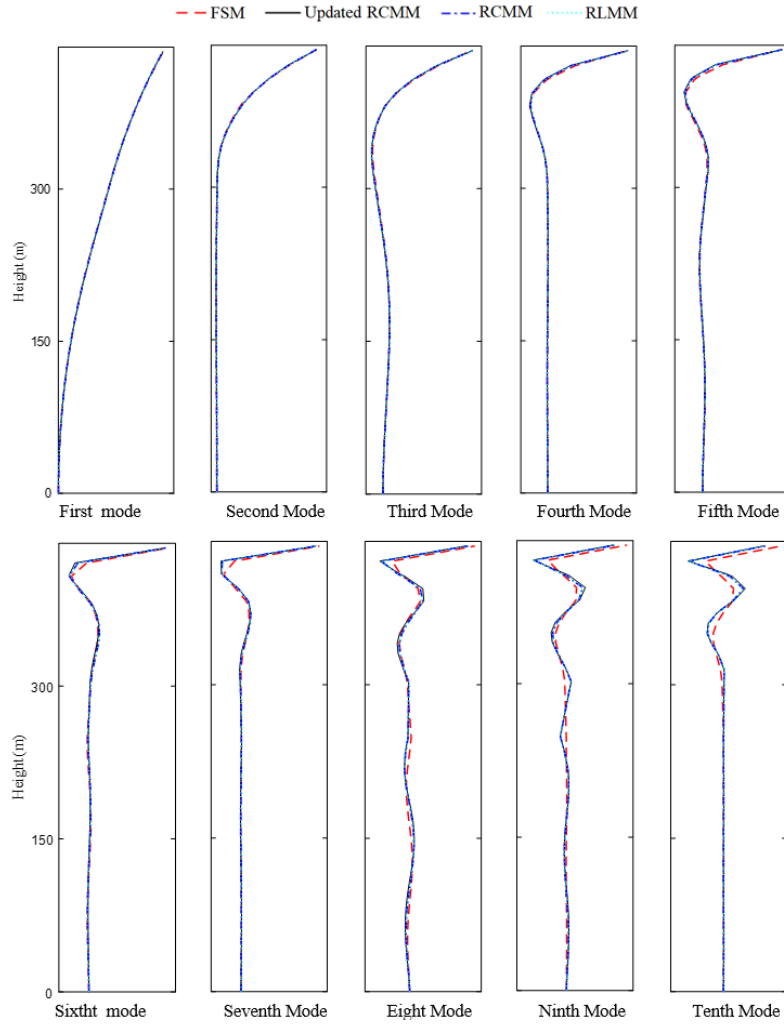


Fig. 6 Comparison of RCMM-based ROM with FSM using first 10 lateral mode shapes.

To conduct a more comprehensive study, the predicted mode shapes by the FSM and both ROMs (RLMM and RCMM) are compared based on the Modal Assurance Criterion (MAC) index defined by Eq. (6) as:

$$MAC(i, j) = \frac{\|(\phi_{ROM})_j^* (\phi_{FSM})_i\|^2}{\|(\phi_{FSM})_i^* (\phi_{FSM})_i\| \|(\phi_{ROM})_j^* (\phi_{ROM})_j\|} \quad (6)$$

Where $(\phi_{FSM})_i$, $(\phi_{ROM})_i$, are the i th mode shapes of the full-scale and ROM models, respectively, and the superscript * denotes the transpose complex conjugate of the i th mode shape. Table 2 gives the calculated values.

The MAC values in Table 2 indicate that the predicted mode shapes for the FSM and ROM based RCMM are very consistent. It also shows that the presented RCMM model is more accurate than RLMM.

Table 2 Comparison of RLMM & RCMM-based ROMs with FSM using MAC values of different mode shapes

Index	Model	Mode No.									
		1	2	3	4	5	6	7	8	9	10
MAC	RLMM	1.000	1.000	0.971	0.990	0.985	0.867	0.866	0.437	0.156	0.025
	RCMM	1.000	1.000	0.999	0.997	0.991	0.986	0.974	0.947	0.897	0.799

Table 3 Comparison of RLMM & RCMM-based ROMs with FSM using COMAC values of mode shapes at different nodes

DOF (Node No.)	RLMM	RCMM	DOF (Node No.)	RLMM	RCMM
1	0.587	0.979	18	0.618	0.923
2	0.531	0.964	19	0.301	0.816
3	0.520	0.958	20	0.475	0.997
4	0.478	0.939	21	0.760	0.980
5	0.405	0.904	22	0.855	0.962
6	0.280	0.837	23	0.934	0.987
7	0.112	0.748	24	0.931	0.994
8	0.343	0.995	25	0.889	0.964
9	0.642	0.947	26	0.802	0.919
10	0.681	0.998	27	0.659	0.925
11	0.503	0.964	28	0.775	0.990
12	0.352	0.886	29	0.825	0.993
13	0.186	0.774	30	0.819	0.943
14	0.322	0.952	31	0.599	0.982
15	0.695	0.904	32	0.844	0.971
16	0.763	0.973	33	0.828	0.909
17	0.824	0.988	34	0.811	0.991

To compare the accuracy of different models locally, the Coordinate Modal Assurance Criterion (COMAC) index is defined by Eq. (7), where i is the node number and L is the number of mode shapes.

$$COMAC(i) = \frac{\int_{l=1}^L |(\phi_{FSM})_{il}(\phi_{ROM})_{il}^*|^2}{\int_{l=1}^L (\phi_{FSM})_{il}^2 \int_{l=1}^L (\phi_{ROM})_{il}^2} \quad (7)$$

The COMAC values at 34 nodes of the structure are shown in Table 3, which indicates the precision of the RCMM. The COMAC values show the deformation at a specific level obtained by the full-scale and reduced-order models for different mode shapes that are close. COMAC index values closer to 1.0 indicate higher accuracies at a DOF.

It must be emphasized that a coarse mesh might not be able to represent the structural behavior at higher modes, and a finer mesh is required. Hence, it is expected that the MAC values at the higher modes will be less than those at the lower modes. Also, COMAC values are sensitive to the local behavior of the structure. More points are required to capture the real behavior in the higher curvature regions. Hence, the discrepancy between ROMs and FSM can increase in such regions.

Model Updating and Damage Detection

a. Weighted-COMAC damage index

The Modal Assurance criterion (MAC) and the Coordinate Modal Assurance Criterion (COMAC) are widely used for structural damage detection and model validation. For a large structure like Milad Tower, the sensitivity of these indices with respect to variation of structural parameters is the main issue. In such cases, the variation of indices due to structural damages might be overshadowed by the variation of indices due to measurement errors. The COMAC index values depend on the technique selected for normalization (Weighting) of the mode shapes. The partially measured mode shape is scaled (weighted) by its second norm as a practical choice. Although there is no difference between the MAC values for the arbitrary scaled or mass-normalized mode shapes, the values for COMAC are quite different. The values of COMAC indices

using normalized mode shapes and error-contaminated data are compared with the exact values in Figure 7. It is reminded that the COMAC values for the intact model without errors are equal to 1.0.

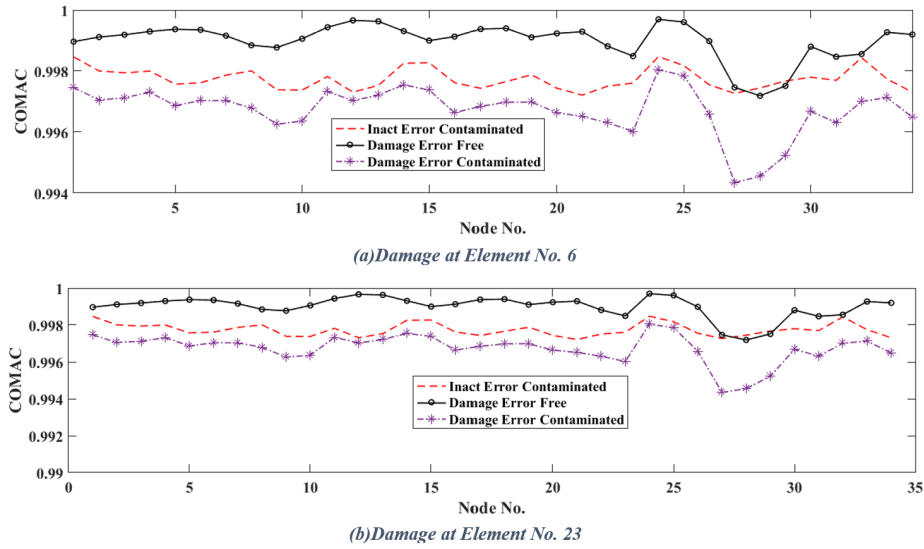


Fig. 7 COMAC values for different cases with 10 percent of stiffness variations (Normalized by 2^{nd} norm).

Figure 7 shows that COMAC indices are highly sensitive to measurement errors, which might overshadow their sensitivity to stiffness variations. This may cause errors in a damage detection process. To provide the ability to observe more changes in COMAC index values due to damage and less sensitivity to measurement errors, arbitrary scaled mode shapes are used to calculate the COMAC values. For this purpose, the mode shape value at a specific node is considered for mode shape normalization, resulting in the weighted-COMAC index values.

Figure 8 shows the weighted-COMAC index values for normalized mode shapes with respect to the mode shape value at node 10. It is notable that the weighted-COMAC values of the error-contaminated damaged and intact models are very close to each other. As shown below, using the weighted-COMAC index values provides excellent observability of damage with less sensitivity to measurement errors.

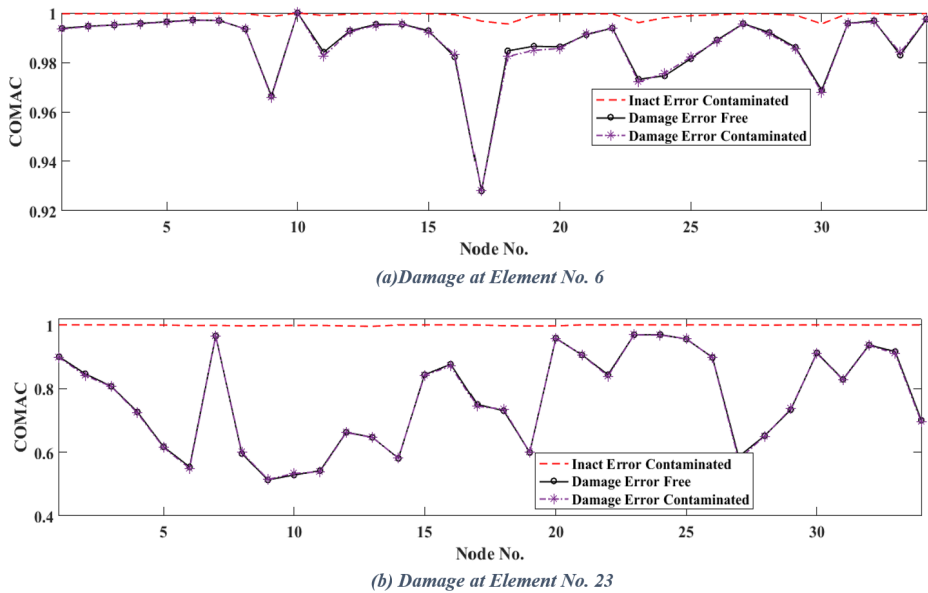


Fig. 8 Weighted-COMAC values for different cases with 10 percent of stiffness variations (Normalized by node 10).

b. Model updating algorithm

Although the focus of this paper is on presenting a ROM for general purposes, the ROM might be used for structural model updating and damage detection. Hence, the presented ROM based on the consistent mass matrix is used for structural model updating. The proposed RCMM model is accurate enough for structural response evaluation and conceptual analysis. However, a more accurate reduced model might be essential for structural model updating and damage detection. Hence, the ROM can be tuned through a model parameter estimation process. Since the extraction of the stiffness matrix is based on the static response and is straightforward, in this study, the extracted reduced consistent mass matrix is tuned for a better match of the modal data of the ROM and the full-scale model.

For an n degree of freedom structure, the eigenvalue equation is expressed as:

$$(\mathbf{K} - \omega_i^2 \mathbf{M}) \phi_i = 0 \quad (8)$$

where $\mathbf{K}$ and $\mathbf{M}$ are $(n \times n)$ stiffness and mass matrices of the structure, respectively; ω_i is the i th circular natural frequency, and ϕ_i is the corresponding mode shape (eigenvector) of the structure. Assuming that structural damage causes changes in the stiffness and mass matrices by the amount of $\delta \mathbf{K}$ and $\delta \mathbf{M}$, the eigenvalue equation for the damaged structure is rewritten as:

$$(\mathbf{K} + \delta \mathbf{K} - \omega_{id}^2 (\mathbf{M} + \delta \mathbf{M})) \phi_{id} = 0 \quad (9)$$

where the subscript "d" indicates that the variable in equation is associated to the damaged states. Rearranging Eq. (10) yields:

$$\phi_{id} = -[\mathbf{K} - \omega_{id}^2 \mathbf{M}]^{-1} \delta \mathbf{K} \phi_{id} + \omega_{id}^2 [\mathbf{K} - \omega_{id}^2 \mathbf{M}]^{-1} \delta \mathbf{M} \phi_{id} \quad (10)$$

Eq. (11) evaluates the i th mode shape of the damaged structure as a function of the changes in mass and stiffness parameters. However, this equation requires the measured mode shapes of the damaged structure. The complete response measurements at all DOFs requires an extensive sensor network, which is impractical. Furthermore, some degrees of freedom embedded within the structure's body are inaccessible for sensor installations. To establish Eq. (10), we approximated the mode shapes of the damaged structure on the right-hand side by the corresponding mode shapes of the intact structure. Then we updated it through an iterative model updating process.

The stiffness and mass matrix of the intact model is the sum of the stiffness matrices of all individual elements:

$$\delta \mathbf{K} = \sum_r^{n_E} \delta k_r \mathbf{K}_r \quad \text{and} \quad \delta \mathbf{M} = \sum_r^{n_E} \delta m_r \mathbf{M}_r \quad (11)$$

Where $\mathbf{K}_r$ and $\mathbf{M}_r$ are the respective contributions of the r th element to the global stiffness and mass matrices of the structure and n_E is the number of elements. The scalar multiplier δk_r and δm_r present the proportional changes in the stiffness and mass of the r th element in the damaged state from their values in the intact state. Substituting (11) in (10), the i th mode shape of the damaged structure can be related to the changes of stiffness and mass parameters as:

$$\phi_{id} = [\mathbf{S}_{\phi_i}^K] \{\delta k\} + [\mathbf{S}_{\phi_i}^M] \{\delta m\} \quad (12)$$

where, r th columns of the sensitivity matrices $\mathbf{S}_{\phi_i}^K$ and $\mathbf{S}_{\phi_i}^M$, which expresses the relation of the i th mode shape at all degrees of freedom to the change of r th stiffness and mass parameter, are evaluated as:

$$\begin{aligned} \mathbf{S}_{\phi_i}^K(:, r) &= -[\mathbf{K} - \omega_{id}^2 \mathbf{M}]^{-1} \mathbf{K}_r \phi_i \\ \mathbf{S}_{\phi_i}^M(:, r) &= +\omega_{id}^2 [\mathbf{K} - \omega_{id}^2 \mathbf{M}]^{-1} \mathbf{M}_r \phi_i \end{aligned} \quad (13)$$

The proposed method extracts the mass matrix of each super-element. Hence, any entries of the RCMM can be the subject of model calibration which could be a challenging issue. Hence, an unknown parameter is assigned to all entries of the consistent mass matrix and the mass matrices of all defined super-elements are updated. The number of unknown mass parameters is 34. The modal data of the first seven mode shapes of the full model are used as the input data for model updating. The updated mass parameters and comparison of the natural frequencies are given by Figure 9 and Table 4.

As the extracted natural frequencies based on the updated mass matrix show, there is an excellent match between the natural frequencies of the updated RCMM model and the full model. To assess the capabilities of the tuned RCMM model to identify simulated structural damages, the stiffness of the super-element 10 (of the shaft from 84 to 96 m height) is reduced by 30 percent. The modal data are extracted from the FSM and used as the input for model updating based on the introduced parameter estimation method. The predicted damage location and severity are shown in Figure 10.

The predicted structural stiffness parameters demonstrated the superior capabilities of the proposed ROM based on the RCMM for structural model updating and health monitoring.

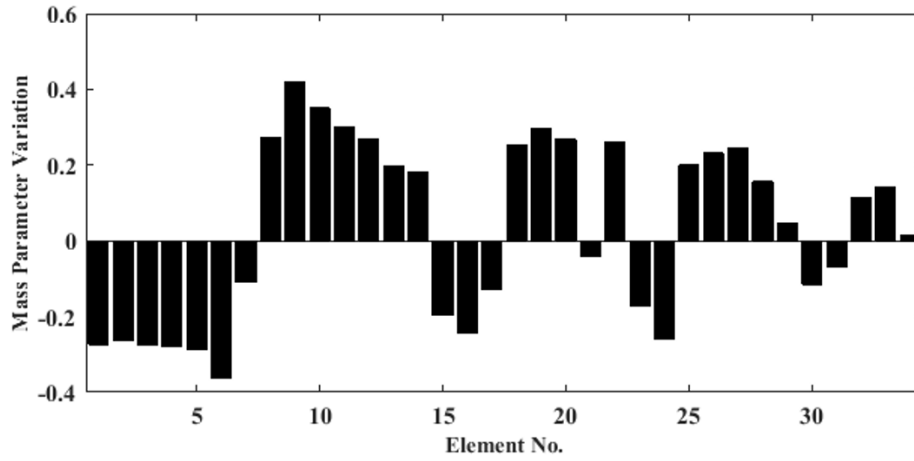


Fig. 9 Updated Mass Parameters of RCMM-based ROM using Mode shape Data.

Table 4 Comparison of Initial and Updated natural frequencies after RCMM mass parameter modification.

Mode No.	FSM	RCMM (Initial)	RCMM (Updated)	Relative Difference w.r.t FSM (%)	
				Initial	Updated
1	0.150	0.148	0.1500	1.214	0.0001
2	0.507	0.495	0.5070	2.511	-0.0016
3	0.742	0.709	0.7416	4.727	0.0560
4	1.456	1.410	1.4560	3.226	-0.0002
5	1.782	1.706	1.7695	5.221	0.7034
6	2.828	2.832	2.8383	-0.514	-0.3651
7	3.210	3.131	3.2060	2.666	0.1238

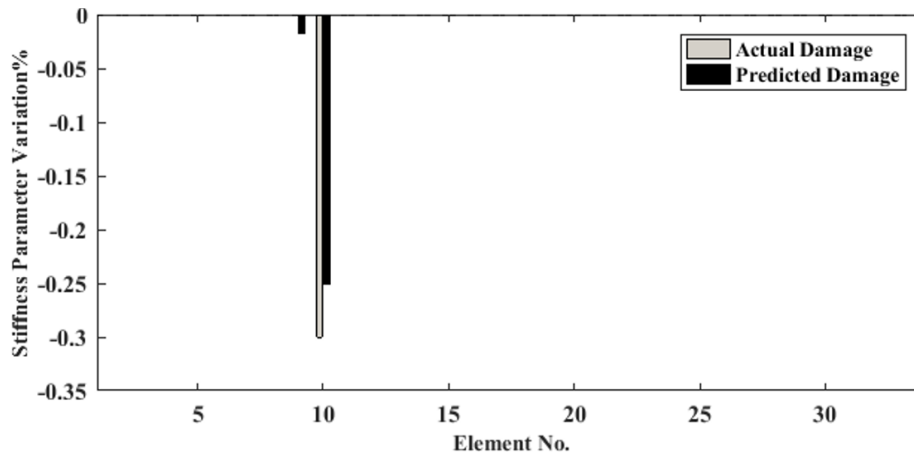


Fig. 10 Updated Stiffness Parameters of RCM-based ROM using Mode shape Data.

Conclusions

Reduced-order models (ROMs) are desired for structural analysis of large structures. These reduced structural models can be used instead of full-scale models (FSMs) for fast computing, conceptual assessment of different models, and model updating for structural health monitoring. ROMs must preserve the inherent properties of FSMs. Unavoidable errors in lumped mass modeling by ROMs may reduce the accuracy of structural responses, leading to magnified errors in model updating methods. In this research, an accurate method for extraction of the ROM of large structures is proposed. Extraction of an accurate mass

matrix for ROMs can be challenging. To reduce modeling errors induced by lumped mass matrices, the reduced consistent mass matrix (RCMM) is extracted based on the evaluated stiffness and natural frequencies of the FSM. The proposed method is applied to the Milad Tower.

The comparison of the predicted structural static and dynamic behavior and modal properties by the proposed ROM (RCMM) demonstrated its ability to reduce large structural models with high accuracy. Additionally, a weighted-COMAC index is proposed, which is more sensitive to structural damage and less sensitive to measurement errors. For damage detection, the updated consistent mass matrix accurately identified the damaged element. Using ROM with RCMM improved modeling accuracy with significant savings in computing time. The proposed ROM with RCMM, in conjunction with the weighted-COMAC index, can be used to investigate structural behavior further and update the structural model for structural health monitoring.

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Chapter 10

Vibration Serviceability Evaluation of an All-Steel Floor System using the High-Frequency Floor Procedure

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Abstract The increasing trend towards open and lightweight office spaces has led to a growing concern about floor vibration serviceability. This study investigates the vibration serviceability of two all-steel floor modules, denoted as SM68.RAF and SM94.RAF, using the high-frequency floor (HFF) procedure outlined in American Institute of Steel Construction's Design Guide 11 (AISC DG-11). The two specimens are 10 ft $\times$ 40 ft modules that would form a larger 30 ft $\times$ 40 ft bay for a new type of modular floor system made entirely of structural steel. The specimens have the same configuration but vary in beam and plate sizes. Finite element models are created to implement the HFF procedure, which involves determining the walking frequency, calculating peak acceleration responses, and obtaining equivalent sinusoidal peak acceleration (ESPA) values as defined in DG-11. The study highlights the impact of damping on the response, with two damping ratios of 0.75% and 2.25% considered to represent bare-floor laboratory conditions and in-service conditions, respectively. The vibration serviceability of the two specimens is evaluated by comparing the computed ESPA values to the acceleration tolerance limits provided in DG-11. The results indicate that the peak accelerations of the light specimen (SM68.RAF) tend to exceed the allowable limit for both damping ratios, while the heavy specimen (SM94.RAF) remain below the limit for in-service conditions. Even though these results cannot be considered for the overall acceptability of floor concept, the findings of this study provide valuable insights into the vibration serviceability of light-weight floor systems as well as the applicability of the HFF procedure to the non-traditional floor systems.

Keywords Structural floor systems · Floor vibration serviceability · Modular construction · High-frequency floors · Damping

Introduction

Floor vibration serviceability has emerged as a significant concern in modern office buildings, primarily due to the evolution of office layouts and equipment. The trend towards more open and lightweight spaces, characterized by fewer partitions and reduced furniture mass, has contributed to an increase in floor vibration complaints [1]. Human activity, such as walking, can induce resonant responses in floors when the step frequency coincides with a subharmonic of the floor's natural frequency [2, 3, 4]. This can lead to discomfort due to footfall-induced vibrations. This issue is particularly relevant to the all-steel floor system being discussed in this paper. The floor system, which is known as FastFloor, exhibit a combination of relatively light-weight and low inherent damping, which distinguishes them from traditional composite floors. As a result, meeting acceptable vibration performance criteria can be challenging, especially in modern building designs that often require longer beam and girder spans, as noted in [5].

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The FastFloor system discussed in this paper is a new type of modular floor system made entirely of structural steel, designed to increase construction speed by 30-50% compared to traditional composite floors [5, 6, 10]. A typical full-bay FastFloor module measures 30 ft $\times$ 40 ft on plan and consists of multiple wide-flange steel beams connected to a steel plate using bolts and staggered welding. The modular units of FastFloor are prefabricated off-site and then transported and erected on steel girders with field connections, allowing for easy installation and significant time and labor cost savings. One of the advantages of FastFloor is its reduced weight and lower carbon footprint [5]. Additionally, the system can accommodate a raised access floor (RAF) installed on top of the steel plate, which provides space for various mechanical system components.

In this study, the vibration serviceability of two FastFloor specimens is investigated using the high-frequency floor (HFF) procedure outlined in American Institute of Steel Construction's Design Guide 11 (AISC DG-11 or simply DG-11). The two specimens are compared, and their predicted vibration levels due to walking are assessed against the acceleration tolerance limits provided by DG-11.

FastFloor Specimens

Two 10 ft $\times$ 40 ft floor specimens (Figs. 1 and 2) were fabricated and transported to the West Virginia University for dynamic testing to evaluate their vibration serviceability. Each specimen represents a version of a single module designed to represent a relatively light (SM68_RAF) and a relatively heavy (SM94_RAF) structural framing [6]. The specimens consist of three W24 steel beams topped with a steel plate as shown in Table 1. The beams are spaced 5 ft on center with a clear span of 40 ft between supports (Fig. 3) and the plate is connected to the top flange of the beam using Shuriken blind bolts on one side and through staggered welds at the remaining beam-to-plate connection locations (Fig. 4).

To ensure lateral stability for dynamic testing, the W24 beams had web stiffeners around the support locations (Figs. 1 and 2) to prevent sideways movement. The beams were secured to the bearings using C-clamps (Fig. 1) or bolts (Fig. 2), simulating a pinned connection at support locations. Raised access floor (RAF) units were also installed on top of the steel plates, comprising 100 (2x2 ft) raised floor panels on each module supported by 125 steel pedestals with adjustable heights attached to the steel plate. The RAF units are shown in Figs. 1, 2 and 4.

Table 1 Components of the two FastFloor specimens investigated in this paper.

Specimen	SM68_RAF (Light specimen)	SM94_RAF (Heavy specimen)
Beam Sections	W24 $\times$ 68	W24 $\times$ 94
Plate Thickness	3/8 in.	1/2 in
RAF	Standard	Standard



Fig. 1 The light specimen (SM68_RAF) with W24x68 beams, a 3/8 in. thick plate, and raised access floor (RAF).



Fig. 2 The heavy specimen (SM94_RAF) with W24x94 beams, a 1/2 in. thick plate, and raised access floor (RAF).

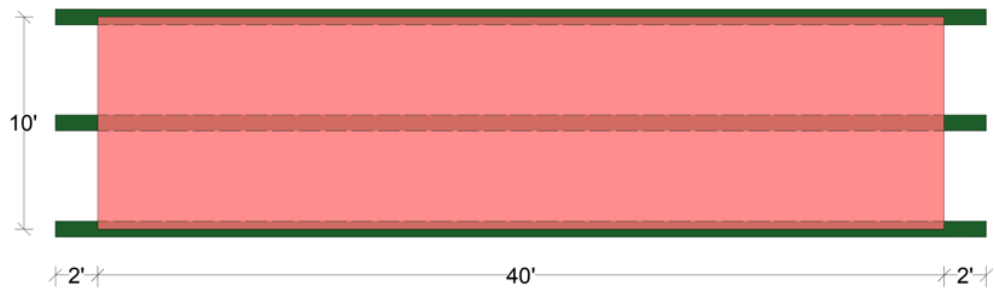


Fig. 3 Plan layout of a typical 10 ft $\times$ 40 ft single FastFloor module. The distance between the supports is 40 ft.

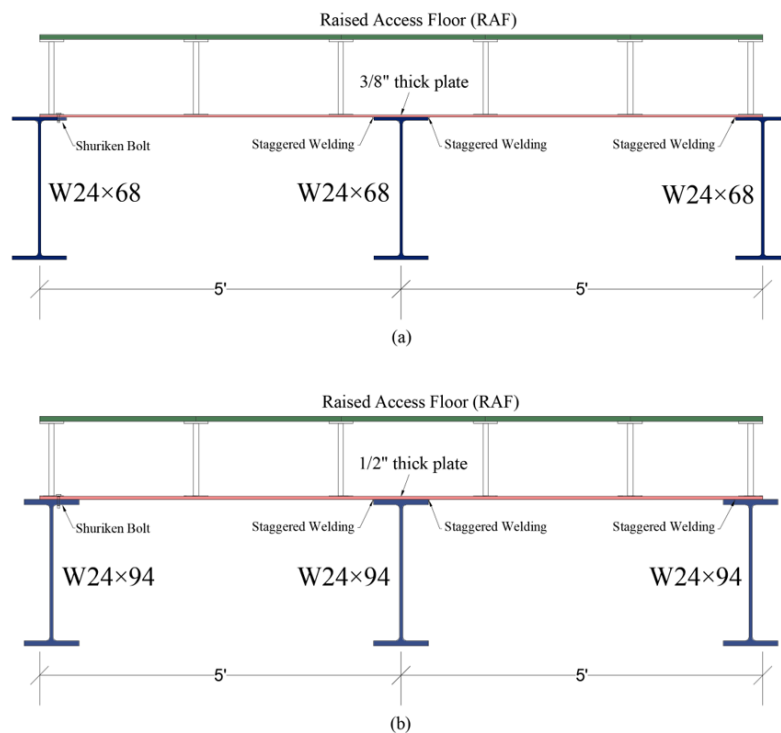


Fig. 4 Sectional view of the 10 ft $\times$ 40 ft single FastFloor modules used in vibration serviceability evaluation. (a) Section of the light specimen (SM68_RAF). (b) Section of the heavy specimen (SM94_RAF).

AISC Design Guide 11's high-frequency floor (HFF) procedure for peak acceleration prediction

DG-11 offers a methodology for predicting and evaluating floor vibrations caused by human activity, with a primary focus on walking. As walking is a common activity in most buildings, engineers need to predict vibration levels to ensure floor serviceability. To facilitate this, DG-11 classifies floors and pedestrian bridges into two categories based on their natural frequencies:

1. Low-Frequency Floors (LFFs): floors with at least one natural frequency below 9 Hz.
2. High-Frequency Floors (HFFs): floors with all natural frequencies above 9 Hz.

DG-11 provides two approaches for predicting peak acceleration responses to human activity: equation-based methods and finite-element analysis (FEA) based computational methods [1]. These tools enable engineers to assess floor vibration levels and design floors that meet specific vibration criteria. However, it is important to note that the prediction approaches provided in DG-11 were developed based on data collected from traditional composite floor systems consisting of a concrete deck on steel framing. This raises questions about the applicability of these methods to non-traditional floor systems, such as the all-steel FastFloor system discussed here in this paper.

To predict the peak acceleration caused by a single footstep, the effective impulse method is used, which is based on research by [7, 8]. However, the resulting peak acceleration value is not directly comparable to the sinusoidal peak acceleration limits, so it needs to be converted to an equivalent sinusoidal peak acceleration [1].

To implement the HFF procedure, a finite element (FE) model is created, and then the Frequency Response Function (FRF) magnitude is calculated for a unit load at the walking location (i) and acceleration at the occupant location (j) with the goal to identify the mode with the highest FRF magnitude. Engineering judgment is predominantly required to determine the optimal walking path and locations on the floor being assessed. The walking load location (i) should be placed near the mid-length of a 5-10 ft long unobstructed walking path, which is typically near the maximum mode shape value. If the affected occupant location is unknown, it should be placed near the maximum mode shape value. For regular floor bays or pedestrian bridges, the walking load and affected occupant locations can be conservatively placed at the mid-bay or midspan location [1]. The following steps are typically followed to apply the HFF procedure:

- 1) The natural frequencies and mode shapes at the walking and occupant locations are determined. The Frequency Response Function (FRF) is then used to identify the dominant mode, which corresponds to the highest FRF peak (not the mode with the lowest frequency).
- 2) The peak acceleration due to each mode (m) is calculated using Eq. (1), where $f_{n,m}$ represents the step frequency of mode m. The effective impulse ($I_{n,m}$) is computed using Eq. (2), assuming a body weight (Q) of 168 lbs. The step frequency (f_{step}) is determined by dividing the fundamental frequency by the harmonic number, as specified in Table 7-1 of DG-11

$$a_{p,m} = 2\pi f_{n,m} \phi_{i,m} \phi_{j,m} I_{eff,m} \quad (1)$$

$$I_{eff,m} = \left(\frac{f_{step}^{1.43}}{f_n^{1.30}} \right) \left(\frac{Q}{17.8} \right) \quad (2)$$

- 3) The individual time series responses for modes with frequencies below 20 Hz are superposed using Eq. (3), which represents the total response between two consecutive footsteps, to obtain the overall response of the floor system.

$$a(t) = \sum_{m=1}^N a_{p,m} e^{-2\pi f_{n,m} \beta t} \sin(2\pi f_{n,m} t) \quad (3)$$

- 4) The peak value of the total response after supposition is not directly compared to the established tolerance limits for sinusoidal peak acceleration in DG-11. For this reason, the equivalent sinusoidal peak acceleration (ESPA) is calculated using Eq. (4), which involves multiplying the root-mean-square (RMS) of the acceleration time history $a(t)$ between two footsteps by a factor of $\sqrt{2}$, representing the peak-to-RMS ratio for a sinusoidal signal.

$$a_{ESPA} = \sqrt{2} \sqrt{\frac{1}{T} \int_0^T [a(t)]^2 dt} \approx \sqrt{2} \sqrt{\frac{1}{N} \sum_{k=1}^N a_k^2} \quad (4)$$

N represents the number of discrete data points in the acceleration time series $a(t)$ for a single footstep which spans a time period T equal to $1/f_{step}$.

Evaluation of the two FastFloor specimens for vibration serviceability

The high-frequency floor (HFF) procedure described earlier is implemented to obtain the equivalent sinusoidal peak accelerations (a_{ESPA}) due to human walking excitations. These values are then compared to the tolerance limits provided in DG-11. First, finite element models of the two floor specimens are created in SAP2000 and updated to ensure that the experimental and analytical frequencies are in close agreement for multiple modes. Fig. 5 shows the 3D finite element model of the light specimen (SM68_RAF) in SAP2000 [9], highlighting the locations of the walking and occupant points that result in the highest mode shape at point (j).

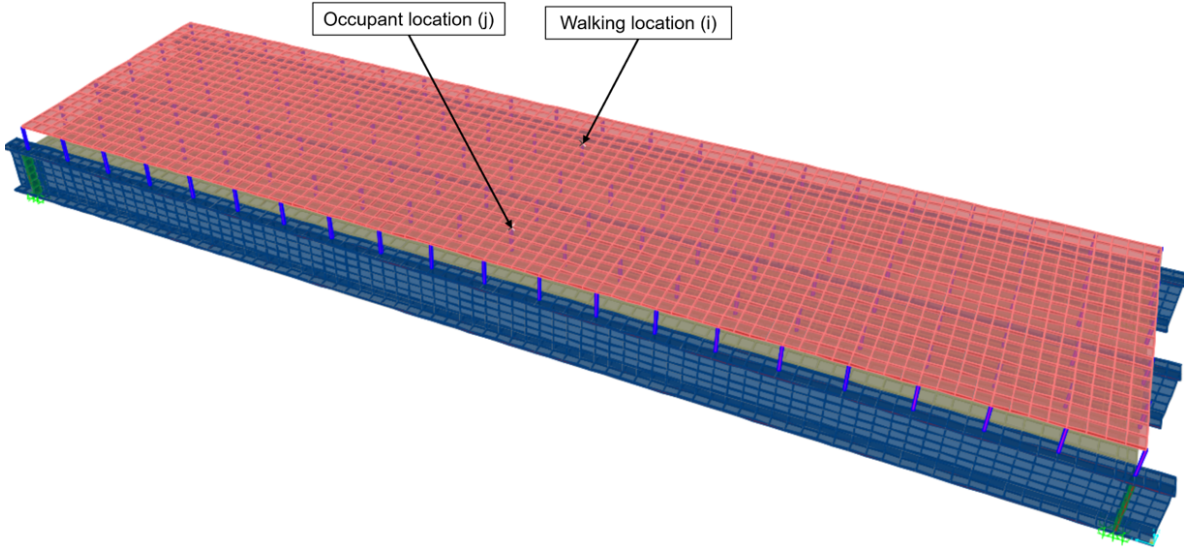


Fig. 5 3D view of the SAP2000 model of the floor specimen, showing the walking and occupant locations.

Notably, two damping ratios are employed in the calculations: $\beta = 0.75\%$ intended to represent the bare test conditions in the laboratory, while $\beta = 2.25\%$ intended to represent the in-service conditions. The dominant frequencies (frequencies of the highest FRF peak) are identified, and the corresponding step frequencies are determined, as presented in Table 2. Following the natural frequency calculations and mode shape determination at the walking and occupant locations, the peak responses and individual time series responses are computed for all modes up to 20 Hz. The time history plots for the dynamic responses are illustrated in Fig. 6 and Fig. 7 for the light specimen (SM68_RAF) and heavy specimen (SM94_RAF), respectively. These figures highlight the significant contribution of the dominant mode to the total response. For both damping ratios, the predicted peak accelerations are higher for SM68_RAF than for SM94_RAF.

Table 2 Dominant and step frequencies used in the HFF procedure.

Specimen	Light specimen(SM68_RAF)	Heavy specimen (SM94_RAF)
Dominant frequency (f_n)	19.25 Hz	16.93 Hz
Harmonic number (h)	9	8
Step frequency (f_{step})	2.14 Hz	2.12 Hz

To be able to compare walking acceleration results of Figs. 6 and 7 with the acceleration tolerance limits of DG-11, the peak accelerations are obtained using the dominant frequencies of the specimens, as shown in Fig. 8. The computed peak acceleration (a_{ESPA}) values are then compared to the obtained tolerance limits. A specimen is considered acceptable for vibration serviceability if its a_{ESPA} value is less than the tolerance limit. Table 3 presents a comparison of these values. The results indicate that the peak accelerations of the light specimen (SM68_RAF) tend to exceed the allowable limit for both damping ratios, rendering it unacceptable for vibration serviceability with the HFF procedure defined in DG-11. In contrast, the heavy specimen (SM94_RAF) has a peak acceleration that slightly exceeds the tolerance limit for $\beta = 0.75\%$ (bare test conditions), but is acceptable for $\beta = 2.25\%$ (in-service conditions), per the same procedure.

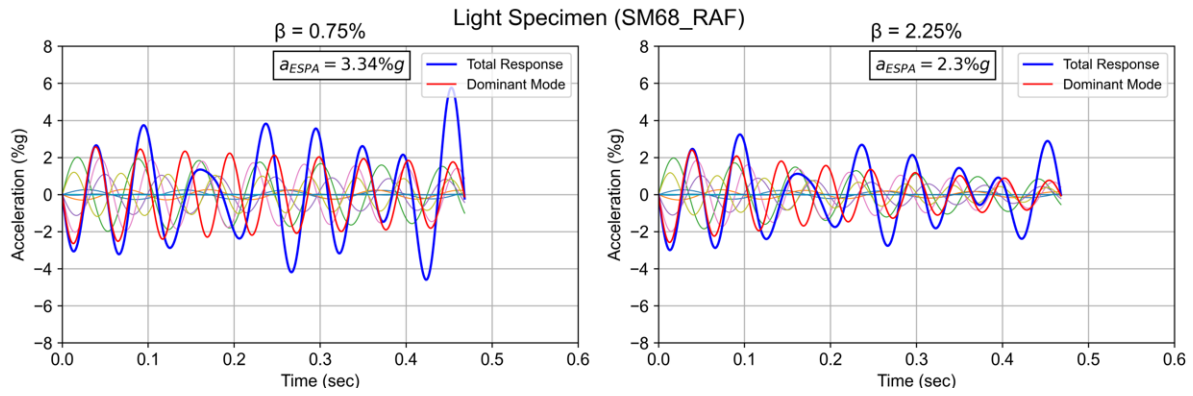


Fig. 6 Acceleration responses and a_{ESPA} values for the light specimen (SM68_RAF) due to walking excitations with laboratory-condition (left) and in-service (right) damping ratios. Dynamic responses of non-dominant modes are not indicated on the legend.

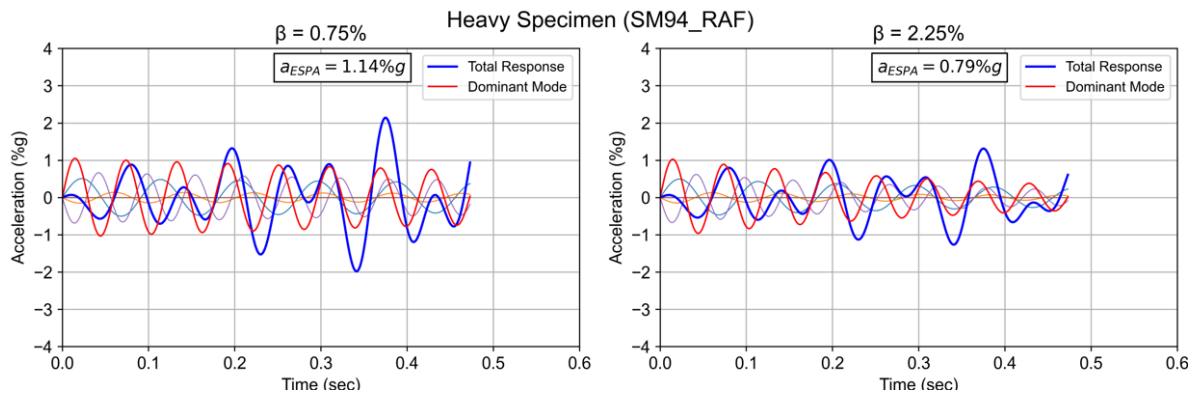


Fig. 7 Acceleration responses and a_{ESPA} values for the heavy specimen (SM94_RAF) due to walking with laboratory-condition (left) and in-service (right) damping ratios. Dynamic responses of non-dominant modes are not indicated on the legend.

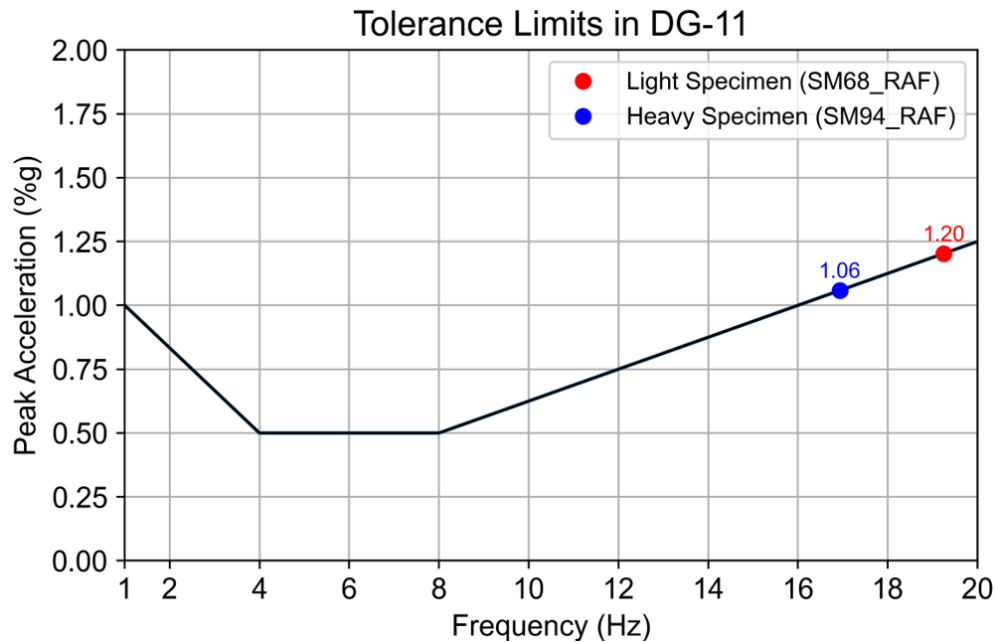


Fig. 8 Acceleration tolerance limits according to DG-11 for offices and residences, calculated for the two specimens.

Table 3 Comparison of predicted peak accelerations with DG-11 tolerance limits for acceptability evaluation.

Specimen	Light specimen(SM68.RAF)		Heavy specimen(SM94.RAF)	
Tolerance Limit	1.20%g		1.06%g	
Damping	$\beta = 0.75\%$	$\beta = 2.25\%$	$\beta = 0.75\%$	$\beta = 2.25\%$
a_{ESPA}	3.34%g	3.34%g	1.14%g	0.79%g
Acceptability	Not acceptable	Not acceptable	Not acceptable	Acceptable

Conclusions

This study presented the vibration serviceability investigation of two all-steel floor modules, utilizing the high-frequency floor (HFF) procedure described in AISC Design Guide 11 (DG-11). Finite element models are created to implement the HFF procedure, which involves determining the walking frequency, calculation of peak acceleration responses, and then equivalent sinusoidal peak accelerations (ESPA). Two damping ratios were considered for bare floor laboratory conditions (0.75%) and in-service floor conditions (2.25%). The vibration serviceability of the two specimens per HFF procedure is investigated by comparing the computed ESPA values to the acceleration tolerance limits provided in DG-11. As a result, the peak accelerations determined for the light specimen (SM68.RAF) tend to exceed the allowable limit for both damping ratios, while the peak accelerations determined for heavy specimen (SM94.RAF) remain below the limit for in-service conditions.

It is important to note that the analysis presented in this paper has limitations. The results cannot be considered for the overall acceptability of the floor concept, due to several reasons. First, the evaluated specimens are one module only (10 ft × 40 ft) instead of a larger floor bay (30 ft × 40 ft) which requires three of the modules carried on girders. Second, the analysis of the 10 ft × 40 ft modules as described in this paper neglects the influence of the girders on the frequencies and overall dynamic response. However, the findings of this study provide valuable insights into the vibration serviceability evaluation of light-weight floor systems as well as the applicability of the HFF procedure to the non-traditional floor systems.

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Chapter 11

It's All About That Bass! Isolating For Low-Frequency Ground-Borne Vibrations

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Abstract Floor motions can disturb occupants, leading to frequent complaints and loss of functionality. In especially vibration-sensitive facilities, this issue can be more critical, as high-resolution imaging equipment with stringent vibration criteria are often employed. As urban intensification increases, new buildings are being constructed in close proximity to existing vibration sources, such as rail lines and busy roads. In many cases due to a number of site-specific factors, the low-frequency content of the ground-borne signal can be significant, which can only be properly mitigated with very low-frequency isolation systems. These systems can be extremely challenging to design, with limited options available on the commercial market for components.

In this paper, two case studies are presented of buildings that required significant effort to isolate for low-frequency rail-induced vibrations. The first project involved a low-rise residential building directly adjacent to a heavily used freight and passenger rail line. At this specific location, a bolted track joint was present with a significant misalignment which caused intense low-frequency impulsive vibrations to be felt on the project site. Base isolation was not reasonably feasible due to the foundation type required for the local soil conditions, so a complex isolation system was devised that would be installed below the second floor of the structure. The second project involved several townhouse blocks that were constructed immediately adjacent to a freight rail line, where site-specific soil conditions led to very low-frequency ground vibrations. Based on measurements taken on site it was determined that the foundation for the crash wall had significantly reduced the vibrations, but that whole building isolation was still required. The main challenge overcome in this case was designing a system that would have a low-frequency isolation system while being constrained by the relatively light-weight wood-framed constructions above.

Keywords Vibration Serviceability · Ground-borne Vibration · Vibration Isolation · Vibration Measurements · Model Validation · Sensitive Floors · Dynamic Loading

Introduction

As urban intensification increases, new buildings are being constructed in close proximity to existing vibration sources, such as rail lines and busy roads. In many cases due to a number of site-specific factors, the low-frequency content of the ground-borne signal can be significant, which can only be properly mitigated with very low-frequency isolation systems. These systems can be extremely challenging to design, with limited options available on the commercial market for components.

Vibration is generated by the contact between train wheels and the steel rails. The resulting vibration is caused by the inherent roughness and irregularities in the rail, which forces the wheels to move up and down, thus imparting a force in the rail. The vibration generated by the resulting forces propagates through the underlying structure of the transit system that supports the track and subsequently into the surrounding soil until it encounters nearby buildings, at which point the vibration is transmitted into a building through its foundation.

Ground vibration produced by rail transit systems can be annoying to human occupants in residential, office, and commercial buildings when they perceive some combination of feelable vibration, re-radiated sound, and vibration-induced rattling of building interior items. Passing railway traffic also can be disruptive to operation of sensitive equipment in manufacturing, medical facilities, and recording studios.

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Vehicles running on tracks give rise to vibration with predominant components in the frequency range of 4 Hz to 250 Hz. Vibration propagates parallel to the surface as wave modes of the layered soil and has a low rate of attenuation with distance. In the frequency range of 4 Hz to 80 Hz, the vibration may be perceived as whole-body vibration in buildings near to the line. At higher frequencies, in the range from about 20 Hz to about 250 Hz, ground vibration may excite bending resonances in the floors and walls of buildings, which then radiate a rumbling noise directly into the rooms. A third source of disturbance is due to movement of building objects, shaking of items on shelves or hanging on walls or by the rattling of windowpanes, glassware, and computer screens. In all these cases the problem of ground-borne vibration is important at frequencies typically up to about 250 Hz.

Two case studies are presented in this paper of structures that have vibration-sensitive occupancies that are directly adjacent to rail lines where measurement indicated that without significant modification to the structure, significant low-frequency ground-borne vibrations would render the spaces inadequate for their intended uses.

Vibration Criteria

For human comfort, vibration criteria are normally expressed as the root mean square (RMS) response of each one-third octave band from 1 Hz to 80 Hz [1]. For sensitive equipment, the criteria may be expressed in one-third octave bands, or other formats, including power spectral densities, peak-to-peak levels, etc. Over the past 35 years, generic vibration limits have been developed which provide frequency-dependent sensitivities for wide classes of equipment and are used extensively in design of healthcare and research facilities [2]. These vibration criterion (VC) curves are internationally accepted as a basis for designing and evaluating the performance of vibration sensitive equipment and the structures that support them. The VC curves range between Workshop (least stringent) through VC-G (most stringent). Most laboratories target at least VC-A as a maximum baseline for the majority of the facility, with certain areas requiring more stringent criteria, depending on the expected use of the space.

These curves were originally based on the ISO 2631-2 (1989) [3] base curve for human response to whole body vibration, which is considered the threshold of human perception, but have since evolved. The ISO base curve is often referred to as the ISO-Operating Room criteria. The above noted criteria are also specified as RMS velocities in one-third octave bands. The VC curves should not be used to replace manufacturers' specifications for vibration requirements, but are beneficial where manufacturers' specifications are non-existent, incomplete, or where specific equipment has not yet been selected.

The VC and ISO curves will be primarily used as a basis of evaluating the performance of the structures in this paper.

Case Study #1 – Low-Rise Residential Building

The first case study is a project that was to be constructed in the greater Chicago area, directly adjacent to a major commuter and freight rail corridor, located to the west of the site. The existing site contained an abandoned commercial strip mall. Initial screening-level measurements had indicated an exceedance of the ISO Residential Night criteria near the western property boundaries, so a more comprehensive measurement scheme was devised to determine the attenuation of vibrations into the site (Figure 1).

A total of three freight trains and four commuter trains pass-bys were measured. Sample one-third octave band spectral plots at the measurement locations due to freight train pass-by are shown in Figure 2.

The measurement results show that:

- Freight Train Pass by – the measured vibration levels exceeded the recommended “Residential Day” and “Residential Night” limit at Location #1 and Location #2. At Location #3, the measured vibration exceeded the “Residential Night” limit. The frequency of peak ground-borne vibration was measured to be at 6.3 Hz as seen in Figure 2. It should be noted that this is a considerably lower frequency than what is typically seen for rail vibration, and was attributed to a misaligned bolted joint in the rails adjacent to the site, which caused a large impulsive vibration to be generated.
- Commuter Train Pass by – the measured vibration levels at Location#1 exceeded the “Residential Day” and “Residential Night” guideline limit. At Location#2 and Location#3, the measured vibration levels exceeded the “Residential Night” limit. The frequency of peak ground-borne vibration was 20 Hz, which was less of a concern that the peak frequencies observed for the freight trains.

Additionally, the maximum structure-borne noise due to freight train pass-by was predicted to be 39 dBA which exceeded the maximum allowable sound level of 35 dBA recommended by the industry-standard guidelines for residential building.

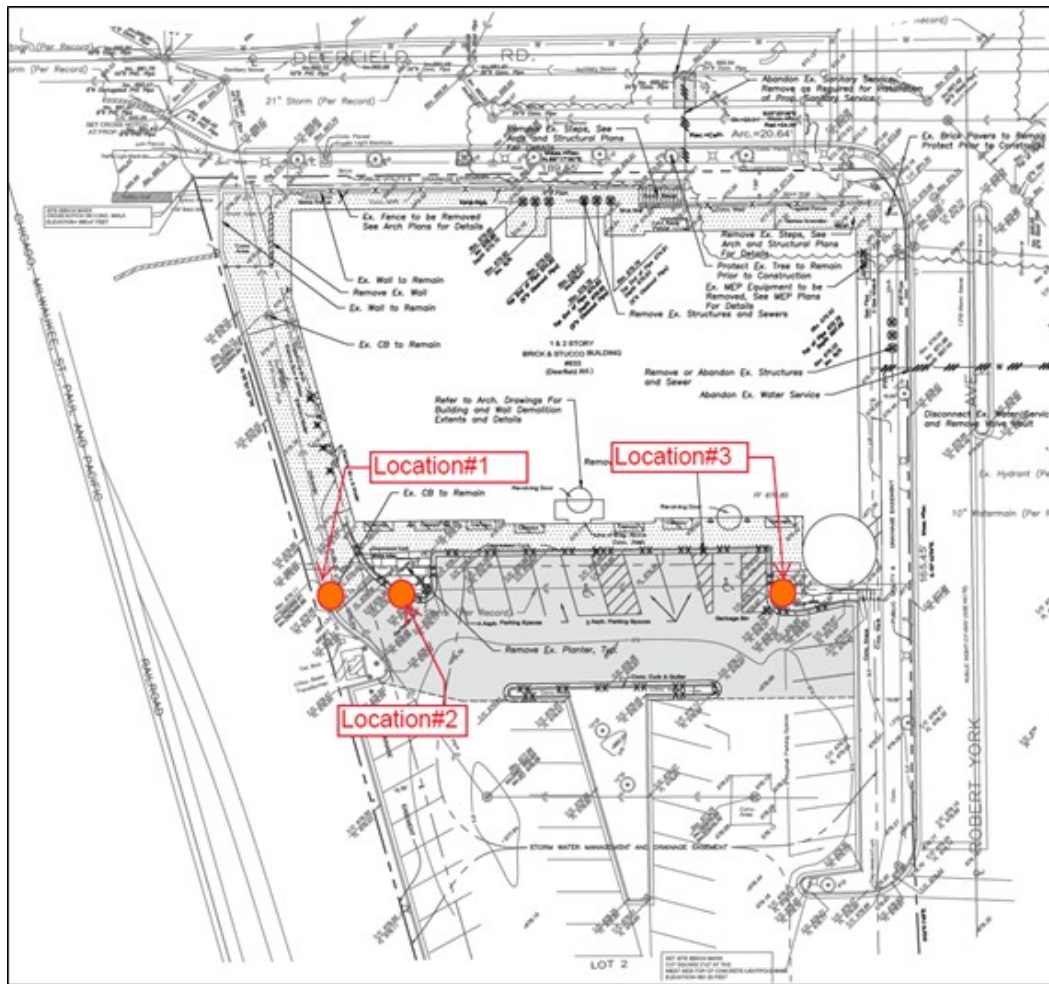


Fig. 1 Vibration Measurement Locations.

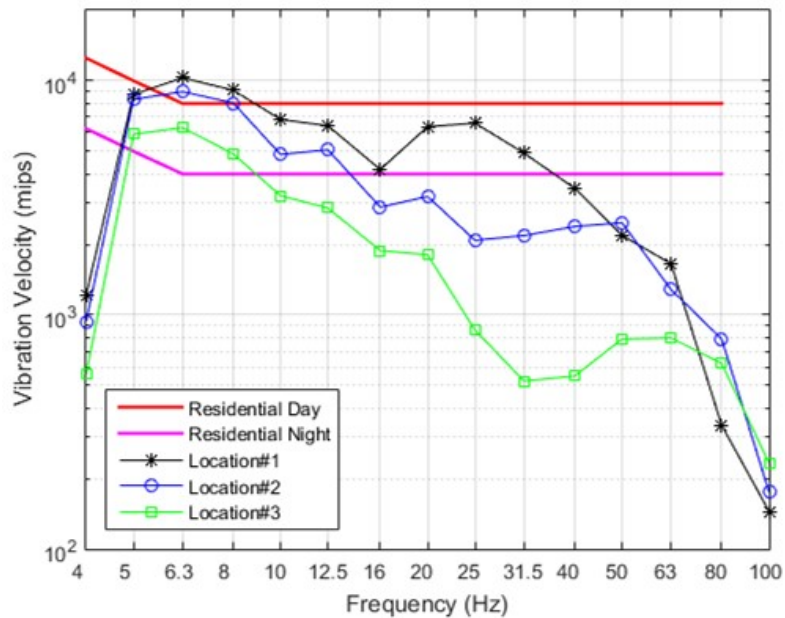


Fig. 2 One-Third Octave Band Spectral Level – Freight Train Pass-by.

Due to commuter train pass-by, the maximum sound level was predicted to be 34 dBA which is marginally lower than the recommended guideline limit.

Since the measured vibration levels (and predicted structure-borne noise inside the residential spaces) exceeded the recommended guideline limits, vibration mitigation would be required. A Finite Element (FE) model was then created in order to determine the level of attenuation in the building through the foundation, and to determine the optimal location for isolation devices. The soil conditions on the site were poor, requiring the use of aggregate piles. This complicated the ability to use more traditional base isolation due to the interface of these piles, the ground floor slab and the bearing soil around them. Since the ground floor of the structure was going to have a commercial occupancy, and was thus exempt from the Residential criteria, it was decided to install the isolation systems on the top of the first-floor columns, directly below the second floor structure (Figure 3). Owing to the low frequency of the ground-borne excitation, extremely low-frequency pre-compressed mechanical spring isolators were required, having a natural frequency of 3.5 Hz.

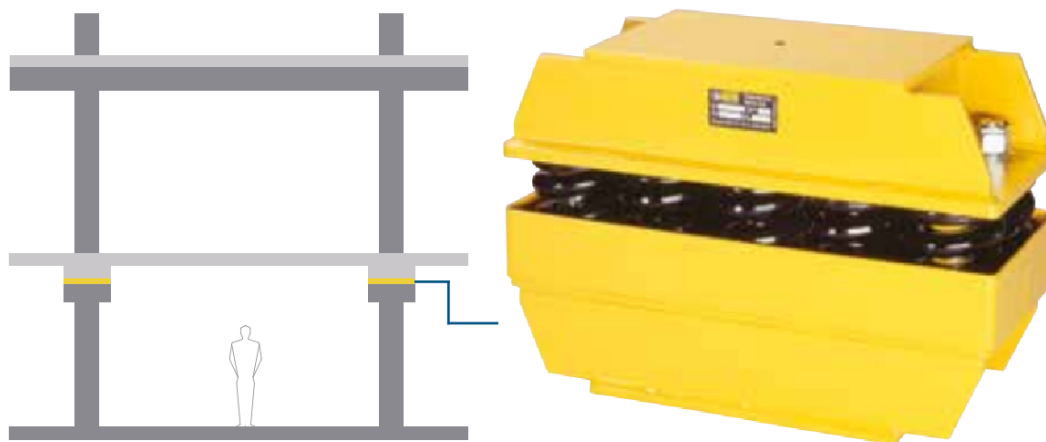


Fig. 3 Schematic Location and Configuration of Isolators.

It should be noted that once this phase of the design was completed, the project was suddenly cancelled to financial reasons stemming from the increased cost of construction materials due to the Covid pandemic.

Case Study #2 – Townhouse Residential Complex

The project used for Case Study #2 is a proposed residential development, consisting of eleven townhouse blocks. A Canadian National Railway (CNR) track is located immediately adjacent to the north side of the site. In order to establish the vibration levels and spectrum caused by train passes, continuous vibration measurements were recorded at two setback distances from the tracks, corresponding to the two closest rows of townhouses (Figure 4).

Currently, there are no guidelines for the impact of railway vibration in the land use approval process in Ontario. However, in May 2013, the Federation of Canadian Municipalities (FCM) and the Railway Association of Canada (RAC) issued "Guidelines for New Development in Proximity to Railway Operations" [4] to address developments in close proximity to railway operations. The FCM/RAC guidelines identify dwellings within 75 meters from railways alignments as susceptible to vibration impact and recommend an overall maximum vibration limit of 0.14 mm/sec root-mean-square (RMS). The limit should be based on a one-second averaging time between 4 Hz and 200 Hz. Mitigation is prescribed when limit is exceeded.

It is our understanding that CNR has adopted the use of this guideline and the criterion for train vibration on this proposed new residential development is an overall vertical vibration level of 0.14 mm/sec (RMS) between 4 Hz and 200 Hz. This is comparable to the ISO Residential Night criteria described in Section 2.

Eight train passes were identified during the measurement period. Time histories of the vibration velocity produced by each train pass-by were band-pass filtered between 4 Hz and 200 Hz, and then processed using an RMS (root-mean-square) averaging routine with a time constant of one second. The maximum overall vertical vibration level from each train pass at each measurement location are shown in Table 1. The CNR criteria was exceeded by the average all train measurements at Location #1 (block A, B, and C). The criteria was met by the average of all trains at Location #2 (block J, K, L, and D). Spectral analysis of the results indicated that the peak frequencies of measured responses were in the 8-10 Hz range, which is considered to be lower than typical rail-induced vibration excitation.

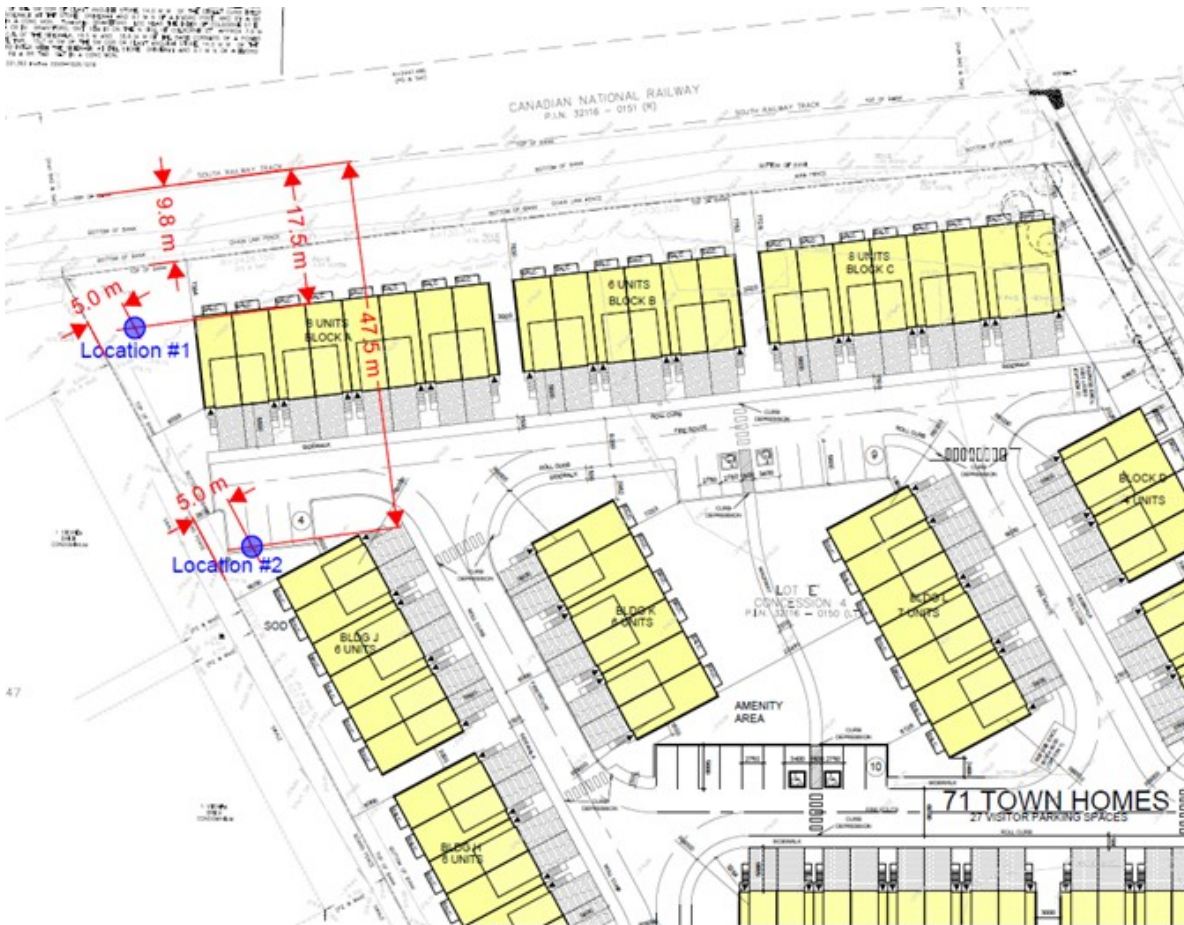


Fig. 4 Vibration Measurement Locations.

Table 1 Measured Vibration Levels for Train Passes.

Date	Time	Maximum Overall Vertical Vibration Level (mm/s RMS)	
		Location #1 (17.5m from tracks)	Location #2 (47.5m from tracks)
Feb. 15, 2021	13:18	0.21	0.13
Feb. 15, 2021	14:23	0.20	0.13
Feb. 17, 2021	10:34	0.22	0.15
Feb. 17, 2021	11:55	0.18	0.12
Feb. 19, 2021	11:54	0.23	0.12
Feb. 19, 2021	12:42	0.23	0.15
Feb. 22, 2021	11:03	0.20	0.13
Feb. 22, 2021	11:48	0.19	0.11
Feb. 24, 2021	14:19	0.19	0.13
Feb. 24, 2021	15:12	-	0.11
Feb. 26, 2021	10:34	0.18	0.12
Feb. 26, 2021	11:35	0.22	0.15
Average		0.20	0.13

When a projected vibration level exceeds the applicable criterion, it is necessary to consider vibration mitigation. Potential mitigation measures are evaluated to determine their effectiveness in reducing the projected vibration and reradiated noise. A mitigation measure's performance is referred to as its Insertion Loss (IL) in dB, which represents the difference in vibration with mitigation relative to the vibration without mitigation.

There are a number of differences in the dynamic behavior of elastomeric pads and steel springs. The most important difference is the natural frequency at which can be achieved under structural loading conditions. Elastomeric pads have typically been used in building isolation systems with natural frequencies in 5 to 15Hz. Steel spring systems for building isolation are most cost efficient in the range of 3Hz to 5Hz natural frequency. Figure 4 illustrate the achievable insertion loss (vibration attenuation) of various isolation frequencies.

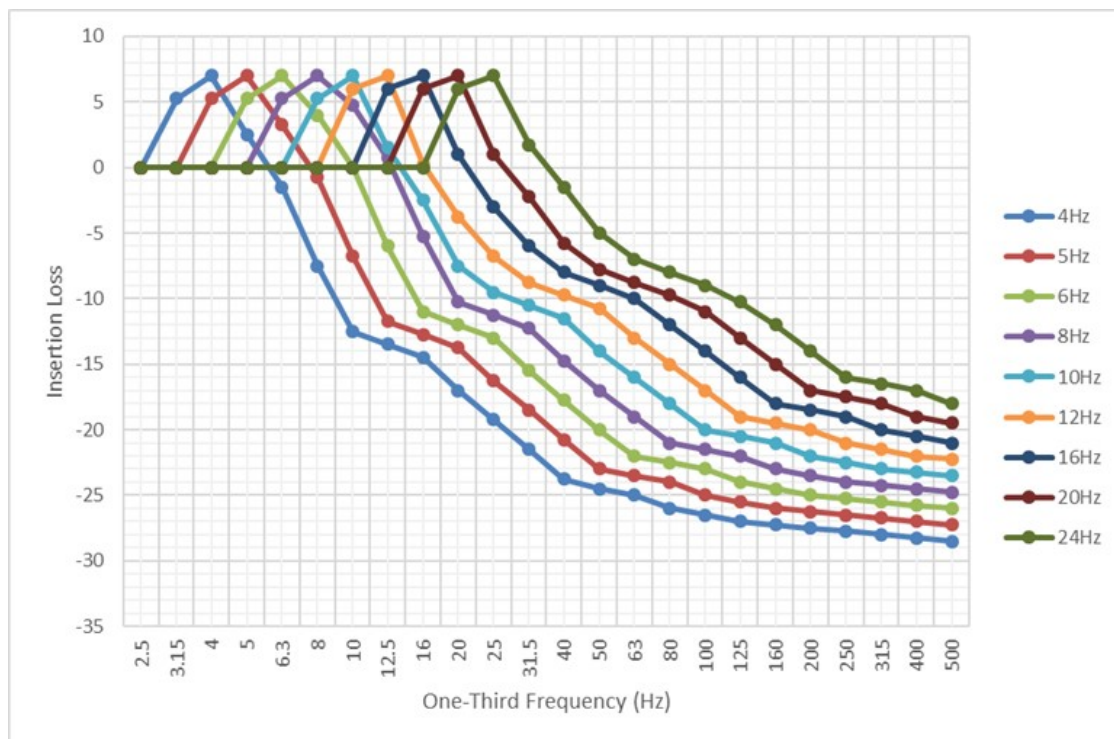


Fig. 5 Insertion Loss for Various Isolation Frequencies.

The expected performance of the base isolation was analyzed using the one-third octave vibration spectra of the measurements and adjusted by foundation coupling loss and floor amplification factors. Table 2 provides a summary of vibration predictions of the 5 Hz system. The base isolation mitigates the vibration levels to an average of 0.09 mm/sec (RMS) which is below the vibration limit.

Table 2 Measured Vibration Levels for Train Passes and Predictions of Isolation System.

Date	Time	Maximum Overall Vertical Vibration Level (mm/s RMS)	
		As Measured on Soil	With 5 Hz Base Isolation
Feb. 15, 2021	13:18	0.21	0.11
Feb. 15, 2021	14:23	0.20	0.12
Feb. 17, 2021	10:34	0.22	0.11
Feb. 17, 2021	11:55	0.18	0.05
Feb. 19, 2021	11:54	0.23	0.05
Feb. 19, 2021	12:42	0.23	0.05
Feb. 22, 2021	11:03	0.2	0.05
Feb. 22, 2021	11:48	0.19	0.14
Feb. 24, 2021	14:19	0.19	0.13
Feb. 24, 2021	15:12	-	-
Feb. 26, 2021	10:34	0.18	0.10
Feb. 26, 2021	11:35	0.22	0.12
Average		0.20	0.09

Difficulty was experienced in locating a commercially available neoprene-based isolation material that would meet the 5 Hz requirements for an isolation frequency. After an exhaustive search, one supplier was located that was able to provide the required material. Additional complications arose in the proper implementation of the selected pads to ensure no flanking pathways were created, given the complexity of physically placing the isolators in a townhouse structure. Figure 6 shows a typical detail for one isolator location. Ultimately, the isolators were successfully implemented.

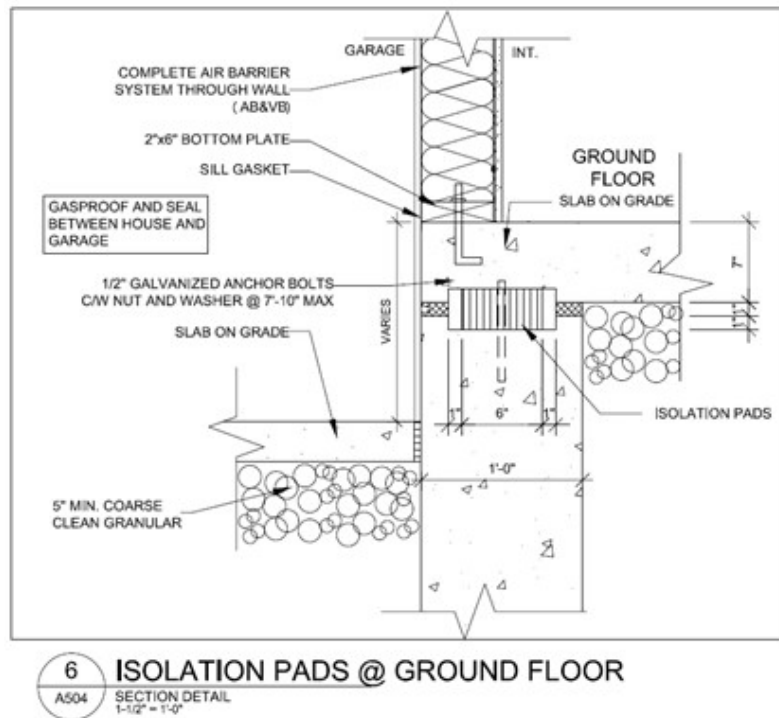


Fig. 6 Typical Detail for Isolators at the Ground Floor Level.

Conclusions

Two case studies have been presented that illustrate the significant complications arising when rail-induced vibrations have significant low-frequency components.

The following conclusions have been made:

1. In both cases the ground-borne vibrations had significant energy in the low-frequency bands.
2. This low-frequency energy requires extremely low isolation frequency materials and systems that can be challenging to procure
3. Unusual foundation configurations further complicates the appropriate integration into the structural system.
4. With careful design and detailing, it is possible to accommodate these unusual systems to successfully isolate the structures.

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Chapter 12

Machine Learning based Automated Operational Modal Analysis for monitoring of large infrastructure

André Tavares, Susana Dias, Marco Barreira, Emilio Di Lorenzo, Bram Cornelis, Bart Peeters, Carlos Moutinho, Filipe Magalhães, Wim Desmet, and Konstantinos Gryllias

Abstract Modal analysis is a valuable technique for understanding a structure's natural vibration characteristics by analyzing modal parameters such as natural frequencies, damping ratios, and mode shapes. However, traditional modal analysis is time-consuming, subjective, and challenging with complex structures, making long-term monitoring impractical. To address these challenges, this work introduces an Automated Operational Modal Analysis (AOMA) methodology using DBSCAN, a density-based machine learning clustering technique, combined with modal verification steps inspired by expert judgment. This approach minimizes user interaction, reduces human error and decreases processing time, facilitating long-term structural monitoring. The AOMA methodology was applied to the Infante Dom Henrique bridge in Porto and an onshore wind turbine for long-term monitoring. A modal tracking procedure was implemented to correlate a reference set of modes for each structure throughout the measurement campaigns. The results demonstrated high accuracy and successful long-term monitoring, confirming the applicability of this AOMA methodology to different structures.

Keywords Operational modal analysis · Arch-bridge · Onshore wind turbine · Automated modal analysis · Machine learning

Introduction

Operational Modal Analysis (OMA) is a method employed in structural dynamics to identify the modal parameters of a structure (natural frequencies, damping ratios, and mode shapes) while it is functioning. Unlike conventional modal analysis, which necessitates the use of external excitation methods such as shakers or impact hammers, OMA utilizes the natural vibrations produced by the structure during its normal operation. These vibrations can result from environmental influences, including wind, waves, traffic, machinery, or human activities. OMA is especially advantageous for large structures, such as bridges [1], buildings [2], or wind turbines [3], where applying artificial excitation may be impractical or pose safety risks.

The automation of modal analysis allows to process the large measurement campaigns of structural monitoring without user interaction and with repeatability. In recent years, several automated methods for conducting modal analysis have been developed. Most of these automated modal analysis (AMA) techniques utilize clustering algorithms due to their effectiveness as versatile unsupervised learning tools. Applications of these methods have been implemented using various clustering approaches, including fuzzy clustering [4, 5], hierarchical clustering [6, 7, 8], and density-based clustering [9, 10].

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This study introduces an automated modal analysis (AMA) methodology that employs the Density-Based Spatial Clustering of Applications with Noise (DBSCAN) algorithm, along with mode verification steps based on the expert judgment of modal analysis. The methodology is applied to monitor two different structures: a) the Infante Dom Henrique bridge in Porto - a 280m arch-bridge monitored using two accelerometers and subjected to regular traffic, and b) a 2.0MW wind turbine with a 80m hub height operating in different environmental and operational regimes.

Automated Modal Analysis

Automated Modal Analysis (AMA) consists of three main stages: 1) Modal Parameter Estimation (MPE), where the stabilization diagram is calculated; 2) Automated Modal Parameter Selection (AMPS), which focuses on automatically identifying and selecting poles from the stabilization diagram that best represent the physical resonance modes of the structure; and 3) Modal Tracking, which enables the continuous observation of a set of reference modes over time and throughout extended measurement periods for a specific structure. The third stage, Modal Tracking, is particularly used for long-term structural monitoring.

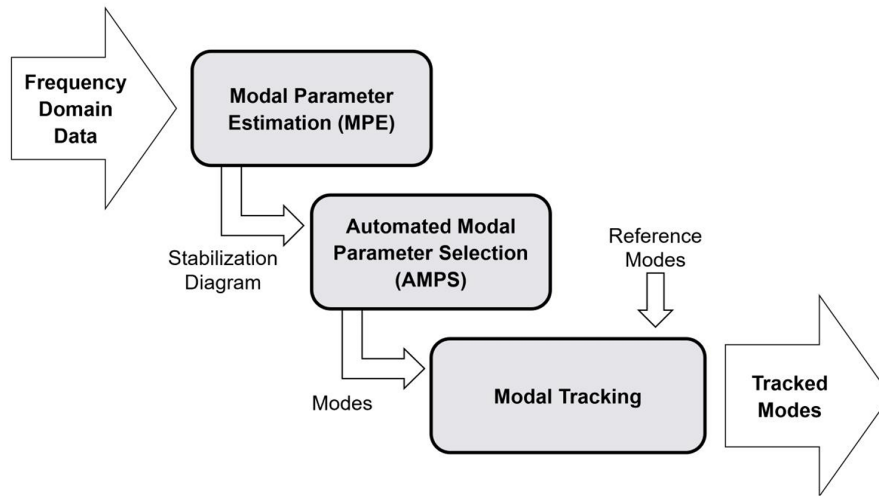


Fig. 1 Automated Modal Analysis (AMA) process for long-term monitoring.

Automated Modal Parameter Selection (AMPS)

The Automated Modal Parameter Selection (AMPS) process, illustrated in Figure 2, involves three main steps. Initially, clusters are formed based on the poles shown in the stabilization diagram, with the estimated modal parameters serving as input. For this task, a density-based clustering algorithm known as DBSCAN is used, which also filters out poles identified as noise during the initial phase. Following this, the resonance modes identified through clustering are subjected to two verification stages. The first ensures that none of the modes are split into multiple clusters, especially in complex structures where certain conditions might apply. The second verification step seeks to detect and remove any spurious or numerical modes by examining the cluster characteristics and their influence on the final modal model synthesis.

Using natural frequencies (f) and damping ratios (ξ) as inputs, the initial clusters of poles in a stabilization diagram are created using the DBSCAN algorithm. A key step in this process is calculating the distance between poles, which is based on the absolute differences in normalized frequency and damping to account for their differing magnitudes and units. To ensure the AMPS methodology is adaptable to various structural characteristics, such as different modal densities and model sizes, DBSCAN hyperparameters are carefully selected. The minimum number of points (MinPts) is set at one-sixteenth of the model size, with a lower limit of 3, while the epsilon distance (ϵ) is defined as 0.06, corresponding to tolerances of 1% for frequency and 5% for damping to classify stable poles. In this way, the distance between two points, p_1 and p_2 , can be described by equation 1.

$$d = d_f + d_\xi = \frac{|f_{p_1} - f_{p_2}|}{f_{p_1}} + \frac{|\xi_{p_1} - \xi_{p_2}|}{\xi_{p_1}} \quad (1)$$

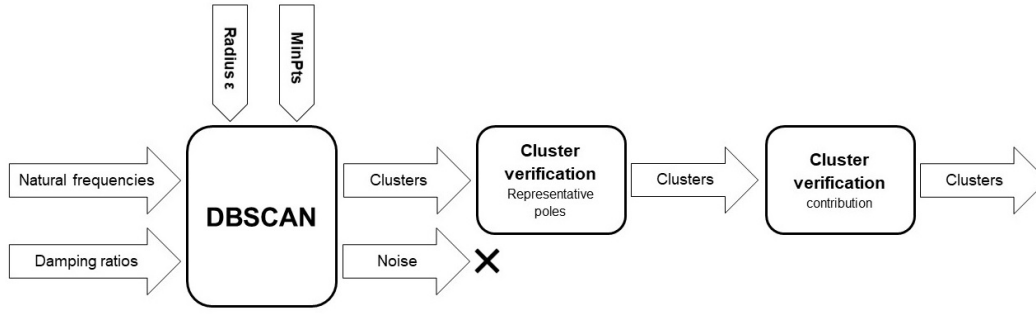


Fig. 2 Methodology for Automated Operational Modal Analysis (AOMA).

DBSCAN assigns each pole either to a specific cluster or labels it as noise if it does not meet the density criterion. After the initial clusters are formed, two verification steps are conducted. The first ensures that no single mode has been divided into multiple clusters, while the second eliminates any spurious modes, which could be numerical artifacts from the stabilization diagram rather than actual physical modes. In the first step, a similarity check is performed between the representative poles of each cluster (the pole closest to the median frequency and damping of the cluster is chosen as the representative). This check ensures that representative poles fall within a 2% frequency range and a 20% Modal Assurance Criterion (MAC) range between modal participation factors (L) (see equation 2). If these conditions are met, the corresponding clusters are merged. The second step evaluates how each representative pole contributes to the final modal model synthesis. If a representative pole has a marginal contribution to the modal model fit or results in a highly complex mode shape (suggesting spurious poles), the corresponding cluster is flagged as spurious and excluded from the final mode set.

$$d_{MAC(L)} = 1 - \frac{|L_{p_1} L_{p_2}|^2}{(L_{p_1} L_{p_1})(L_{p_2} L_{p_2})} \quad (2)$$

Modal tracking

Modal tracking is the process of identifying similarities between resonance modes by assessing their modal parameters. This technique is particularly important for long-term structural monitoring, as it helps establish a connection between the resonance modes calculated for specific measurements and a set of reference modes evaluated over time. By tracking the evolution of modal parameters, engineers can detect any structural changes, thereby gaining insights into the health and behavior of the structure under observation.

Modal tracking is a crucial step in the overall Automated Modal Analysis (AMA) process, following Modal Parameter Estimation (MPE) and Automated Modal Parameter Selection (AMPS). Once the stabilization diagram is created, the AMPS step automatically evaluates the poles to derive a final set of resonance modes. For structures monitored over multiple measurements, modal tracking is employed to follow a set of reference modes over time by calculating the distances between respective modal parameters. A correlation between modes is established if both the natural frequency (equation 3) and Modal Assurance Criterion (MAC) for mode shapes (ϕ) (equation 4) distances exceed certain thresholds, which may vary based on the specific application and the number of Degrees-of-Freedom (DOF) measured.

$$d_f = \frac{|f_{p_1} - f_{p_2}|}{f_{p_1}} \quad (3)$$

$$d_{MAC(\phi)} = 1 - \frac{|\phi_{p_1} \phi_{p_2}|^2}{(\phi_{p_1} \phi_{p_1})(\phi_{p_2} \phi_{p_2})} \quad (4)$$

Infante Dom Henrique bridge

In this section, the Infante Dom Henrique bridge, located in Porto, Portugal, will be analysed with the AOMA procedure, as explained in the "Automated Modal Analysis" section. These developments are further detailed in [11].

The Infante Dom Henrique Bridge, a concrete arch bridge, consists of two interacting components: a highly rigid pre-stressed reinforced concrete box girder, 4.50 meters deep, supported by a very shallow and slender reinforced concrete arch,

1.50 meters thick, as shown in figure 3. The arch and the deck converge in the 70 meters central segments, forming a 6m-deep box beam. While the arch maintains a constant thickness, its width gradually increases from 10 meters at the central span to 20 meters at the springs. The arch spans 280m between the abutments and rises 25 meters at its crown, giving it a shallowness ratio greater than 11:1. The structure behaves as a beam bridge, defined by the abutments and intermediate elastic supports 35 meters apart, due to the high stiffness of the deck compared to the slenderness of the arch [12].

Dataset characterization

The monitoring system installed on the bridge consists of two acceleration nodes developed at FEUP, located in sections S2 and S4, as shown in figure 3. These sensors are cost-effective and have the advantage of being battery-powered, which eliminates the need for installing cables throughout the structure.

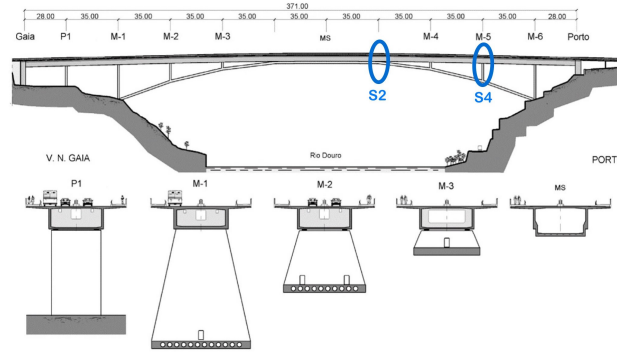


Fig. 3 Elevation and cross-sections of the bridge, including position of the monitoring system.

The modules in operation are built around an Arduino-based microprocessor, which processes the data acquired by an ADXL355 MEMS accelerometer from Analog Devices. The sampling frequency for this application was set at 60 Hz. Data was collected in 10-minute intervals and stored locally on a microSD card [13].

Different operational conditions

The Infante Dom Henrique Bridge serves as one of the primary entrances to Porto's city center. As a result, it experiences a high volume of vehicle traffic, particularly during the morning and afternoon rush hours. This leads to two distinct periods, as shown in figure 4: one at night, characterized by lower excitation, and the other during morning and afternoon rush hours, marked by increased traffic on the bridge.

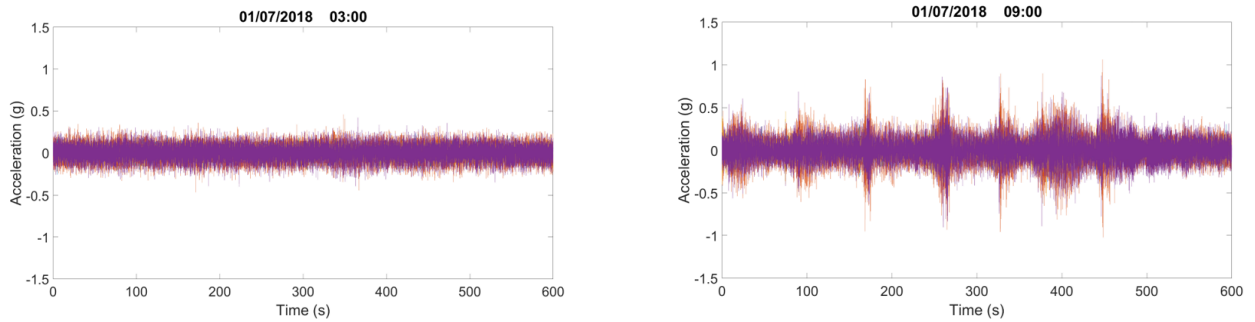


Fig. 4 Acceleration time series during the night (left) and the morning (right).

Automated Modal Analysis results

The frequency-domain Modal Parameter Estimation (MPE) technique, Polymax [14], is used throughout this work to compute the stabilization diagrams. In this section, the stabilization diagrams processed using the Automated Modal Parameter Selection (AMPS) methodology, as explained in the previous section, will be analyzed for the bridge dataset. Following this,

the Automated Modal Tracking methodology, also presented before, will be applied to the complete dataset for the month of July 2018.

Stabilization diagram

Figure 5 presents a stabilization diagram processed using the AMPS methodology. The measurement shown in this diagram corresponds to a rush hour period, specifically at 5:10 PM, with a sampling duration of 10 minutes. The stabilization diagrams were obtained using a model size of 152, time lags of 7168, and an exponential window of 1% to minimize leakage errors [15].

The stabilization diagram shows the thirteen prominent natural frequencies clearly identified by the method, in agreement with the modes previously analyzed in the following reference study [6]. Additionally, some noise points, identified by DBSCAN as -1, are present and do not represent a physical cluster. The resonance modes are highlighted with pink crosses, which indicate the selected representative poles.

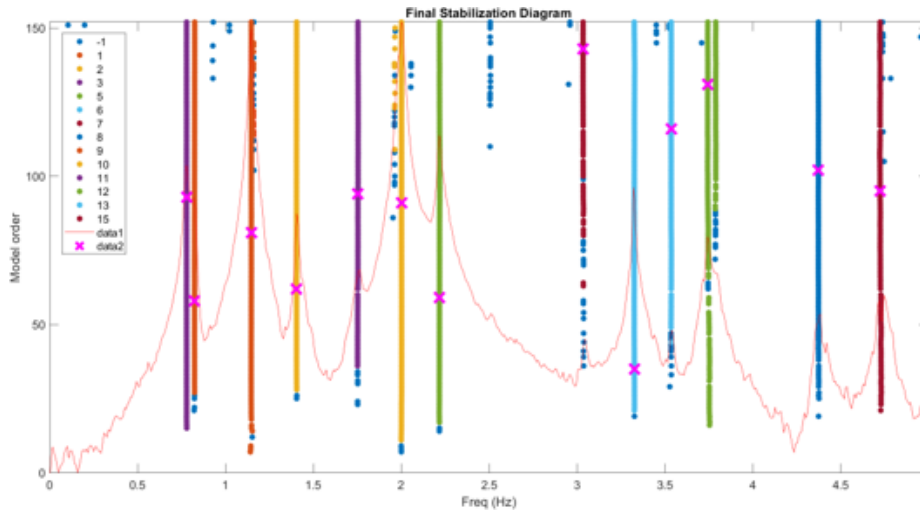


Fig. 5 Stabilization Diagrams of the bridge.

Due to the lower excitation during the night, the methodology is unable to identify all thirteen natural frequencies, particularly the higher frequencies, as shown in Figure 6, which represents a stabilization diagram from 2:30 AM. This issue is further compounded by the high noise levels from the accelerometers, which makes difficult the capture of higher frequencies. This is a limitation of the sensors adopted in this work (to explore a more challenging application), the use of force balance accelerometers allows a more continuous identification of the modal properties [6].

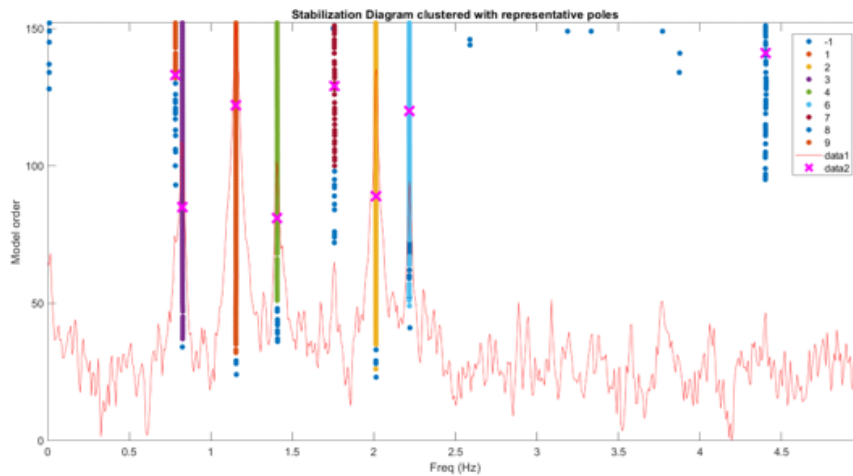


Fig. 6 Representative stabilization diagram of a low rush hour.

Modal tracking

Figure 7 shows the tracked frequencies resulting from the AMT methodology, applied to one month of data (July 2018).

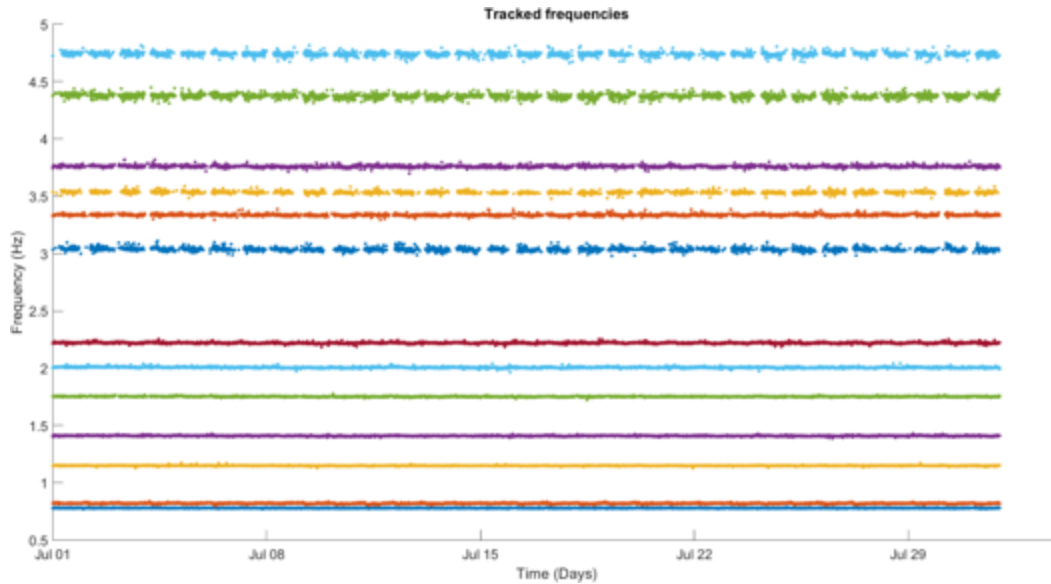


Fig. 7 Tracked frequencies for the Infante Dom Henrique Bridge in July 2018.

With the AMPS results for the entire month and a set of reference modes, the Automated Modal Tracking (AMT) procedure can be applied, using the modeset presented in figure 5 as the reference modeset.

For the AMT methodology, only the frequency distance, as explained in the subsection “Modal Tracking” was considered. The MAC value was excluded due to the simplicity of the monitoring system, which makes this value unreliable because of the spatial aliasing phenomenon. Additionally, an update of the reference modes was applied, based on the previous 10 points in the analysis, to account for small variations that may occur.

Analyzing figure 7, a clear definition of the first seven natural frequencies can be observed. However, noticeable gaps appear in the higher frequencies. These gaps are mainly due to the lower excitation during off-peak hours, as explained in the previous section. Conversely, in the higher frequencies, areas with a good concentration of poles are successfully tracked due to the high excitation present during rush hours, as shown in figure 5.

Overall, there was a good identification of the thirteen natural frequencies, despite the challenges associated with identifying the higher frequencies due to the constraints of the monitoring system.

Wind turbine 2.0 MW

In this section, the AOMA procedure previously explained in the section ‘Automated Modal Analysis’ is implemented to analyze the structural dynamics of a 2MW onshore wind turbine. These developments are reported in [16].

The wind turbine under study is located in northern Portugal. It has an up-wind rotor with a radius of 41 meters and three pitch-controlled blades. The tower is steel tubular with 80 meters in height and comprises three segments assembled by bolted flanged connections.

Dataset characterization

Six uni-axial accelerometers were combined in duos to measure the accelerations in two orthogonal directions and distributed over three levels along tower height, as presented in Figure 8. The sensors have a range of 140 dB and can measure acceleration ranges from ± 0.1 g to 4.0 g. The acquisition system, composed of a computer and a digitizer, recorded acceleration data with a sampling frequency of 50 Hz within a 10-minute time series to pair with the SCADA data. 2401 data sets were recorded over a one-month measuring period.

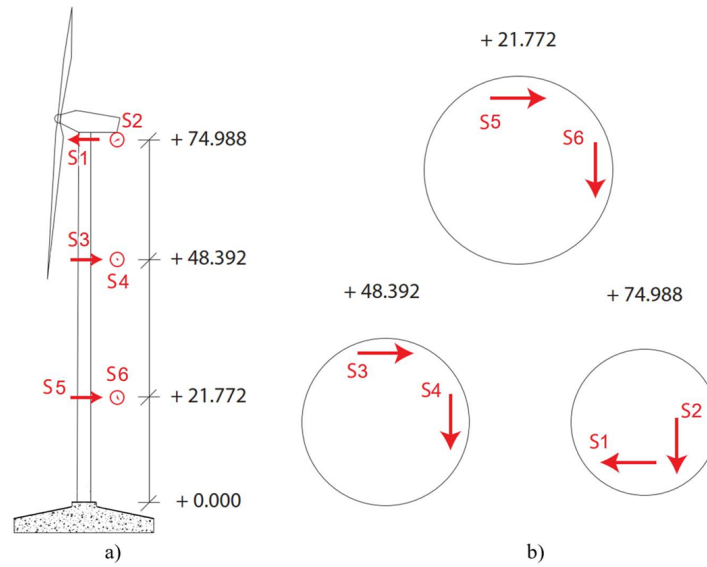


Fig. 8 Accelerometers located at three levels of the wind turbine: a) front view; b) cross-sectional views.

Different operating conditions

Figure 9 presents the wind turbine power curve defined with measured data. Five main regions were defined according to the operating conditions and are further described.

The rotor is parked or idling for wind speed inferior to 3 m/s while the environmental conditions are evaluated to determine whether the operation is feasible. The second region corresponds to the initiation of power production, if the rotor torque is consistent, and a rotor speed augmentation is observed. In this region and the following one, the pitch angle is minimal to optimize the power generated, while maximizing the lift force. The rotor torque rated value achievement defines the end of the third region. In the following region, for enhanced wind speed, the blade's pitch angle decreases to maintain the rotor torque within operation limits while the power production enhances. The last region observed in the operational data, for wind speed above 13 m/s, corresponds to the increasing blades pitch for enhanced wind speed maintaining the rotor torque and power production at rated values. These limits ensure the wind turbine's operational safety and structural integrity, preventing mechanical and electrical overloading.

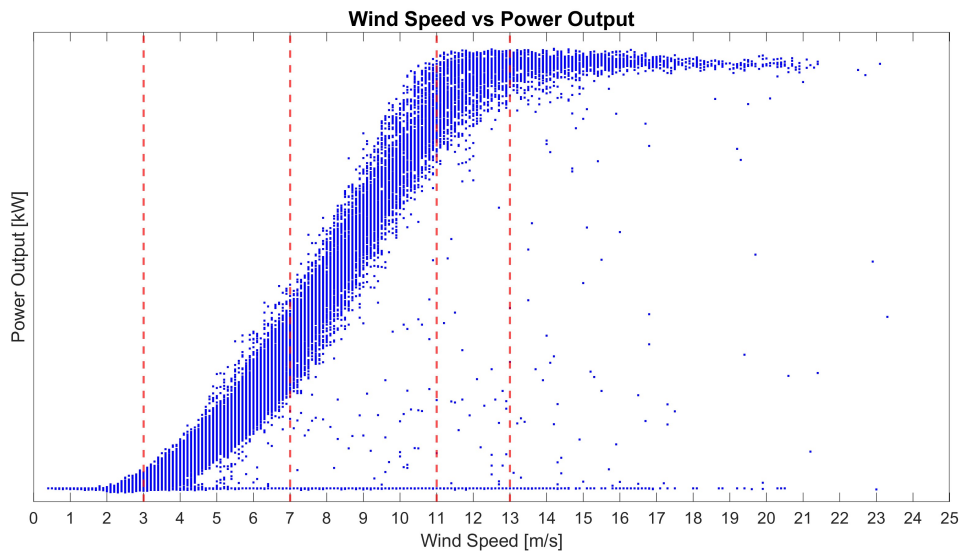


Fig. 9 Wind turbine power curve based on real measured data over one year.

Automated Modal Analysis results

In this section, the impact of the wind turbine rotating conditions is observed in the stabilization diagrams. Therefore, the Automated Modal Parameter Selection (AMPS) is applied to the complete dataset for February 2014, followed by the implementation of the Automated Modal Tracking methodology, presented in the section titled 'Modal tracking'.

Stabilization diagrams

Polymax is applied to each 10-minute acceleration dataset to estimate the modal parameters, followed by the AMPS implementation to define accurately the resonance modes. The first algorithm considered 4096-time lags, 1% exponential window, and was applied to a frequency range between 0 and 5 Hz for a maximum model order of 76.

Referring to Figure 10, two stabilization diagrams are presented, the first while the wind turbine is parked and the second while the rotor is rotating at 12 rpm. The structural modes identified have positive integer identifiers, with pink crosses signaling each reference pole. The poles labeled '-1' were classified as spurious by the DBSCAN clustering algorithm and the clusters designated '-2' correspond to spurious modes defined by the second verification step. The red function represents the sum of the cross-power spectral densities.

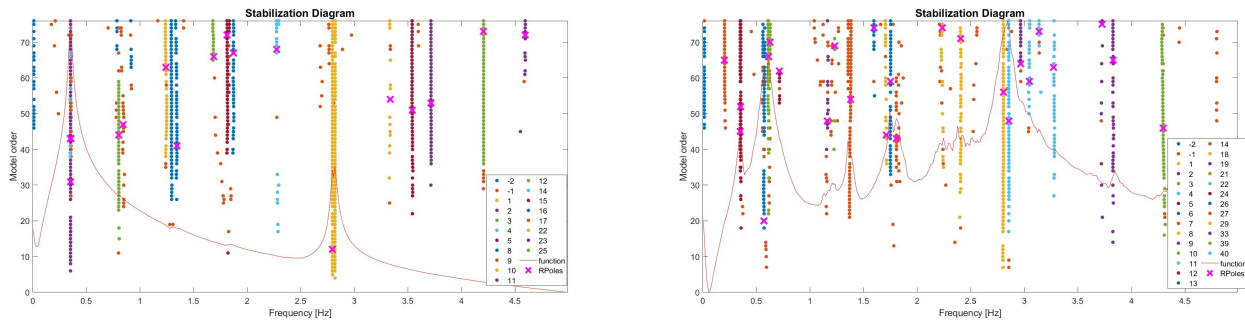


Fig. 10 Wind Turbine Stabilization Diagrams: Parked Condition (left) and Operating at 12 rpm Rotor Speed (right).

The stabilization diagram in parked conditions presents a modal model graph with greater stability, with the identified modes appearing cleaner and more distinctly defined compared to when the rotor is in operation. In rotating conditions, a higher number of modes are identified due to the aerodynamic rotor excitation. Additionally, the rotor speed variation over the 10-minute time series analyzed introduces greater complexity, leading to more dispersed poles in the stabilization diagram.

Modal tracking

Posterior to obtaining the stabilization diagram for each 10-minute acceleration dataset, the resulting resonance modes identified were subjected to the tracking procedure to evaluate their correspondence with the reference modes defined.

Random measurement setups were investigated, and the planar mode shapes corresponding to the expected were identified to establish the reference, presented in Figure 11. This method leads to ten reference modes defined mainly while the rotor is close to its maximum speed.

To track a resonance mode, the frequency variation must not exceed 3.5%, and the mode must maintain a maximum 30% MAC deviation from the reference mode. If multiple modes meet these criteria, the mode with the smallest distance is selected for tracking. The results are visually presented in Figure 12, where the datasets are organized in ascending order of rotor speed, emphasizing their effect on mode frequency response.

The majority of modes are successfully tracked in more than half of the datasets analyzed. However, the mode 3SS is detected in less than 10% of the datasets, and three other modes—1FA, 3FA, and 1SS*—show success rates below 50%.

The procedure encounters challenges in tracking the modes when the wind turbine is either parked or idling. Although the reference modes were defined based on near-maximum rotor speed, two factors may explain the reduced tracking success rates observed at zero or low rotor speeds. First, certain modes—specifically, 1SS*, 2SS*, and 3SS*—rely on rotor motion for excitation. Second, as noted in [17], both frequency and mode shape vary with changes in rotor speed. This is observed for modes 1SS* and 2SS* once the frequency of the first one tends to decrease with enhanced rotor speed and the opposite trend is verified for second mode 2SS*.

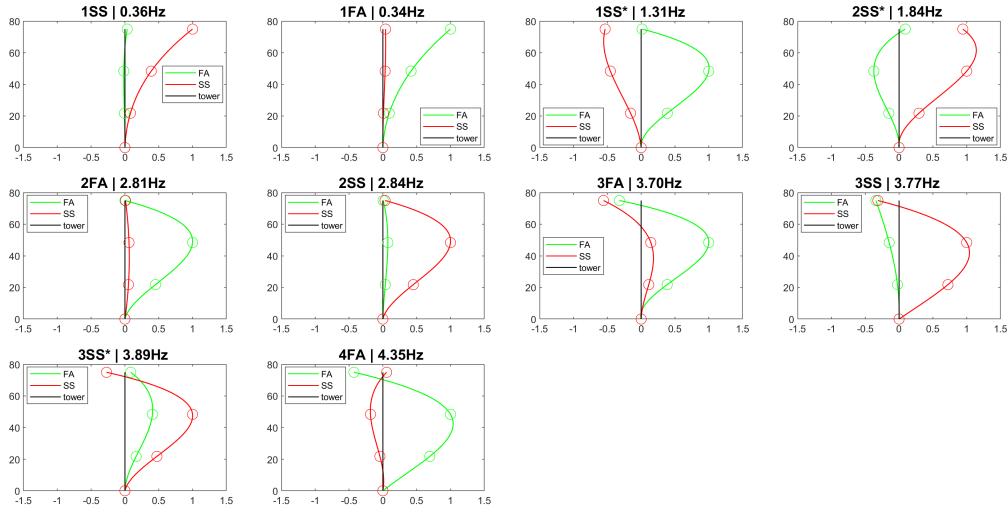


Fig. 11 Wind turbine reference mode set, regarding frequencies and mode shapes. The following abbreviations are FA - fore-aft vibration mode, SS - side-side vibration mode, and SS* - side-side vibration mode dependent on rotor motion.

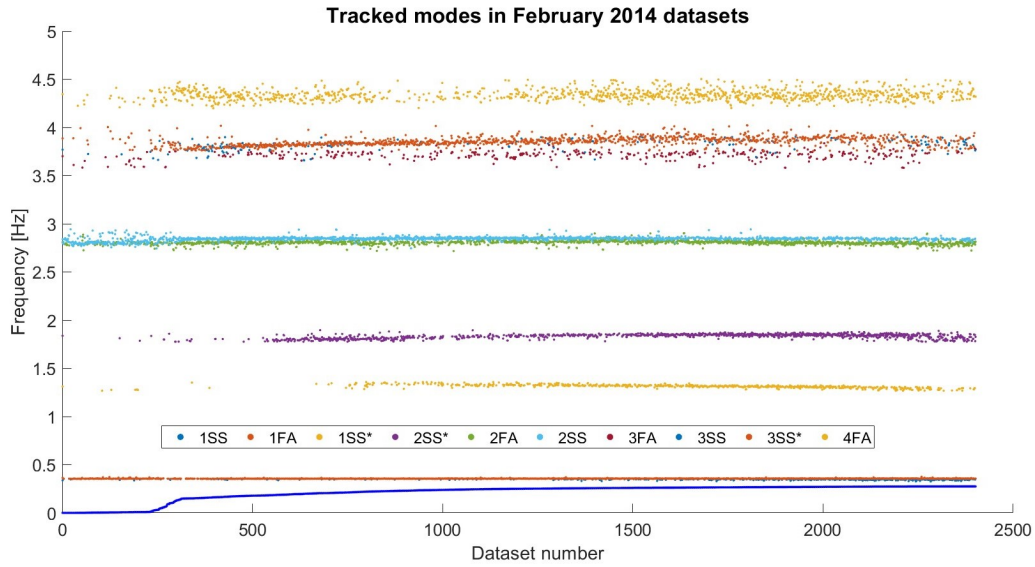


Fig. 12 Tracked frequencies for the 2MW wind turbine in February 2014. The continuous blue line at the bottom of the graph represents the rotor speed corresponding to each measurement.

On average, the MAC between each tracked mode and its reference counterpart exceeds 85% and the frequency variation is generally inferior to 7%. These results suggest a high level of similarity between the tracked and reference modes, indicating strong overall performance, despite the lower success rates for certain modes.

Conclusion

In this work, an automated modal analysis methodology was developed by combining the DBSCAN, density-based clustering algorithm, with mode verification steps inspired on modal analysis expert judgement. This methodology was applied to automatically calculate the resonance modes from the stabilization diagrams obtained with the Polymax modal parameter

estimator, for the Infante Dom Henrique bridge and a 2MW wind turbine. Both these structures were monitored across time by coupling the AMPS methodology with a modal tracking procedure based on statistical comparisons between modes obtained for a given measurement and a set of reference modes for each structure. For the Infante Dom Henrique bridge, the modal tracking was successful in tracking the reference modes, though for low excitation periods mostly during the night, some of the high-frequency modes were not excited enough to be obtained in the stabilization diagrams. For the wind turbine, the same procedure managed to track ten reference modes. Due to the variability of some modes with respect to different operational conditions of the wind turbine, the modal tracking procedure has difficulties in tracking the more susceptible modes.

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Chapter 13

Characterizing Walking Loads based on Force Measurements on Lightweight Pedestrian Bridges



Ghaffarian Dallali Elyar and Dey Pampa

Abstract With the increasing use of high-strength and lightweight materials in sustainable constructions, vibration serviceability often becomes the governing design criteria, particularly for pedestrian bridges subjected to human-induced walking excitations. For assessing the vibration serviceability of lightweight pedestrian bridges, characterizing the walking-induced ground reaction forces (GRFs), is crucial. While most of the existing walking load models are based on direct GRF measurements on rigid surfaces such as force plates or treadmills, there exists very limited studies based on direct force measurements on pedestrian bridges, that can represent real walking behaviour including the human-structure interactions (HSI) due to the vibrating surface of the structure. This study characterizes vertical GRFs induced by a single pedestrian, using force measurements collected at the support level of a lightweight aluminum pedestrian bridge with distinct vibration characteristics. We introduce a new probabilistic framework for estimating GRF values to simulate walking loads from a single pedestrian. By utilizing both time and frequency domain analyses, the proposed approach simultaneously accounts for HSI and uncertainties in walking activities. Future work will be expanded to groups of pedestrians, where interaction effects are expected to be more pronounced.

Keywords Human-structure Interaction · Lightweight Pedestrian Bridges · Ground Reaction Force · Human Induced Vibrations

Introduction

Given the increasing demands for architectural designs—particularly regarding length, slenderness, and the use of innovative lightweight materials—many new pedestrian structures are experiencing excessive vibrations, which can lead to discomfort for pedestrians. Current methods for analyzing human-induced vibrations typically model the pedestrian as a force moving at a constant speed across the structure [1]. The time-history of this force, known as the ground reaction force (GRF) is usually represented as periodic load considering as a sum of up to certain numbers of Fourier harmonic components [2]. However, this approach has been shown to frequently overestimate structural responses, specially in the resonant scenario where the amplitude of the bridge response is significant [3]. Increasing evidence suggests that this overestimation is partly due to the mechanical interaction between the structure and the pedestrian, known as the human-structure interaction (HSI) effect. This effect typically arises in structures with high levels of vibration [4]. Specifically, in terms of GRF, the HSI reduces the contact force exerted by the pedestrian on a vibrating structure compared to walking on a rigid surface [5].

There are several methods to estimate the GRF in the vertical direction, however, only a few effectively account for HSI which can significantly alter the GRF's shape and magnitude [6]. For example, methods using force plates assume a rigid surface, therefore, these methods are incapable of capturing interactional behavior between pedestrian(s) and bridge. On the other hand, the GRF can be also estimated via instrumented shoes or indirectly using bridge acceleration measurements that potentially can capture HSI on flexible structures; however, they may lack precision, with smoothing effects of pressure

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integration in instrumented shoes and complexity of calculations in indirect method [7] and [8]. Attaching force plates to a vibrating bridge is a more recent approach, but it introduces potential errors due to frequency and phase mismatches between the bridge and the force plates [5]. Additionally, three-dimensional recordings using inertial measurement units can be compromised by "tissue artifacts," caused by relative movement between body segments, reducing the accuracy of GRF estimation [9]. Among the various methods, using load cells at the bridge support level is distinctive because it can account for HSI effects without influencing pedestrian behavior, as it does not require additional equipment on the pedestrian or complex calculations. However, analyzing the GRF components measured through load cells in resonant and near-resonant scenarios can be challenging due to the overlaps of bridge's energy with the harmonics of pedestrian walking.

Despite the semi-periodic nature of walking mechanisms, walking is assumed to be perfectly periodic in current design practices which recommends deterministic values of the dynamic load factors (DLFs) to estimate the GRF. Such assumption ignores the inherent uncertainties and random behavior of walking [7]. Consequently, there is an inherent bias in the DLF values, as they often represent only a single footfall, which does not encompass the full variability of walking forces at a given frequency. Nevertheless, a probabilistic approach, which accounts for uncertainties in walking behavior, could provide a more realistic and accurate model, though it has received limited attention in the literature and code provisions [10], [9], [11], [12], and [13].

Given these challenges, this study introduces a novel probabilistic framework that quantifies dynamic load factors (DLF) up to the 5th harmonic of walking load, offering a more precise representation of walking-induced loads while accounting for HSI. This framework derives the probability density functions (PDFs) for each DLF value based on force measurements collected at the supports of a lightweight aluminum pedestrian bridge. The estimated DLF values accounts for the inherent variability in pedestrian walking patterns, as well as uncertainties introduced by the HSI effects.

Background

In the deterministic framework, it is assumed that the successive footfalls are identical and thus, the vertical force induced by walking pedestrian is represented by a Fourier series as follows [2]:

$$G(t) = G + \sum_{i=1}^n G\alpha_i \sin(i2\pi f_p t + \Phi_i) \quad (1)$$

where, G is the pedestrian's weight in [N], α_i is the dynamic load factor corresponding to the i^{th} harmonic of the walking frequency, f_p is the pacing frequency of the pedestrian in [Hz], Φ_i is the phase shift of the i^{th} harmonic, and n is the number of contributing harmonics, respectively. Over the last few decades, extensive research has been dedicated to quantify the α_i values based on direct or indirect measurements of walking forces [14].

Traditionally, α_i values for each harmonic of the walking load are calculated by extracting the maximum values from the Fast Fourier Transform (FFT) response, known as the peak picking method [7]. However, as mentioned earlier, walking is a semi-periodic activity with significant uncertainties. These uncertainties, combined with the effects of HSI, can cause energy leakage into neighboring frequencies. As a result, a narrow-band stochastic representation of the GRF provides a more accurate quantification. This implies that a single α_i value may not accurately represent complex walking and associated GRF [7]. To address this, Peters et al. [7] introduced the time-domain averaging method. In this approach, instead of proposing a constant value associated with each integer of the walking load, contribution of the frequency bands around the integers of the walking load is taken into account. By band-pass filtering each harmonic within its associated frequency range and converting the signal back to the time domain, the average of the peak values is calculated, representing the α_i value for that harmonic. This method called time domain averaging accounts for the uncertainties in walking load, including HSI and the semi-periodic nature of human locomotion. However, it has only been applied to GRF data collected from instrumented treadmills, which are not capable of capturing the natural walking of a pedestrian or the HSI effects due to the vibrating surface of the bridge [7].

Several models are available in the literature for calculating the vibration response of pedestrian bridges to the GRF induced by pedestrians [15]. One widely used approach is the moving force (MF) model, which simplifies the pedestrian and their load as a concentrated force moving along the bridge at a constant velocity, v [1], as shown in Figure 1. This method is also adopted by the current design standards for its simplicity and ease of calculation [16], [17], and [18]. However, this model lacks accuracy, particularly in resonant scenarios where the effects of HSI is significant [1]. Nevertheless, there is ongoing debate in the literature about whether the effects of HSI can be accounted for by adjusting the GRF shape and values, without resorting to complex interaction models for incorporating HSI into the bridge's response estimation [19].

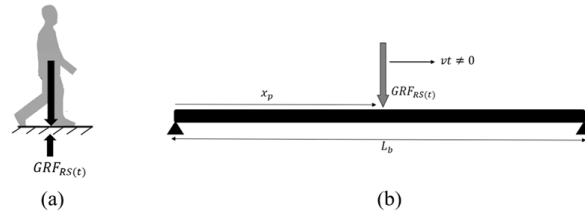


Fig. 1 (a) Ground reaction force exerted by a walking pedestrian (b) Moving force (MF) model on an equivalent beam.

Analysis

Experimental data

This study is based on the experimental study that was performed at the University of Waterloo (Canada) on a 22.9 m aluminum pony truss pedestrian bridge with a natural frequency of 4.58 Hz, total mass of 1735 Kg and modal damping of 0.8% [20]. More details of the tests can be found in [21] and [22]. During the experimental study, the bridge was instrumented with 4 triaxial load cells at four corner supports of the bridge to measure force time histories. 12 uniaxial accelerometers were also installed to measure vertical and lateral accelerations at both side of the bridge span (L) at $L/4$, $L/2$ and $3L/4$ points. Numerous walking tests based on individual, and groups of pedestrians were conducted. In order to estimate the pedestrian-induced walking loads and validate them, this study is limited to the load cell data as well as the acceleration data at the middle of the bridge for a test subject representing a pedestrian with mass of 70 kg. During the walking tests, the test subject walked in five different pacing frequencies ranging from 1.67 Hz to 2.33 Hz with an interval of 0.167 Hz. This frequency range represents non-resonant, near-resonant, and resonant scenarios ensuring comprehensive coverage of different vibration-response conditions as classified in Table 1. In addition, each walking test for each frequency was repeated 30 times to account for the intra-subject variabilities. Figure 2 (a) illustrates the schematic layout of the pedestrian bridge, highlighting the positions of the load cells and accelerometer at the mid span. Additionally, Figures 2 (b) and 3 (c) present the force signals recorded from each load cell during an example trial, along with the total force, which is the sum of these signals, representing the pedestrian's impact and the resulting bridge vibrations.

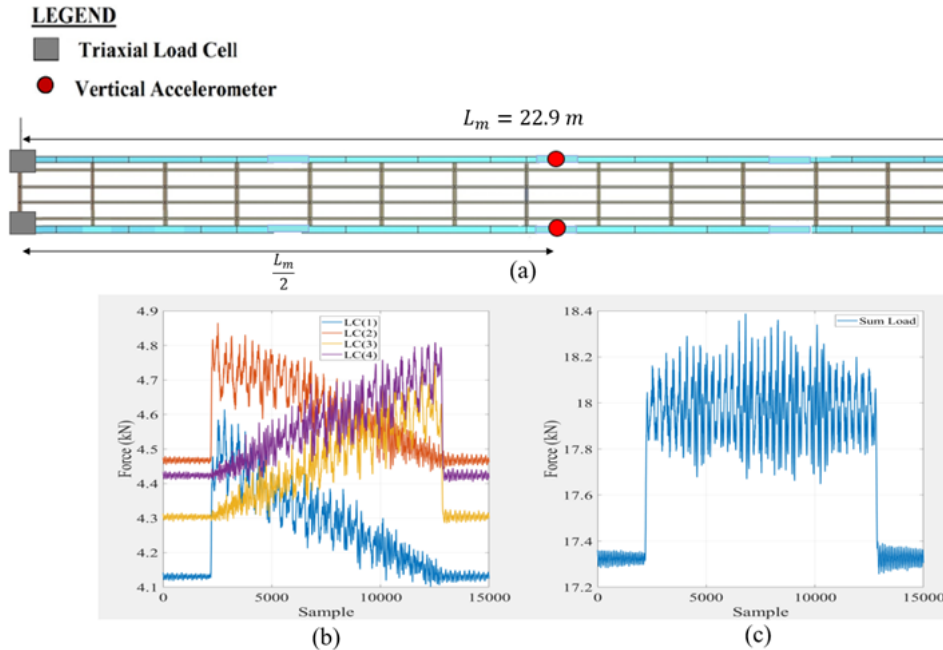


Fig. 2 (a) Schematic of the pedestrian bridge (modified from [17]); (b) raw force signals collected from load cells in the vertical direction; (c) Total force in vertical direction including all load cells.

Table 1 Classification of non-resonant, near resonant, and resonant scenarios based on the five pacing frequencies.

Pacing frequency	$f_p = (1.67) Hz$	$f_p = (1.83) Hz$	$f_p = (2.00) Hz$	$f_p = (2.17) Hz$	$f_p = (2.33) Hz$
Scenarios	NR-(H3) ²	Non-resonant	Non-resonant	NR-(H2) ³	R-(H2) ⁴

f_p ¹: Pacing frequency of the pedestrian;

NR-(H3)²: near resonance with harmonic 3rd harmonic of walking load.

R-(H2)³: near resonance with harmonic 2nd harmonic of walking load.

R-(H2)⁴: resonance with harmonic 2nd harmonic of walking load.

Estimation of DLF (α_i) values

In this section, the effects of uncertainty and HSI on the DLF values are examined separately. In the first subsection, the DLF values for non-resonant scenarios are evaluated using both the peak picking and time-domain averaging methods. Since the bridge's acceleration response is low in these scenarios, the accuracy of each method can be attributed primarily to the uncertainties in the walking activity itself. In the second subsection, the DLF values for resonant and near-resonant scenarios are introduced using a probabilistic approach, which accounts for both the effects of HSI and the uncertainties in the GRF induced by pedestrians.

Non-resonant scenarios

In this section, the DLF values representing the GRF are calculated using both the traditional peak picking [23] and time-domain averaging methods [7], up to the 5th harmonic of the walking load, based on data extracted from load cells. These values correspond to non-resonant scenarios (i.e., $f_p = 1.83$ and 2.00 Hz). The mean DLF values on each method, considering all trials, are presented in Table 2. To evaluate the effectiveness of these DLF extraction methods, the maximum acceleration response of the bridge at mid-span is simulated using the MF modeling approach, incorporating the estimated DLF values. To do so, the bridge is represented as an equivalent single-degree-of-freedom (SDOF) system based on its mode of vibration with frequency of 4.58 Hz and damping of 0.8% . In the Figure 3, the simulated acceleration response is compared with the measured acceleration of the bridge at mid span. The maximum acceleration response is also simulated through the existing DLF values as proposed in the literature [17], [24], and [7], considering both pacing frequencies associated with non resonant state.

Table 2 Extracted DLF values for the first 5 harmonics of walking load using both peak picking and time domain averaging scenarios

	Frequency	α_1	α_2	α_3	α_4	α_5
Pick Picking	1.67	0.2441	0.0357	0.0607	0.0203	0.0062
	1.83	0.3416	0.1011	0.0532	0.0174	0.0060
Time Domain Averaging	2.00	0.2619	0.0553	0.0777	0.0253	0.0089
	2.17	0.3576	0.1190	0.0466	0.0208	0.0079

Using equation (2), the relative maximum acceleration response of the pedestrian bridge considering two current methodologies as well as the proposed values by two authors and CSA S7 is calculated. As evident from Table 3, the average relative maximum acceleration calculated using the DLF values extracted from load cell data via time-domain averaging provides the closest match between experimental observations and numerical results in the non-resonant scenarios. It is important to note that, in non-resonant conditions, the bridge's acceleration levels are relatively low, meaning the effects of HSI are less pronounced. Therefore, the accuracy of time-domain averaging in predicting the bridge response is likely not due to HSI effects on GRF values but rather to its ability to capture the uncertainties in natural walking behavior using load cell data. In contrast, the inaccuracy of the peak picking method supports the hypothesis that human walking activity resembles a narrow-banded stochastic signal, which cannot be accurately represented by fixed α_i values. Therefore, this conclusion can be drawn that the uncertainties in walking behavior causes energy leakage into neighboring frequencies, while the amplitude of these neighbor frequencies might not be prominent in the frequency domain, still contribute to the accuracy of pedestrian-induced walking load prediction.

$$\beta = \left(\frac{a_{num} - a_{exp}}{a_{exp}} * 100 \right) \quad (2)$$

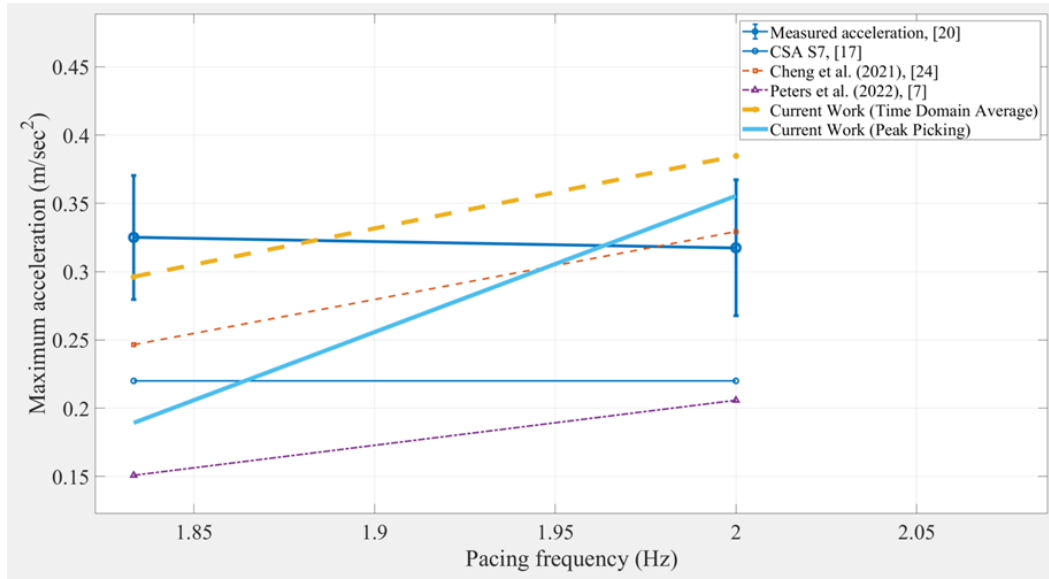


Fig. 3 Comparisons of the measured maximum accelerations at the mid span of the bridge with the simulated ones using the DLF values extracted from two methods as well as proposed in the literature [20], [17], [24], and [7].

Table 3 Relative acceleration response of the bridge using peak picking and time domain averaging as well as the relative simulated response of the bridge using DLF values introduced by two other authors and CSA S7.

	$\beta_{f_{1.83}}$	$\beta_{f_{2.00}}$	$\frac{\beta_{f_{1.83}} + \beta_{f_{2.00}}}{2}$
CSA S7 [16]	-32.30	-30.59	-31.45
Chen et al. [22]	-24.30	3.78	-10.26
Peters et al. [7]	-53.84	-35.33	-44.58
Current Work (Peak Picking)	-41.84	11.98	-14.92
Current Work (Time Domain Averaging)	-8.92	21.13	6.10

Resonant and near-resonant scenarios

In this subsection, an improved DLF estimation framework is proposed for both the resonant and near-resonant scenarios that combines a modified time domain averaging method to the approach proposed by Ahmadi et al. [5] for separating bridge contributions from the force signals based on synchronized force and acceleration signals measured on the bridge. It should be noted that the DLF value corresponding to the resonant harmonic is only determined for resonant scenarios, as it absorbs the highest energy in the system, with negligible contributions from other harmonics.

Since the acceleration at the middle of the bridge and the force signals at the supports as collected by Dey et al. [22] were not synchronized due to the use of different acquisition systems, the first step of this approach involves synchronizing the force and acceleration signals using cross-correlation technique [25]. The next step involves calculating the mean inertial mass of the bridge by dividing the force signal by the synchronized acceleration response at the center of the bridge during the decay phase of vibration, i.e., when the pedestrian has left the bridge, and it vibrates freely without external forces. The inertial mass of the bridge was estimated to be 1082 kg, which is slightly higher than half of the total bridge mass, a result consistent with modeling the bridge as an equivalent simply supported beam. Next, both the force signal from the load cell and the acceleration response are filtered using a band-pass filter to isolate the frequency band of the resonant or near-resonant harmonic during the forced phase of vibration, while the pedestrian is still on the bridge. The filtered acceleration during this phase is then multiplied by the previously calculated inertial mass, representing the bridge's modal force in the selected frequency band where the harmonic of the walking load and the bridge's modal response overlap. Finally, by subtracting the load cell data during the forced phase from the calculated modal force, the force values associated with the resonant or near-resonant harmonics can be obtained, without the contribution of the bridge's modal response. Once the force signals are extracted, a modified time-domain approach as described in Figure 4 is applied to determine the DLF values. The traditional Time Domain Averaging proposed by Peters et al. [7] used the mean values of the signal peaks (as indicated by

green line in Figure 4) in the time domain to propose DLF values associated with each harmonic of walking load. In this study, instead of using mean peak value, all the peaks of the force signals (as shown in red dots in Figure 4), associated with a specific frequency band corresponding to a walking harmonic, is extracted. All these peaks for all walking trials are then presented in a probabilistic framework as discussed in the next section.



Fig. 4 Modified approach to extract DLF through Time Domain Averaging method.

Statistical distribution of estimated DLF values

Once the DLF values for each walking trial and each frequency are calculated, the statistical distribution of the DLF values are determined. It should be noted that similar to the resonant and near-resonant cases, the modified time domain averaging method is also applied to extract each peak of the GRF signals corresponding to a harmonic which are then modelled probabilistically including all walking trials. It has been observed that in case of non-resonant scenarios, only the first harmonic of walking load follows a specific statistical distribution. Hence, the α_1 is presented within a probabilistic framework, while the mean DLF values are reported for the remaining harmonics. Similarly, in the near-resonant scenarios, a statistical distribution is introduced for both the first and resonant harmonics, with mean values presented for the remaining harmonics. Finally, in the resonant scenario, the statistical distribution and PDF are provided only for the resonant harmonic, as it absorbs the highest energy in the system, with negligible contributions from other harmonics.

To identify the most suitable probability density function (PDF) for representing the DLF values, the Akaike Information Criterion (AIC) was applied to various statistical distributions. The results indicated that the t-distribution, characterized by its three parameters—location (μ), scale (σ), and degrees of freedom (ν)—provided the best fit for all PDFs. For instance, Figure 5 illustrates the PDF for the second harmonic of the walking load in a resonant situation with a pacing frequency of 2.17 Hz. Additionally, Table 3 summarizes the mean DLF values for harmonics that do not require a PDF representation, while Table 4 presents the PDF parameters for all pacing frequencies.

Table 4 Mean DLF values for non-resonant scenarios for 5 harmonics and different pacing frequencies.

Frequency (Hz)	α_1	α_2	α_3	α_4	α_5
1.67	t-student	0.1190	t-student	0.0208	0.0079
1.83	t-student	0.0553	0.0777	0.0254	0.0089
2.00	t-student	0.1190	0.0466	0.0208	0.0079
2.17	t-student	t-student	0.0297	0.0231	0.0185
2.33	0	t-student	0	0	0

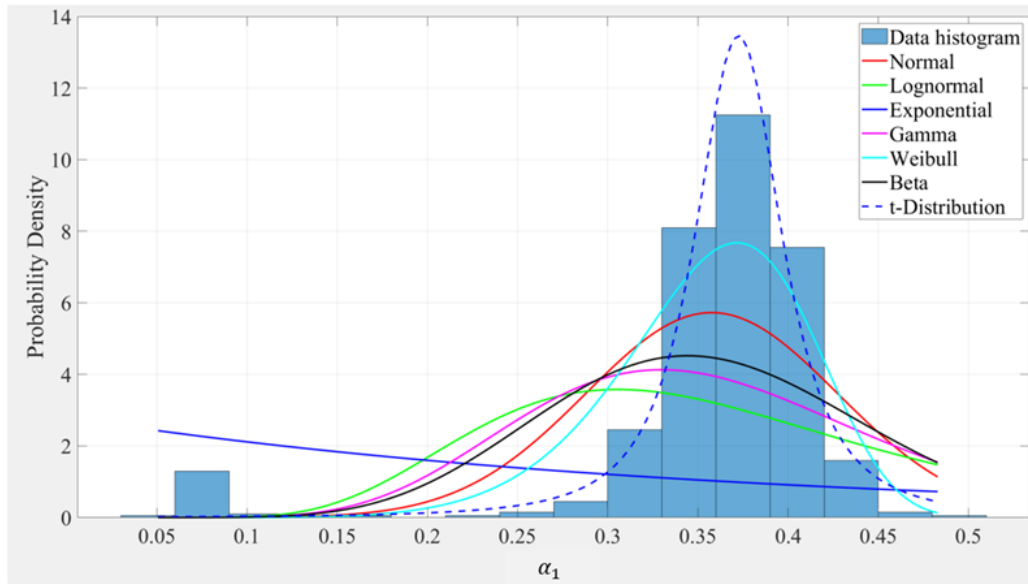


Fig. 5 Different PDFs of the first DLF value for pacing frequency equal to 2.17 Hz.

Table 5 t-student parameters for DLF values of resonant and near resonant scenarios considering all pacing frequencies

Frequency (Hz)	α_i	μ	σ	ν
1.67	α_1	0.2522	0.0162	1.7398
	α_3	0.0324	0.009	12.0619
1.83	α_1	0.2728	0.0189	1.732
2.00	α_1	0.3725	0.0261	1.9506
2.17	α_1	0.4154	0.0446	1.6941
	α_2	0.0392	0.0129	4491372.8
2.33	α_2	0.0597	0.0188	763697.6

Comparison of maximum responses in a probabilistic framework

This section assesses the performance of the estimated DLF values and their statistical distributions to simulate bridge response probabilistically. Based on the 30 walking trials during the experimental study, the histogram of the measured acceleration is plotted in Figure 6. For the predicted response based on the estimated DLF values, a total of 6000 simulations were performed using Monte Carlo simulation and are presented in Figure 6 to compare and assess its performance with respect to the experimental measurements.

As shown in Figure 6, there is strong agreement between the simulated and experimental results. However, for non-resonant scenarios ($f_p = 1.83$ and 2.00 Hz), the maximum acceleration response of the bridge is more widely distributed. This suggests that when the excitation energy is not concentrated in a specific frequency range, the coupled bridge-pedestrian system exhibits more scattered responses. This could also indicate a higher level of uncertainty in the pedestrian's walking activity in non-resonant scenarios, as the pedestrian is not synchronizing their steps with the bridge's vibrations, leading to varying and scattered output in each trial. Conversely, in resonant and near-resonant scenarios, the maximum response of the bridge is more concentrated. This could be attributed to the stronger effects of HSI and to the fact that the pedestrian is more inclined to modify their steps with respect to the oscillatory surface. Moreover, for the resonant scenario ($f_p = 2.33$ Hz), there is a good agreement between the experimental and simulated responses. However, in near-resonant scenarios, the predictions slightly underestimate the system's response. This discrepancy could be due to inaccuracies in the method developed by Ahmadi et al. for near-resonant conditions or unique dynamic characteristics of lightweight aluminum bridges. However, further investigation is required to understand why the DLF values extracted from the same bridge in near-resonant conditions underestimate the system response.

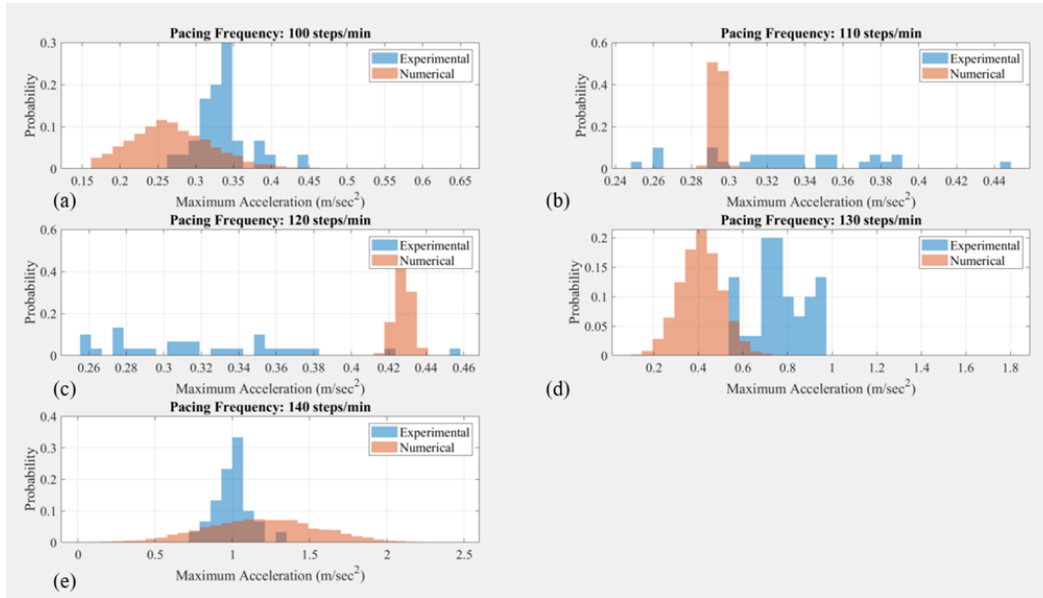


Fig. 6 Probability distributions of simulated and measured acceleration responses for a) $f_p = 1.67$ Hz, b) $f_p = 1.83$ Hz, c) $f_p = 2.00$ Hz, d) $f_p = 2.17$ Hz, and e) $f_p = 2.33$ Hz.

Conclusion

In this study, the ground reaction force (GRF) induced by a single pedestrian was estimated using force signals from load cells, applying a hybrid time-frequency domain representation of the force and estimating the dynamic load factor (DLF) values in a probabilistic framework. In the first part of the analysis, two methodologies—peak picking and time domain averaging—were evaluated by calculating the maximum acceleration response of the bridge in non-resonant scenarios. It is concluded that the time domain averaging technique captured the uncertainties in walking loads more effectively, suggesting it may be a better method for developing GRF values compared to the peak picking approach. An improved method is also proposed to estimate DLF values for resonant and near-resonant harmonics. Finally, all the DLF values for resonant, near-resonant, and non-resonant scenarios are determined probabilistically. The analysis showed that for all scenarios, the first DLF, near-resonant, and resonant harmonics were best described by the t-distribution, while the remaining harmonics were represented by their mean values, as they did not follow a specific statistical distribution. To further assess the performance of the extracted DLF values and their statistical distribution, the predicted maximum acceleration responses through Monte Carlo simulations are compared with the experimental observations. The results imply that the estimated maximum acceleration responses are in good agreement with the measured responses for resonant scenarios indicating better performance of the proposed framework. On the other hand, the experimental results were more scattered as compared to the predicted ones for non-resonant scenarios, implying higher uncertainty in these situations. In fact, during resonant scenarios, since pedestrians tend to synchronize their steps to the bridge oscillations, the uncertainty in walking becomes lower, yielding in narrow band representations of the bridge responses as compared to non-resonant scenarios. Additionally, in near-resonant scenarios, although DLF values were estimated for both resonant and non-resonant harmonics, the proposed methodology underestimates the bridge response, which could be due to the limitations of the DLF extraction method or specific dynamic characteristics of lightweight bridge in near-resonant conditions. However, further study is needed to investigate the near-resonant scenarios. In the future scope of this study, the proposed framework will be extended to groups of pedestrians.

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Chapter 14

Design and Demonstration of an Improved Optical Multiplexer for Vision-based Vibration Monitoring of Civil Structures

Matthew Whelan, Youngjin Park, and Timothy Kernicky

Abstract Computer vision-based approaches to measuring the static displacement and vibration of civil structures offer the potential to reduce the cost and logistical complexity of structural health monitoring systems. However, the application of computer vision for displacement measurement of many civil structures is challenged by the small magnitude of displacements under operational loads relative to the physical size of the structure. Consequently, achieving sufficient measurement resolution of displacement with conventional vision-based systems typically requires imaging of a small field of view rather than the full expanse of the structure. As an alternative to costly synchronization of multiple cameras or the use of reference-based methods, an optical multiplexer can be used to acquire multiple sub-fields of view with a single camera to simultaneously acquire displacement measurements at several locations across a large civil structure. The authors have previously presented a low-cost liquid crystal optical multiplexer assembly with 3D printed fixture and demonstrated the ability of the multiplexer to track the displacement of optical targets within sub-fields of view. However, the opening and closing time of the general-purpose liquid crystal shutter limited the utility of the optical multiplexer for vibration measurement. This paper further explores the use of optical multiplexing for measurement of dynamic displacements by redesigning the instrumentation with a fast optical shutter. The design and operation of the optical multiplexer is presented including the selection and characterization of the optical shutter and imaged sub-fields of view. Additionally, translation of the technology from the laboratory to the field is examined by acquiring vision-based measurements from a structure under ambient conditions.

Keywords Optical multiplexing · Vision-based measurement · Vibration measurement · Full-field displacement measurement · Structural health monitoring

Introduction

Vision-based methods for structural health monitoring of civil structures have received significant attention over the past decade, driven by the potential to provide a convenient and relatively low-cost, non-contact approach to monitoring structural response. Significant improvements in image resolution of digital cameras have enabled accurate static and dynamic displacement measurements with consumer-grade cameras [1, 2]. Recent studies have demonstrated the capability of vision-based approaches to acquire structural responses and estimate system properties for structural health monitoring of full-scale bridges or critical bridge components. For example, an influence line for the vertical displacement at a midspan location of a continuous box-girder highway bridge has been constructed with a vision-based system [3], modal parameters of stay cables of full-scale bridges have been estimated using phase-based motion magnification to enable inverse identification of tension forces [4], and natural frequencies and damping ratios for several modes of a large cable-stayed bridge have been estimated using an array of consumer-grade cameras equipped with long focal lenses [5].

The motivation for introducing an optical multiplexer into a vision-based approach for monitoring civil structures stems from the inherent trade-off between field of view and spatial resolution. Wide field imaging is desirable to permit capture of

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as much of a structure into the image to maximize the sampled region. However, the spatial resolution is degraded by the use of a large field of view, since the pixel area of the imaging sensor is applied over a large physical area. The use of long focal length lenses, and subsequent reduction of the field of view to a small region of interest, is often necessary to produce sufficient spatial resolution to measure dynamic displacement of bridges [6]. In the absence of optical multiplexing, multiple cameras are required to acquire vision-based measurements at several regions of interest across a large civil structure. Alternatively, a roving camera technique developed by Wang et al. [7] can be employed where a series of targets or regions of interest are sequentially sampled while acquiring simultaneous images of a reference target with a second, stationary camera. However, this approach requires linear, time-invariant response, either controlled and repeatable or white-noise excitation, and independent compensation of camera motion for each camera. Optical multiplexing offers the potential capability to acquire multiple sub-fields of view using a single, stationary camera, thereby reducing the number of cameras and eliminating the challenges associated with roving techniques.

The current work builds on the design and demonstration of a low-cost optical multiplexer constructed with liquid crystal (LC) shutters and a 3D printed fixture that was previously presented by Whelan and Park [8]. The optical multiplexer uses the technique developed by Treeaporn et al. [9] that enables the superposition of multiple sub-fields of view onto a single imaging sensor, while providing a means to reconstruct the images of the individual sub-fields of view. The superposition of multiple sub-fields of view is achieved using a multiple beamsplitter arrangement, depicted graphically in Figure 1a. A beamsplitter reflects light while simultaneously transmitting light. This allows for the transmission of light reflected by any beamsplitters or mirrors positioned behind a beamsplitter, effectively superimposing multiple fields of view onto the common image sensor of the camera. By adjusting the orientation of the mirror and beamsplitters, the individual fields of view reflected to the camera can be controlled. To enable disambiguation of the composite image to facilitate reconstruction of the images of the individual sub-fields of view, the multiplexer uses LC shutters to acquire a sequence of three non-redundant measurements. This process is illustrated graphically in Figure 1b. First, an image is acquired with all shutters open, resulting in a composite image formed by all sub-fields of view. Subsequently, images are acquired with shutters sequentially closed, starting with the shutter in the first beamsplitter ahead of the mirror. If the shutters completely block all transmitted light, then the individual pixel intensities of the acquired composite images are the simple sum of the sub-image intensities and any measurement noise, which allows for direct inverse determination of the images of the individual sub-fields of view. However, LC shutters are imperfect shutters and do transmit a small fraction of light in the closed state, particularly if the orientation of the shutter is not perpendicular to the orientation of the camera lens. Treeaporn et al. [9] detail a procedure for estimating leakage coefficients experimentally to construct a system model capable of accounting for beamsplitter transmission and reflection coefficients as well as any absorptive losses.

Improvements to Optical Multiplexer Design

The low-cost optical multiplexer design presented in Whelan and Park [8] used two very low-cost 36 mm x 36 mm LC light valves to serve as both shutters and beamsplitters in the multiplexer. Although the design produced a working prototype that demonstrated the successful acquisition of static and dynamic displacement measurements from three sub-fields of view, the LC light valves imposed several practical limitations on the performance. First, the 38 mm x 38 mm size of the light valves was too small relative to the camera lens to acquire images from multiple fields of view with minimal overlap without obstruction by the optical mounts. Second, the low-cost LC light valves had relatively long opening and closing times, thereby limiting the practical switching speeds that could be used. At even a modest frame rate of 50 fps, a noticeable reduction in transmitted light was observed for the fields of view behind the front shutter. The reduction in transmitted light affected the ability to reconstruct the images for these fields of view at this frame rate, thereby limiting the ability of the original multiplexer design to be used for dynamic displacement measurement.

To improve the optical multiplexer design, the low-cost LC light valves were replaced with fast optical shutters in a larger format. Specifically, the 2nd generation fast optical shutter (FOS(G2)-IQ) manufactured by LC-Tec Displays AB with a 45.6 mm x 47.6 mm clear aperture area was selected for the improved multiplexer design. The model selected includes optical quality cover glass laminated to both sides of the shutter, which is recommended for imaging applications. These fast optical shutters feature a specified minimum open state transmittance of 38.5%, minimum contrast of 2000:1, maximum closing time of 1.8 ms, and maximum opening time of 6 ms when driven with a 4 V_{RMS} signal. In theory, the opening time of the fast optical shutter should permit the use of frame rates up to 150 fps without significant attenuation of the transmitted light, provided that the ambient light is sufficiently high enough to limit exposure times to around 1 ms or less. This frame rate was deemed sufficiently high for most envisioned civil structure applications of the optical multiplexer, as it results in a maximum effective sampling rate of 50 fps for each sub-field of view. For other applications requiring higher frame rates, one could use extra fast optical shutters, also available from LC-Tec Displays AB.

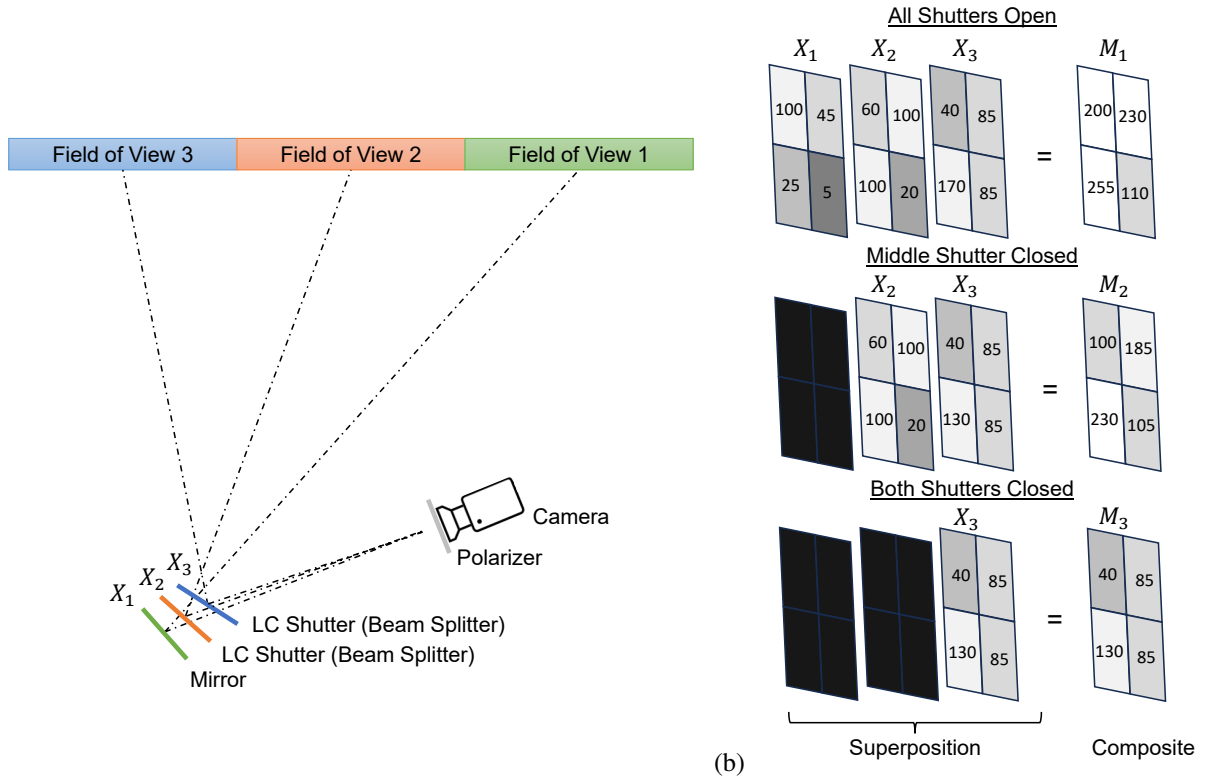


Fig. 1 Optical multiplexer: (a) design; (b) conceptual operation

The LC shutters in the optical multiplexer in Whelan and Park [8] were driven directly by 5V counter outputs from a National Instruments multifunction I/O device. However, driving an LC shutter directly from a 5V counter is not advisable due to the introduction of a DC bias that can degrade the performance of the shutter. Furthermore, the current rating of digital I/O channels is typically lower than the peak current required to achieve the specified switching speeds. To address both of these issues, a small driver circuit was developed to interface the fast optical shutters to the multifunction I/O device. The schematic for the circuit is shown in Figure 2a. The circuit relies on three synchronized counter output signals to establish the timing of the waveforms driving each shutter. One side of each shutter is driven by inverter gates connected to the first counter and wired in parallel to increase the potential to sink and source current. The other side of each shutter is driven by 2-input exclusive-OR gates, also wired in parallel. The inputs to the exclusive-OR gates are the signal driving the other leg of the shutter and additional counter signals that effectively control whether the shutter is driven closed or restored to the open position. Figure 2b depicts the waveforms from the counter signals and the resulting drive voltage across each shutter. Effectively, the Counter 0 signal establishes the polarity of the drive voltage across the shutters, while the Counter 1 and Counter 2 signals control whether a voltage potential is developed across each respective shutter to close the aperture. The SYNC signal is a simple digital output used as a start reference to synchronize the counters. This signal is also connected to the digital camera to synchronize the acquisition of frames. For the prototype developed for this work, a Texas Instruments SN74AC04N was used to provide the inverter gates and a Texas Instruments SN74AC86N was used to provide the 2-input exclusive-OR gates. Both integrated circuits are capable of providing ± 24 mA current output at the supply voltage of 5V.

Lastly, the 3D printed fixture for the optical multiplexer prototype was redesigned to greatly improve the ease by which the arrangement of optical components can be established for a given experimental setup. Figure 3 presents renderings of the 3D printed fixture design in both an assembled view and an exploded view. The design of the slotted fixture that holds the optic mounts, while permitting for adjustment of the position and orientation of the mirror and LC shutters, is largely unchanged from the original design, except for increased sizing to accommodate the larger mirror and shutters. However, a horizontal guide and longitudinal guide were designed and fabricated to allow for the fixture to be secured directly to the camera rather than being independently supported. The adjustments permitted by the guides significantly reduce the time required to establish the positions and orientations of the optics to sample the desired sub-fields of view with the multiplexer.

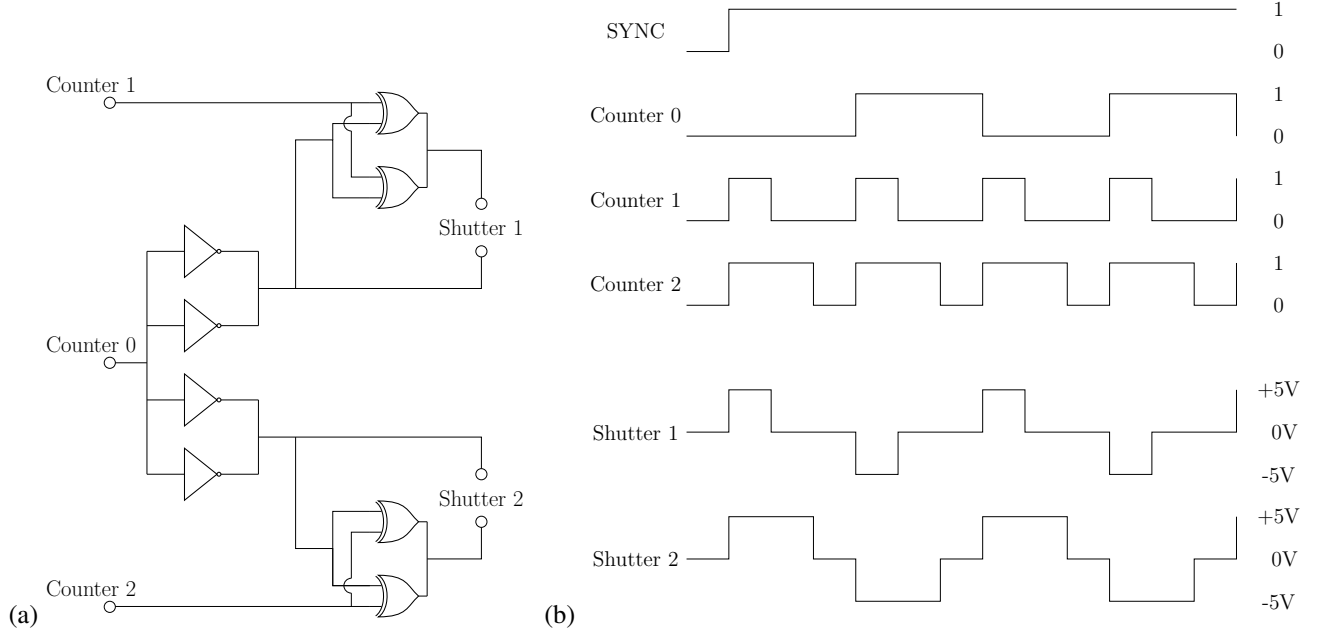


Fig. 2 Driving LC shutters with counter outputs: (a) schematic; (b) waveforms

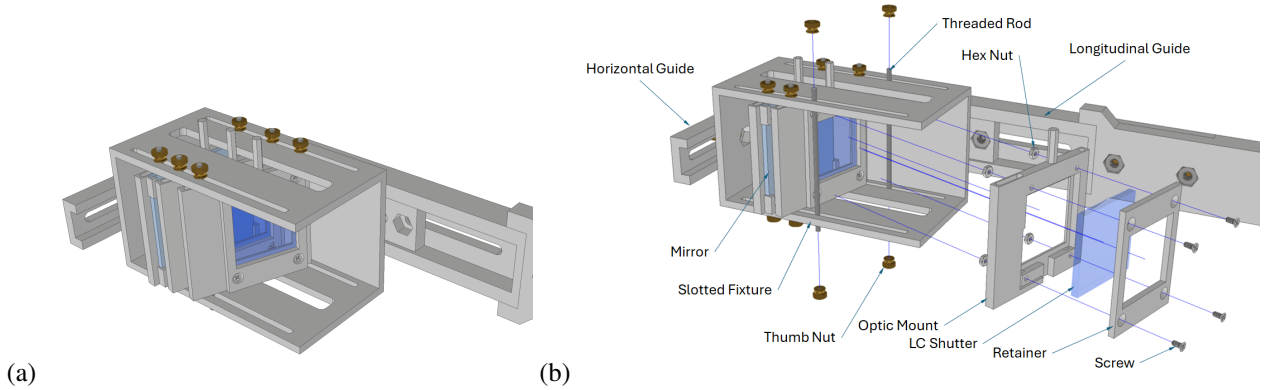


Fig. 3 Rendering of redesigned 3D printed fixture for optical multiplexer prototype: (a) assembled view; (b) exploded view

Field Demonstration

A preliminary field experiment was conducted with the new optical multiplexer prototype to demonstrate its performance in monitoring the dynamic response of a geometrically large structure under ambient conditions. Specifically, a light pole for the football stadium on the University of North Carolina at Charlotte campus served as the structure of interest for this demonstration. Figure 4a presents a photograph of the stadium light pole taken at the time of the testing. A Phantom VEO 640L high-speed color camera was used to acquire the images with a frame rate of 150 fps and 4 Megapixel resolution. A Nikon AF Micro-Nikkor 200mm f/4D IF-ED lens was used with the camera to focus the sub-fields of view on three local regions of the pole: one near the groundline, one around the midheight of the pole, and one containing the supported lights at the top of the pole. Figure 4b provides a photograph of the optical multiplexer configured directly below the long focal length lens attached to the high-speed camera. The multiplexer was supported secondarily by a small tripod to mitigate vibration of the fixture.

A series of composite images obtained from one of the recordings is presented in Figure 5. It is critical to note that a linear polarizing filter of appropriate size was not available for the lens of the camera at the time of the testing and could not be sourced before the submission deadline. Consequently, the image reflected from the front-most LC shutter during the test was comprised of non-collimated light, while the images reflected from the mirror and middle LC shutter were

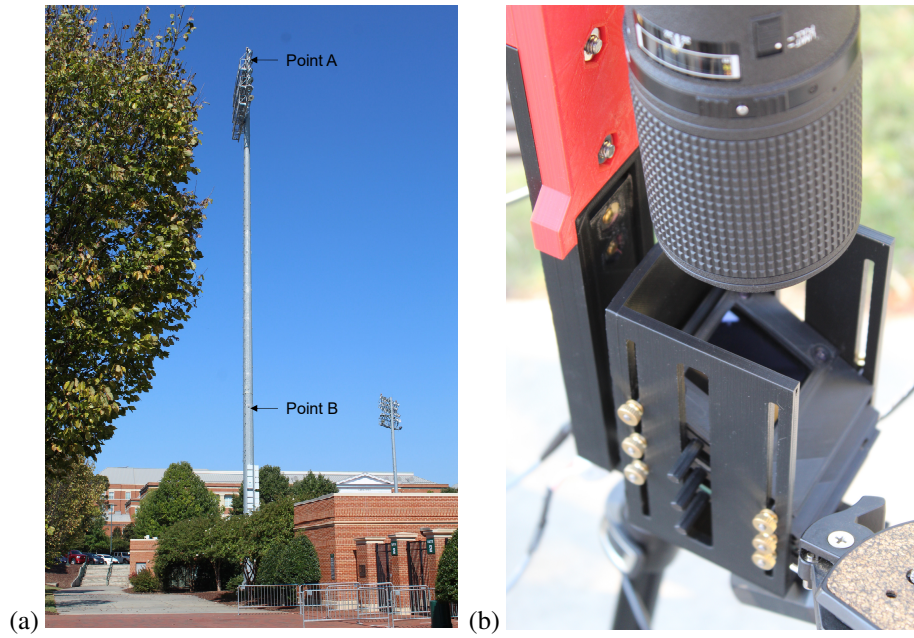


Fig. 4 Photographs of field demonstration: **(a)** stadium light pole; **(b)** tripod-mounted camera with optical multiplexer

linearly polarized by their transmission through the open LC shutters. As a result, the intensity of light from the sub-field of view reflected by the front-most LC shutter, X_3 , was significantly higher than that produced by the other sub-fields of view. This imbalance caused the X_1 and X_2 images to be less apparent in the composite images and degraded the quality of the reconstruction of the individual sub-fields of view. Consequently, reconstructed images of the individual sub-fields of view are not presented in this paper and will be reserved for future publications after the authors repeat the experiment with a linear polarizing filter to improve the relative balance of the sub-field of view image intensities. This future work will be presented at the conference and will serve to formulate conclusions and recommendations related to the performance of the revised optical multiplexer design.

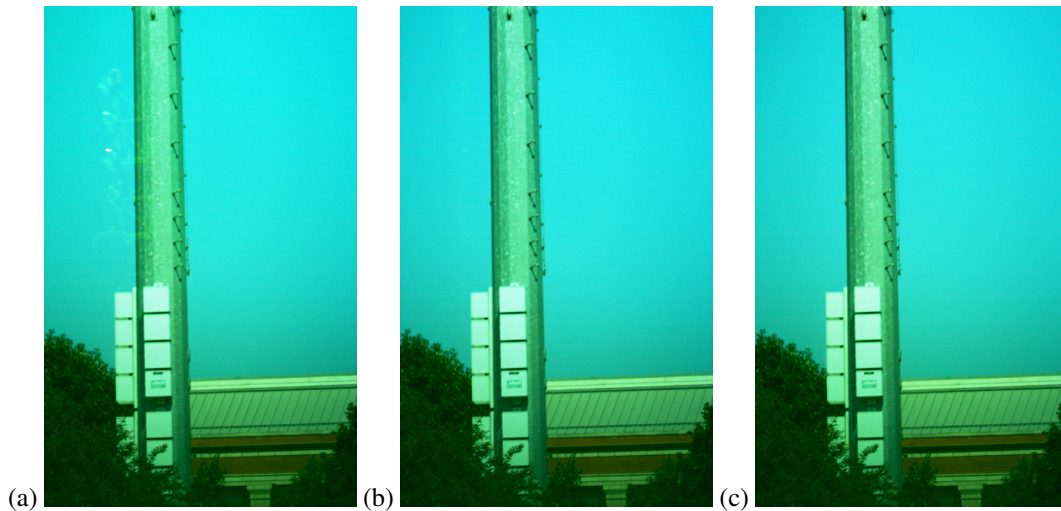


Fig. 5 Composite images obtained with prototype optical multiplexer: **(a)** all shutters open: $X_1 + X_2 + X_3$; **(b)** middle shutter closed: $X_2 + X_3$ +leakage; **(c)** both shutters closed: X_3 +leakage

Despite the issue related to the relative intensities of the sub-fields of view that arose due to the lack of a polarizing filter, the images recorded with the optical multiplexer assembly were post-processed to extract displacement time history estimates to demonstrate the ability to track features from different sub-fields of view. The Sandia National Laboratories

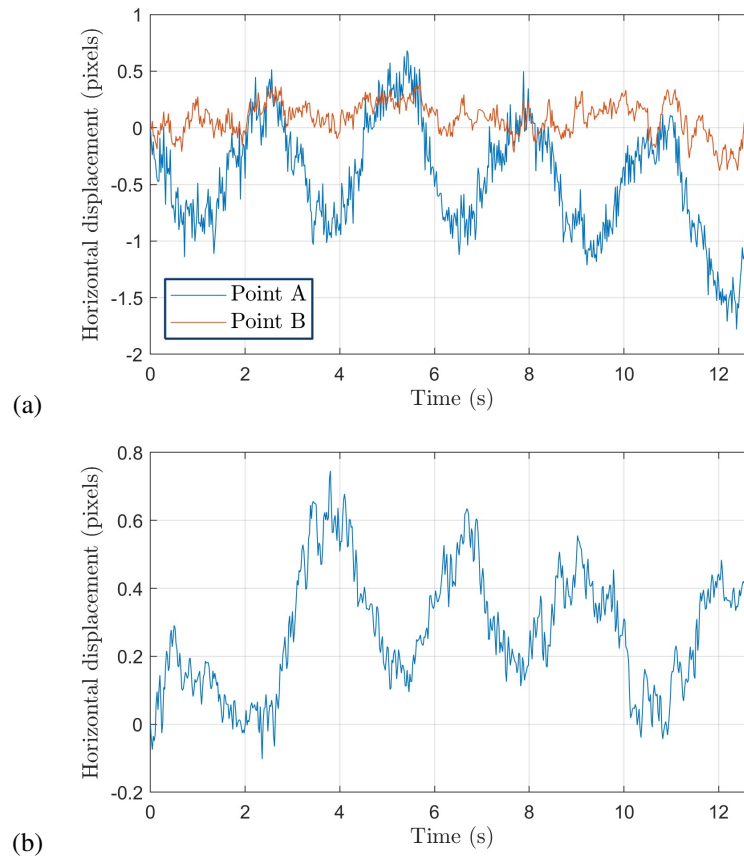


Fig. 6 Displacement time histories obtained from tracking of natural features: **(a)** with optical multiplexer ; **(b)** without multiplexer

Digital Image Correlation Engine (DICE) software application [10] was used to extract displacement time histories from regions with natural features using the tracking analysis mode. For all analyses, the gradient-based optimization method was used after Gauss filtering the images with a 13 pixel filter window. The two locations, denoted as Point A and Point B in Figure 4a, which originate from two different sub-fields of view were tracked over the duration of one of the recordings. Point A originates from the sub-image reflected from mirror, while Point B originates from the sub-image reflected by the front-most LC shutter. During the testing, the weather was mild and the wind speed was estimated to be only around 8 km/hr (5 mph) based on observations from a local weather station. As a result of the limited ambient excitation, the vibration amplitude in the light pole was expected to be low. Figure 6a provides the time histories obtained from tracking natural features of the pole at the locations of interest. The displacements amplitudes are generally less than a pixel in amplitude, but the relative amplitudes at the two locations suggest vibration of the light pole in its fundamental mode. By Fourier transform, the fundamental natural frequency of the pole was estimated to be 0.39 Hz. For comparison, the multiplexer was removed from the camera and the top region of the pole was monitored using a conventional configuration of the high-speed camera with the same long focal length lens. A horizontal displacement time history obtained by trajectory tracking of Point B without the multiplexer is presented in Figure 6b. Since this motion was captured at a different instance in time than the data in the corresponding Figure 6a, the waveforms are not expected to be identical in either amplitude or phase. However, the dominant period of the response is very similar to that observed with the optical multiplexer and Fourier transformation of the data confirmed that the frequency component of greatest amplitude occurred at 0.39 Hz.

Conclusion

Optical multiplexing has been previously shown to provide a practical means of sampling multiple regions of interest across geometrically large civil structures while maintaining the improved spatial resolution obtained with the use of long focal

length lenses. This paper discloses improvements to a preliminary low-cost optical multiplexer design to improve the capabilities for monitoring dynamic displacements. Specifically, the general purpose LC light valves were replaced with large format LC fast optical shutters to significantly reduce the opening and closing times to enable faster frame rates to be used without attenuating the intensity of light from the sub-fields of view transmitted through the LC shutters. A circuit for efficiently driving the LC shutters without introducing DC bias was presented and modifications to the 3D printed fixture to improve the usability of the optical multiplexer assembly were described. A preliminary field experiment was conducted where the vibration of a stadium light pole was monitored with high-speed camera equipped with a long focal length lens and the prototype optical multiplexer assembly. The images acquired during the testing confirm proper functionality of the LC shutters at a frame rate of 150 fps and enabled displacement measurements through feature tracking within different sub-fields of view. Due to the absence of a linear polarizing filter, a significant imbalance of light intensity across the images from the three sub-fields of view was present, which compromised the quality of reconstruction of the individual sub-fields of view. Remedy of this issue is straightforward and will be presented in conference presentation as well as future publications.

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Chapter 15

LSTM – NARX as Emulator for Finite Element Models of Nonlinear Structural Dynamic Systems: From Time-Series Input Vectors to Time-Series Output Vectors

Abdoul Aziz Sandotin Coulibaly, Zhen Hu, and Joel P. Conte

Abstract Performance-based seismic design of structural systems relies on computationally expensive nonlinear time history analyses (NLTHAs) of high-fidelity finite elements (FE) models for predicting structural response to seismic excitation. For risk-based assessment, these FE simulations must be run thousands of times across different realizations of the uncertainty sources, making the computational cost significant and often prohibitive. Even if NLTHAs are reliable and straightforward for assessing civil infrastructure systems, the associated computational cost is too substantial, if not prohibitive, to be disregarded. Consequently, data-driven machine learning surrogate models have gained prominence as fast emulators in replacement of computationally expensive high-fidelity models of engineering, bio-engineering, and biological systems. This paper presents a study of the Long Short-Term Memory (LSTM) networks as global surrogate models for nonlinear inelastic structural dynamic systems, taking a three-dimensional (3D) virtual yet realistic 5-story, 2-by-1 bay reinforced concrete (RC) frame structure subjected to bidirectional horizontal seismic excitation as application example. For high-dimensional, realistic FE structural models with complex nonlinear dynamics, standalone Long Short-Term Memory (LSTM) networks often face challenges in accurately capturing the full range of nonlinear structural behavior. A central approach of our model is a novel approach for selecting earthquake ground motion records with diverse key characteristics to form the input-output pairs for the training set. This method is designed to maximize the accuracy and reliability of predictions from the calibrated surrogate models. This is achieved by combining (i) a convolutional autoencoder (CAE) and (ii) an LSTM network employing the nonlinear autoregressive model with exogenous input (NARX) formulation.

Keywords Nonlinear Inelastic Dynamic Systems · Material nonlinearity · LSTM · Convolutional AutoEncoder · NARX

Introduction

In 2023, a series of record-breaking earthquakes, with intensities ranging from very strong to violent, caused major disruptions in daily life [1–3]. Consequently, there is an urgent operational need to develop risk-based, performance-based seismic design codes to assess and mitigate seismic risks to large-scale civil infrastructure systems and the built environment. Undoubtedly, the use of well-calibrated or even experimentally validated physics/mechanics-based FE models coupled with NLTHAs provides a valuable tool for assessing seismic performance and seismic risk. In risk assessment, even with an accurate and efficient FE analysis method, the computational time remains substantial due to the need for multiple model simulations across varied scenarios. This is compounded by uncertainties arising from (1) uncertainty in the ground motion intensity at the local site, (2) ground motion record-to-record variability, and (3) structural modeling uncertainties, all of which increase the computational demands by orders of magnitude.

To address these demands, data-driven surrogate models serve as fast emulators or meta-models, approximation of the dynamic behavior of structural systems—be it a bridge, or a high-rise building—using either simulated or real-world

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recorded data. Recurrent neural networks (RNNs), distinguished by the directional flow of information through their layers, are specifically designed to handle sequential data, such as time series commonly encountered in fields like structural engineering (e.g., input earthquake excitation and structural response time histories) [4]. LSTM networks have emerged as a more sophisticated and powerful variant of RNNs, with LSTM-based surrogate models documented in the literature for modeling nonlinear dynamic systems [5–11]. The predictive capability of these LSTM surrogate models primarily focuses on sequence-to-sequence (Seq-to-Seq) mapping, necessitating that all pairs of input ground motion and corresponding output response time histories have the same length. Furthermore, the current literature on surrogate modeling of dynamic systems is mostly limited to simplistic mathematical models (e.g., spring-mass-dashpot models) or basic planar nonlinear dynamic frame models of structures, which are often subjected to unidirectional earthquake ground motions. Therefore, it is often unrealistic and lacks comprehensiveness. In this paper, we explore the use of LSTM for surrogate modeling of a 3D RC concrete frame structure subjected to bidirectional earthquake excitation. This RC structure is modeled to account for nonlinear material behavior (through distributed plasticity) and nonlinear geometry.

Application Example: 3D Reinforced Concrete (RC) Frame Structure

Our numerical example consists of a 3D, 5-story, 2-by-1 bay RC frame structure subjected to bidirectional horizontal seismic excitations (see Fig. 1) [12]. The structure is designed for a location in downtown Seattle, in the state of Washington, with Site Class D soil conditions, a short-period spectral acceleration $S_{MS} = 1.37$ g and a one-second spectral acceleration $S_{M1} = 0.53$ g. The building has plan dimensions of $10\text{ m} \times 6\text{ m}$ in the longitudinal and transverse directions, respectively, and a floor-to-floor height of 4.0 m , resulting in a total height of 20 m . The dead and live loads and corresponding seismic masses associated with the structure are computed according to the 2012 International Building Code [13].

Fig. 1 shows the overall geometry of the structure with the dimensions and reinforcement details of the beam and columns cross-sections in both longitudinal and transverse directions.

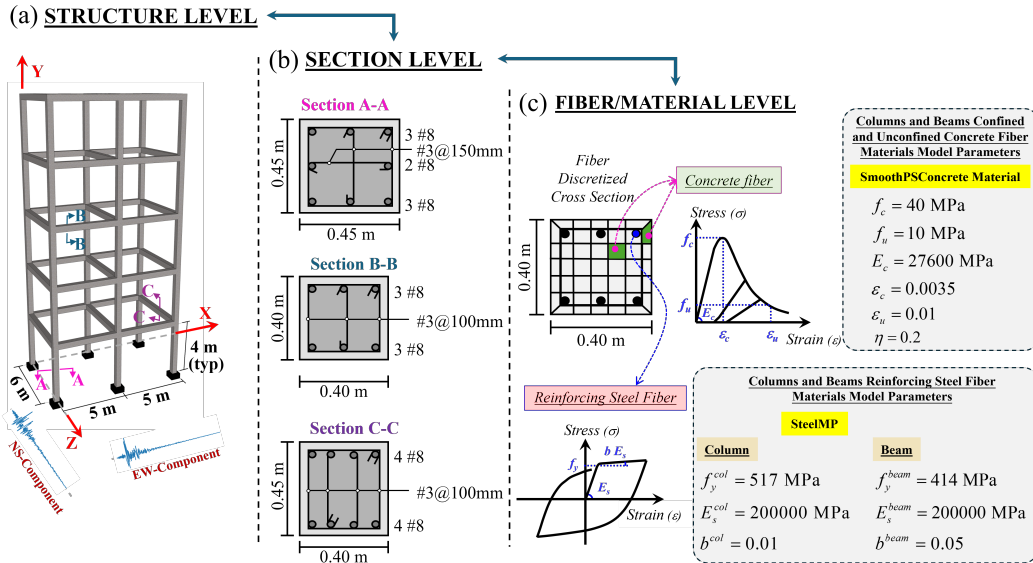


Fig. 1 3D RC frame building: (a) structure level overview, (b) cross-sections of beams and columns, (c) fiber or material level (material model parameters).

The seismic response of the structure is simulated using a mechanics-based nonlinear FE model with displacement-based fiber-section beam-column elements developed in the OpenSees structural analysis framework. The cross-sections of all beams and columns are discretized into longitudinal fibers. At the fiber/ material level, realistic uniaxial material constitutive models are used to represent the steel and concrete materials. Linear elastic shear and torsion models are combined with, but remain uncoupled from, the inelastic axial-flexural behavior at the section level. The effects of nonlinear geometry are accounted for using the corotational formulation for both beams and columns. Other sources of energy dissipation, distinct from the hysteretic energy dissipation due to the nonlinear material behavior, are modeled using mass ($\mathbf{M}$) and tangent-stiffness ($\mathbf{K}_T$) proportional Rayleigh damping, based on the tangent stiffness matrix at the last converged time step of

analysis. A critical damping ratio of 2% is assigned to the first and second vibration modes (obtained using the tangent stiffness matrix after the application of the gravity loads) of the structure in the longitudinal direction. The natural periods of these modes are $T_1 = 1.95$ sec and $T_2 = 0.62$ sec, respectively.

LSTM: Long Short-Term Memory Networks As Emulators Of Nonlinear Structural Dynamic Systems

LSTM neural networks represent a more advanced and powerful variant of RNNs, capable of overcoming the inherent limitations of conventional RNNs in capturing long-term dependencies, which are constrained to approximately 10 time steps, due to vanishing and exploding gradients encountered during the training process [14]. Unlike vanilla RNNs, LSTM can handle up to 1,000 time steps, a capability enabled by LSTM cells that effectively control error flow through internal states [15].

Assume the LSTM model of the 3D RC nonlinear FE model is trained to establish a relationship between two bi-directional earthquake ground-motion input time histories, denoted as $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$, and the corresponding two output relative displacement time histories at the 5th floor of the structure, $\mathbf{Y}^{(1)}$ and $\mathbf{Y}^{(2)}$. These two vector pairs are denoted as $\chi = \{\mathbf{X}^{(1)}, \mathbf{X}^{(2)}\}$ for the inputs and $\psi = \{\mathbf{Y}^{(1)}, \mathbf{Y}^{(2)}\}$ for the outputs. A full-blown version of the dataset looks like:

$$\begin{aligned} \mathbf{X}^{(1)} &= [\ddot{\mathbf{u}}_{1,1:C}^g, \ddot{\mathbf{u}}_{2,1:C}^g, \ddot{\mathbf{u}}_{3,1:C}^g, \dots, \ddot{\mathbf{u}}_{\tau,1:C}^g, \dots, \ddot{\mathbf{u}}_{T_1,1:C}^g]^{(1)} & \mathbf{Y}^{(1)} &= [\mathbf{u}_{1,1:C'}^{rel}, \mathbf{u}_{2,1:C'}^{rel}, \mathbf{u}_{3,1:C'}^{rel}, \dots, \mathbf{u}_{\tau,1:C'}^{rel}, \dots, \mathbf{u}_{T_1,1:C'}^{rel}]^{(1)} \\ \mathbf{X}^{(2)} &= [\ddot{\mathbf{u}}_{1,1:C}^g, \ddot{\mathbf{u}}_{2,1:C}^g, \ddot{\mathbf{u}}_{3,1:C}^g, \dots, \ddot{\mathbf{u}}_{\tau,1:C}^g, \dots, \ddot{\mathbf{u}}_{T_2,1:C}^g]^{(2)} & \mathbf{Y}^{(2)} &= [\mathbf{u}_{1,1:C'}^{rel}, \mathbf{u}_{2,1:C'}^{rel}, \mathbf{u}_{3,1:C'}^{rel}, \dots, \mathbf{u}_{\tau,1:C'}^{rel}, \dots, \mathbf{u}_{T_2,1:C'}^{rel}]^{(2)} \end{aligned} \quad (1)$$

where $\ddot{\mathbf{u}}_{\tau,1:C}^g$ and $\mathbf{u}_{\tau,1:C'}^{rel}$ are respectively the earthquake ground motion and relative displacement values in both directions at time step τ ; T_1 and T_2 denote the total number of time steps in each vector time series; $C = C' = 2$ are the number of channels for the input excitation and output response, respectively. Next, we will delve into the inner workings of the LSTM neural network, followed by outlining the process of constructing our LSTM-NARX emulator using the dataset defined in Eq. (1).

LSTM Cell

The LSTM cell is central to the success of LSTM networks. It functions as a subnetwork with both linear and non-linear connections, utilizing memory states and gate mechanisms to manage and retain information effectively [15]. Fig. 2 shows a graphical representation of an LSTM cell.

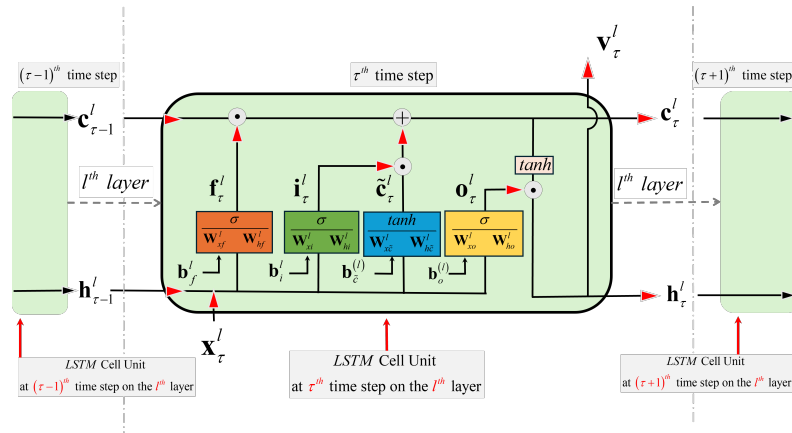


Fig. 2 Graphical representation of an LSTM Cell.

As a subnetwork, an LSTM cell consists of four interacting gates: the input gate $\mathbf{i}_\tau^l$, the forget gate $\mathbf{f}_\tau^l$, the cell state update gate $\mathbf{c}_\tau^l$, and the output gate $\mathbf{o}_\tau^l$ (see Fig. 2). At a specific time step τ and layer l , the four gates selectively retain and update the information from the previous time step $\tau - 1$, regulating the flow of information to the next time step $\tau + 1$ and

layer $l + 1$. In this context, an LSTM cell receives three inputs, notably: (1) an input vector $\mathbf{x}_\tau^l$, which may represent the bidirectional ($C = 2$) earthquake ground motion value at time step τ ($\ddot{\mathbf{u}}_{\tau-1: C}^g$) or/and the response of the structural system at the previous time steps, (2) the hidden state output denoted as $\mathbf{h}_{\tau-1}^l$ and the cell memory state $\mathbf{c}_{\tau-1}^l$ from the previous time step $\tau - 1$. Mathematical relationships between the gates and states are given as:

$$\begin{aligned}
 (a) \quad \mathbf{f}_\tau^l &= \sigma \left(\underbrace{\mathbf{W}_{hf}^l \mathbf{h}_{\tau-1}^l + \mathbf{W}_{xf}^l \mathbf{x}_\tau^l + \mathbf{b}_f^l}_{\text{forget gate}} \right) & (c) \quad \tilde{\mathbf{c}}_\tau^l &= \tanh \left(\underbrace{\mathbf{W}_{hc}^l \mathbf{h}_{\tau-1}^l + \mathbf{W}_{xc}^l \mathbf{x}_\tau^l + \mathbf{b}_c^l}_{\text{candidate cell state}} \right) & (b) \quad \mathbf{i}_\tau^l &= \sigma \left(\underbrace{\mathbf{W}_{hi}^l \mathbf{h}_{\tau-1}^l + \mathbf{W}_{xi}^l \mathbf{x}_\tau^l + \mathbf{b}_i^l}_{\text{input gate}} \right) \\
 (e) \quad \mathbf{o}_\tau^l &= \sigma \left(\underbrace{\mathbf{W}_{ho}^l \mathbf{h}_{\tau-1}^l + \mathbf{W}_{xo}^l \mathbf{x}_\tau^l + \mathbf{b}_o^l}_{\text{output gate}} \right) & (d) \quad \mathbf{c}_\tau^l &= \mathbf{f}_\tau^l \odot \mathbf{c}_{\tau-1}^l + \mathbf{i}_\tau^l \odot \tilde{\mathbf{c}}_\tau^l & (f) \quad \mathbf{h}_\tau^l &= \mathbf{v}_\tau^l = \mathbf{o}_\tau^l \odot \tanh \left(\mathbf{c}_\tau^l \right) \\
 & & & \text{cell update state} & & \text{hidden state output}
 \end{aligned} \tag{2}$$

In Eq. (2), the symbol $\odot$ denotes the Hadamard product, also known as the element-wise product, $\sigma(\cdot)$ and $\tanh(\cdot)$ represent the sigmoid and hyperbolic tangent activation functions, respectively [15].

An LSTM model can automatically retain or discard stored memory using these gates. The parameters of the LSTM model, denoted as θ_{LSTM} , include the kernel weight matrices $\{\mathbf{W}_{xf}^l, \mathbf{W}_{xi}^l, \mathbf{W}_{xc}^l, \mathbf{W}_{xo}^l\}$, recurrent weight matrices $\{\mathbf{W}_{hf}^l, \mathbf{W}_{hi}^l, \mathbf{W}_{hc}^l, \mathbf{W}_{ho}^l\}$, and the bias vectors $\{\mathbf{b}_f^l, \mathbf{b}_i^l, \mathbf{b}_c^l, \mathbf{b}_o^l\}$ that are learned across all layers. The LSTM network architecture consists of stacked layers, with each layer increasingly learning abstract representations of the input data through successive transformations. Training the LSTM network involves minimizing the difference between the normalized predicted LSTM network outputs $\hat{\psi}_{\text{NL}}$ and the normalized ground truth values ψ_{NL} for a given set of input-output ($\chi - \psi$) data pairs, thereby optimizing all parameters θ_{LSTM} . Fig. 3 illustrates a generic LSTM architecture, featuring an input layer with unnormalized input, two LSTM layers ($l = \{1, 2\}$), and an output layer representing the unnormalized targets.

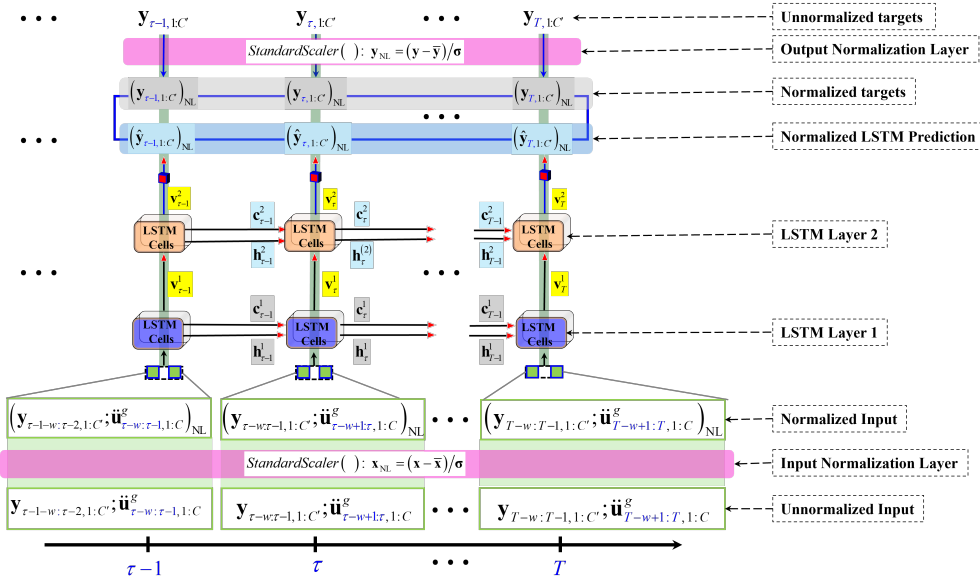


Fig. 3 Typical architecture and dataflow of LSTM Neural Networks.

LSTM-Nonlinear AutoRegressive network with eXogenous inputs Architecture (LSTM-NARX)

When considering a single input-output pair defined by $\mathbf{X}^{(1)} - \mathbf{Y}^{(1)}$, a Nonlinear AutoRegressive eXogenous (NARX) framework in time series modeling regresses the current values of a time series $\mathbf{y}_{t, 1: C'}$ on (1) previous w values of the same time series ($\mathbf{y}_{t-1, 1: C'}, \mathbf{y}_{t-2, 1: C'}, \mathbf{y}_{t-3, 1: C'}, \dots, \mathbf{y}_{t-w, 1: C'}$) and (2) the current ($\ddot{\mathbf{u}}_{t, 1: C}^g$) and past values of the exogenous input time series ($\ddot{\mathbf{u}}_{t-1, 1: C}^g, \ddot{\mathbf{u}}_{t-2, 1: C}^g, \ddot{\mathbf{u}}_{t-3, 1: C}^g, \dots, \ddot{\mathbf{u}}_{t-w+1, 1: C}^g$). The NARX model organizes the input data required for the LSTM architecture as

$$\underbrace{\mathbf{y}_{t, 1: C'}}_{\text{current output sequence}} \approx \mathbf{F}_{\theta}^{\text{NARX}} \left(\underbrace{\begin{matrix} \mathbf{y}_{t-1, 1: C'}, \mathbf{y}_{t-2, 1: C'}, \mathbf{y}_{t-3, 1: C'}, \dots, \mathbf{y}_{t-w, 1: C'} \\ \ddot{\mathbf{u}}_{t, 1: C}^g \end{matrix}}_{\substack{\text{previous output input sequence} \\ \text{current EQs input sequence}}} ; \underbrace{\ddot{\mathbf{u}}_{t-1, 1: C}^g, \ddot{\mathbf{u}}_{t-2, 1: C}^g, \ddot{\mathbf{u}}_{t-3, 1: C}^g, \dots, \ddot{\mathbf{u}}_{t-w+1, 1: C}^g}_{\text{previous EQs input sequence}} ; \underbrace{\theta_{\text{NARX}}}_{\text{parameters}} \right) \tag{3}$$

where $F_{\theta}^{NARX}(\bullet)$ refers to the LSTM-NARX network parameterized by weights and biases θ_{NARX} , which need to be learned during the training process. The parameter w enables the model to capture temporal dependencies and patterns in time series data. It is a key parameter in generating, from the input-output vector time histories, the time series sequences dataset for training of the LSTM-NARX models.

Training the LSTM networks involves performing supervised learning by optimizing the network parameters θ_{NARX} . This is accomplished by minimizing the loss function, as shown in Eq. (4)a, across all training input-output sequences [16].

$$(a) \quad L(\theta_{NARX})^b = \frac{1}{N_{SIB}} \cdot \frac{1}{C'} \cdot \sum_{s=1}^{N_{SIB}} \sum_{k=1}^{C'} [y_{NL, t, k, s} - \hat{y}_{NL, t, k, s}(\theta_{NARX})]^2;$$

$$(b) \quad \theta_{NARX}^{b+1} = \theta_{NARX}^b - \eta \frac{\partial L(\theta_{NARX})^b}{\partial \theta_{NARX}^b} \quad (4)$$

The model parameters θ_{NARX} are updated iteratively using a gradient-based optimization algorithm such as stochastic gradient descent (see Eq. (4)b) or the Adam optimizer. Here, η is the learning rate, b represents the batch number, N_{SIB} denotes the number of time series sequences within each batch. These parameters play a pivotal role in ensuring convergence to optimal weights and biases and enhancing predictive accuracy. The optimizer updates θ_{NARX} accordingly until the entire dataset has been processed multiple times, with each pass corresponding to an epoch.

Predictive Strategies of LSTM-NARX: Recursive Forecasting

The trained model is now denoted as $F_{\theta^*}^{NARX}(\cdot)$ where θ_{NARX}^* (indicated by the asterisk) denotes the optimal weights and biases. Predictions at a specific time step follow the format of Eq. (3). Similar to traditional time-stepping numerical solvers, the LSTM-NARX model's outputs from the previous time steps (in this instance $\hat{y}_{t-1, 1 : C'}, \hat{y}_{t-2, 1 : C'}, \hat{y}_{t-3, 1 : C'}, \dots, \hat{y}_{t-w+1, 1 : C'}$) are reintroduced as inputs to predict $\hat{y}_{t, 1 : C'}$ in Eq. (5).

$$\underbrace{\hat{y}_{t, 1 : C'}}_{\text{current predicted output sequence}} \approx \underbrace{F_{\theta^*}^{NARX}}_{\text{trained LSTM-NARX model}} \left(\underbrace{\hat{y}_{t-1, 1 : C'}, \hat{y}_{t-2, 1 : C'}, \hat{y}_{t-3, 1 : C'}, \dots, \hat{y}_{t-w, 1 : C'}}_{\text{previous output input sequence}}, \underbrace{\ddot{u}_{t, 1 : C}^g, \ddot{u}_{t-1, 1 : C}^g, \ddot{u}_{t-2, 1 : C}^g, \ddot{u}_{t-3, 1 : C}^g, \dots, \ddot{u}_{t-w+1, 1 : C}^g}_{\text{current EQs input sequence}}, \underbrace{\theta_{NARX}^*}_{\text{optimal parameters}} \right) \quad (5)$$

This procedure continues for each subsequent time step within the prediction horizon, employing recursive forecasting to reflect real-world scenarios in which the future structure's responses are unknown. This approach is particularly advantageous for real-time predictions when such data are unavailable.

Selection Of Input-Output Pairs For Training Of The LSTM Surrogate Model

Regardless of the capacity of the LSTM model, training with non-representative earthquake ground motion records will constrain the surrogate model's ability to generalize to unseen events. This section focuses on selecting a diverse set of earthquake records, ranging from low to high intensities, to capture a wide range of structural responses, from quasi-linear to highly nonlinear. This approach aims to improve the training of our LSTM-NARX network.

Engineering Intensity Measures (IM) for Ground Motion Selection in LSTM Training

For our three-dimensional RC frame structure subjected to bidirectional horizontal earthquake ground motion records, specifically along the X and Z axes (see Fig. 1(a)), we combine the spectral ordinates from these two horizontal components into a single value, which is the geometric mean of their respective spectral accelerations: $S_{a-gm}(\xi = 5\%, T_{1-X}, T_{1-Z})$ or S_{a-gm} for short. S_{a-gm} is then used as intensity measure for selecting ground motion records in LSTM training and is defined as

$$S_{a-gm}(\xi = 5\%, T_{1-X}, T_{1-Z}) = \sqrt{S_{a-X}(\xi = 5\%, T_{1-X}) \times S_{a-Z}(\xi = 5\%, T_{1-Z})} \quad (6)$$

where $T_{1-X} = 1.95$ sec and $T_{1-Z} = 1.94$ sec denote the initial fundamental period of the building structure in the X and Z direction, respectively. In Eq. (6), $S_a(\xi = 5\%, T_1)$, S_a for short, is the 5% elastic spectral pseudo-acceleration at the structure's initial fundamental period in a specified direction. S_a , which is structure-dependent and application-specific, is commonly used by structural engineers to assess the severity of earthquake ground motions [17]. For our 3D case, S_{a-gm} is considered a sufficient intensity measure, as it has traditionally been the preferred choice for structures subjected to two horizontal ground motions [18–20].

A database of 3,462 bidirectional earthquake ground motions was accessible from the PEER Strong Motion Database [21]. A subset of these motions was selected such that S_{a-gm} falls between the lower and upper bounds of 0.1g and 1.2g. This range ensures that the structure exhibits linear and nonlinear responses, with 1.2g allowing for significant nonlinear inelastic behavior without nearing the point where the iterative time-stepping scheme (to integrate the equations of motion) fails to converge due to the proximity of physical failure. The ranges of [0.1g, 0.3g], [0.3g, 0.7g], and [0.7g, 1.2g] correspond to quasi-linear, mildly to moderately nonlinear, and highly nonlinear structural responses, respectively. When earthquake ground motions in the high range, typically between 0.6g and 1.2g are underrepresented, randomly selected scaling factors ranging from 1 to 3 are applied to the available unscaled records to ensure sufficient coverage by producing additional ground motions with sufficient intensity to induce significant nonlinear structural response. This process resulted in 366 bidirectional ground motions in the [0.1g, 0.3g] range, 119 in the [0.3g, 0.7g] range, and 34 in the [0.7g, 1.2g] range, totaling 519 records. A Convolutional AutoEncoder (CAE), as described in the following section, is then employed to select a representative subset of these ground motions from each of the three ranges for training the LSTM-NARX model.

CAE for Selection of Earthquake Ground motions in LSTM Training

A CAE-based method is used to select appropriate bidirectional ground-motion records for generating training data for the LSTM-NARX model. Results not shown here indicate that using CAE for ground-motion selection enhances the predictive performance of the surrogate model once trained. Fig. 4 provides an overview of the CAE-based framework for ground motion input selection, with detailed explanations provided below.

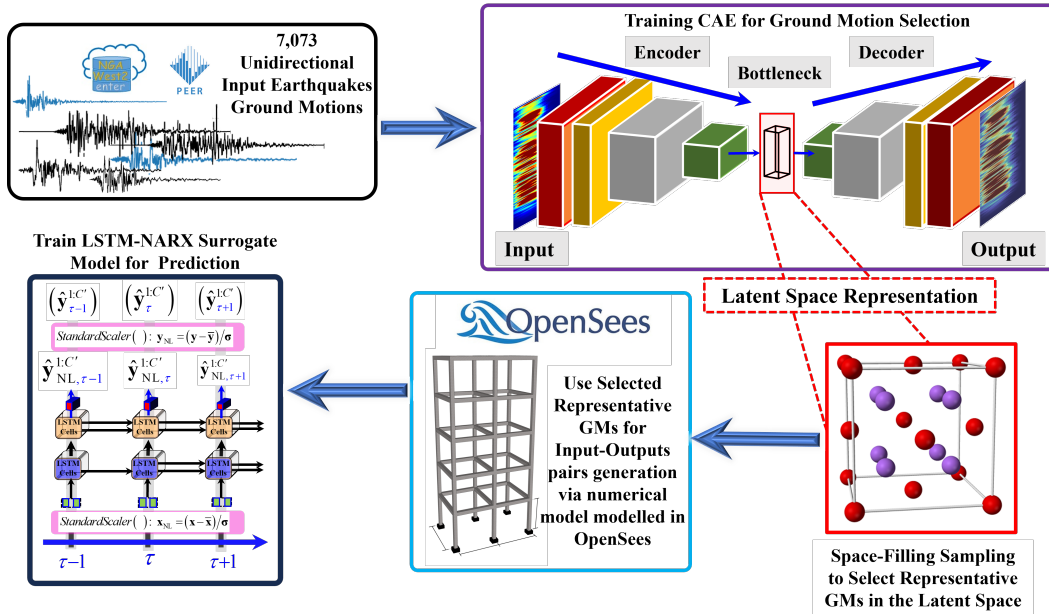


Fig. 4 Overview of the Novel CAE Framework for Ground Motion Selection Using Latent Space for LSTM Surrogate Model Training.

(a) About Convolutional AutoEncoders

Convolutional AutoEncoders (CAE) are unsupervised dimensionality reduction models characterized by convolutional layers, kernel size, and stride sizes adept at extracting salient features from input data [22, 23]. A convolutional layer processes

grid-based input by using adaptive filters, which are parametrized 3×3 or 5×5 matrices, to perform element-wise operations. These filters slide across the input, generating feature maps that capture important patterns and spatial hierarchies, while reducing the dimensionality of the input [24]. The convolution layers are organized in three fundamental components: the encoder, the latent space, and the decoder, resembling the structure of a horizontally oriented hourglass (see Fig. 5). The autoencoder is trained to reconstitute the input grid-like data $\mathbf{H}$ by first mapping $\mathbf{H}$ to a lower-dimensional representation $\mathbf{z}$; known as the latent space or bottleneck, via the encoder, and then using a decoder to restore it to its original form grid-like form $\mathbf{H}'$ [25].

The input data for training the CAE consists of spectrograms generated by applying the Short-Term Fourier Transform (STFT) to 7,073 unidirectional time series from ground motion records. This process yields 7,073 2D grid-like matrices, each containing 1 channel of amplitude data, 251 frequency bins, and 32 time intervals. These 7,073 spectrograms, each of size $1 \times 251 \times 32$, are then fed into the CAE for training, aiming to minimize the loss function in Eq. (7). A single spectrogram of size 251×32 represents 8,032 features (in Eq. (7), $d = 8,032$) that are passed through the encoder for a compressed representation $\mathbf{z}$.

Learning $\mathbf{z}$ involves minimizing a reconstruction loss in Eq. (7), which quantifies the fidelity of the autoencoder in reproducing the original input data $\mathbf{H}$ from its compressed latent representation $\mathbf{z}$, serving as a pivotal metric for optimizing the model via a search for the optimal weights and biases θ_{CAE} during training.

$$L_{CAE}(\theta_{CAE}) = \frac{1}{N_{SIB}} \sum_{s=1}^{N_{SIB}} \|\mathbf{H}_{NL,s} - \mathbf{H}'_{NL,s}(\theta_{CAE})\|_2^2 = \frac{1}{N_{SIB}} \cdot \frac{1}{d} \sum_{s=1}^{N_{SIB}} \sum_{i=1}^d [h_{NL,s,i} - h'_{NL,s,i}(\theta_{CAE})]^2 \quad (7)$$

where $\mathbf{H}'_{NL}$ is the normalized reconstructed output data after passing the normalized input data $\mathbf{H}_{NL}$ through the encoder; thus $\mathbf{H}'_{NL} = Dec_{\theta_{CAE}}(Enc_{\theta_{CAE}}(\mathbf{H}_{NL}))$. In Eq. (7), N_{SIB} is the number of grid-like matrices in a batch, d is the total number of features in $\mathbf{H}_{NL}$, and θ_{CAE} represents the set of CAE model weights and biases to be optimized. Through trial and error, testing the number of latent variables from 6 to 20, a dimension of 10 (i.e., $\mathbf{z}$ is taken as a 10-D vector) yielded the smallest reconstruction error of approximately $L_{CAE}(\theta_{CAE}) = 30$.

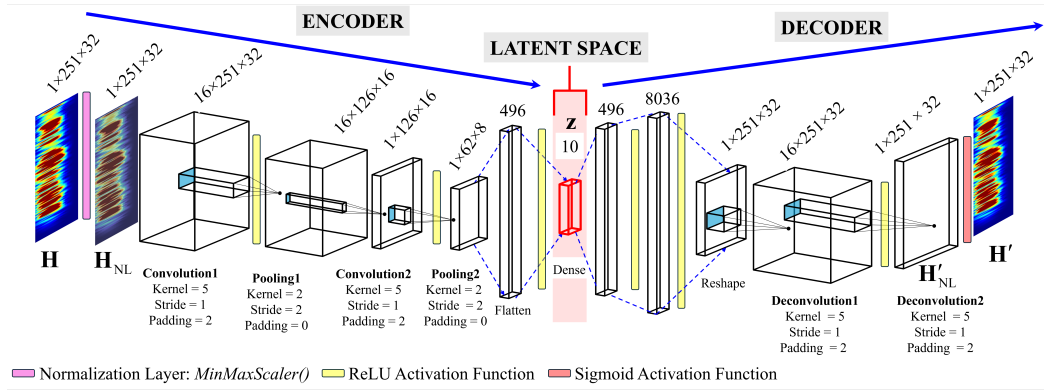


Fig. 5 Convolutional AutoEncoder Architecture.

(b) Selection of Ground Motions in the Latent Space

After training the CAE, we generate uniform-sized spectrogram of $2 \times 251 \times 32$, where "2" corresponds to the North-South and East-West components of the bidirectional EQ ground motions. In the latent space, each spectrogram channel is encoded as a 10-dimensional vector, with both components concatenated into a 20-dimensional representation. This process results in a 519×20 representation of the 519 (unscaled and scaled) earthquake ground motions defined in Section 4.1, followed by "space-filling" to select a representative set of 30 bidirectional earthquake ground motions from each of the 3 ranges specified in Section 4.1, totaling 90 earthquake bidirectional ground-motion records for LSTM model training [26, 27]. Fig. 6 shows the distribution of the original 519 bidirectional earthquake ground motions with S_{a-gm} between 0.1g and 1.2g (red diamonds) and the 90 ground motions selected via the CAE approach using the space-filling algorithm (blue circles).

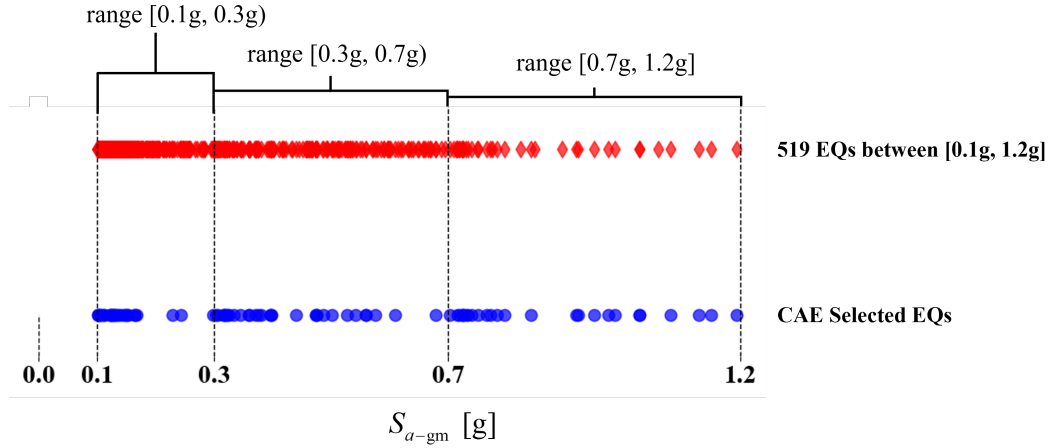


Fig. 6 Distribution of S_{a-gm} for the ensemble of 519 bidirectional earthquake ground motions defined in Section 4.1 (red diamonds) and after selection by CAE (90 ground motions represented by blue circles).

LSTM Surrogate Models And Results

Training of LSTM-NARX Surrogate Models for 3D Reinforced Concrete Frame

Table 1 summarizes the architecture of the LSTM-NARX neural network developed for the virtual yet realistic 3D reinforced concrete frame structure.

Table 1 LSTM-NARX Model Architecture for the 3D 5-story 2-by-1 bay RC frame structure

Layer Name	Number of units	Activation	Trainable Parameters (θ_{NARX})
LSTM Layer 1	100	elu	44,000
LSTM Layer 2	100	elu	80,400
LSTM Layer 3	100	elu	80,400
Fully Connected	1	linear	1,515
		Total	206,315

Ninety (90) input-output pairs are used to train the univariate LSTM-NARX models. The inputs consist of bidirectional earthquake ground motions, while the outputs represent relative displacements at each of the five floors of the 3D structure in both the X and Z directions. These 90 input-outputs pairs generate 192,060 input-output time series sequence samples using $w = 99$. The training process involves 3,000 epochs (the number of times the model iterates through the 192,060 samples), with a batch size of 1,280. Thus, for each epoch, the model processes 151 batches of 1,280 input-output time series sequence samples. The Adam optimizer is employed, with an initial learning rate of 6×10^{-4} , a final learning rate of 1×10^{-4} , and a weight decay of 1×10^{-5} . The mean squared error is used as the loss function, as defined in Eq. (4)a.

Fig. 7 and Fig. 8 show the LSTM-NARX prediction performance for two new, previously unseen bidirectional ground motion records, referred to as testing earthquakes. Both figures provide a detailed breakdown as follows:

- (A) displays the time histories of the bidirectional input ground motion records.
- (B) shows the level of nonlinearity at the structural level (total base shear normalized by the structure weight vs roof drift ratio defined as the fifth-floor relative displacement normalized by the total height of the structure), the section level response (bending moment vs curvature plot), and the stress-strain fiber response at the extreme fibers of the sections. All quantities are obtained from nonlinear time history analysis of the FE model.
- (C) compares the ground truth relative displacement time histories obtained from the FE model with the LSTM-NARX predicted displacement responses in both the X and Z directions across all floors of the 3D structure, under the bidirectional earthquake excitation shown in (A).

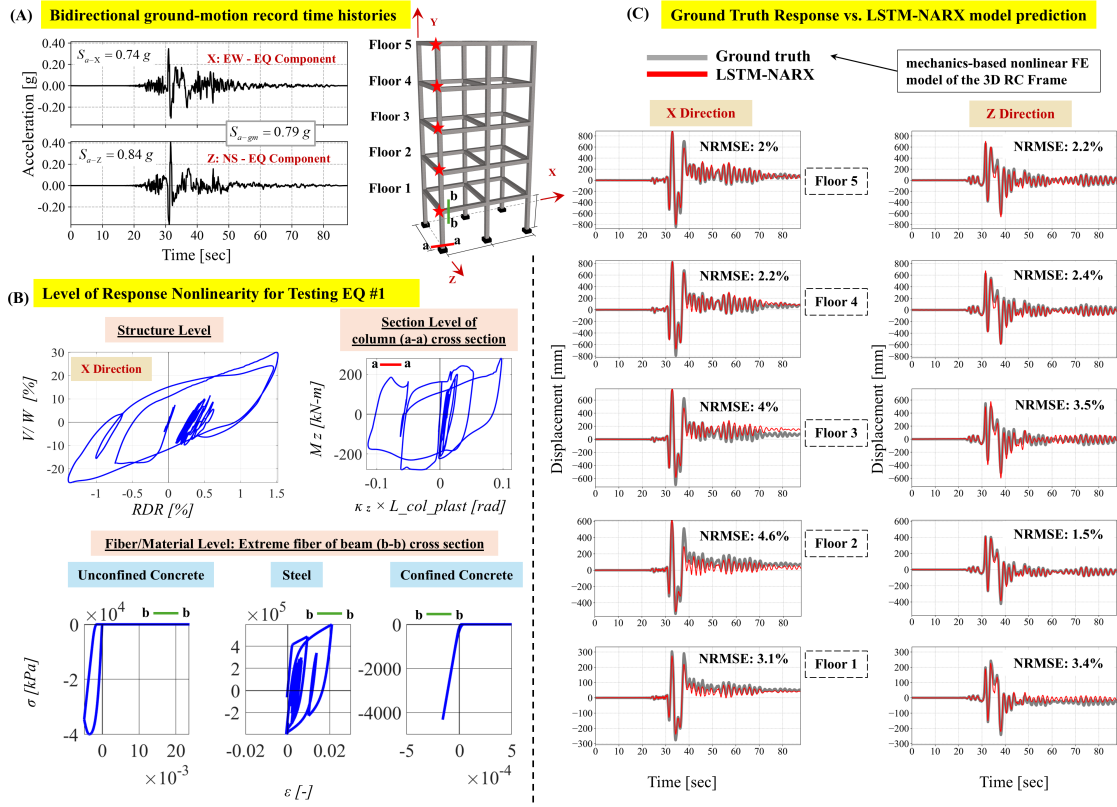


Fig. 7 LSTM-NARX predictions for testing bidirectional earthquake record #1.

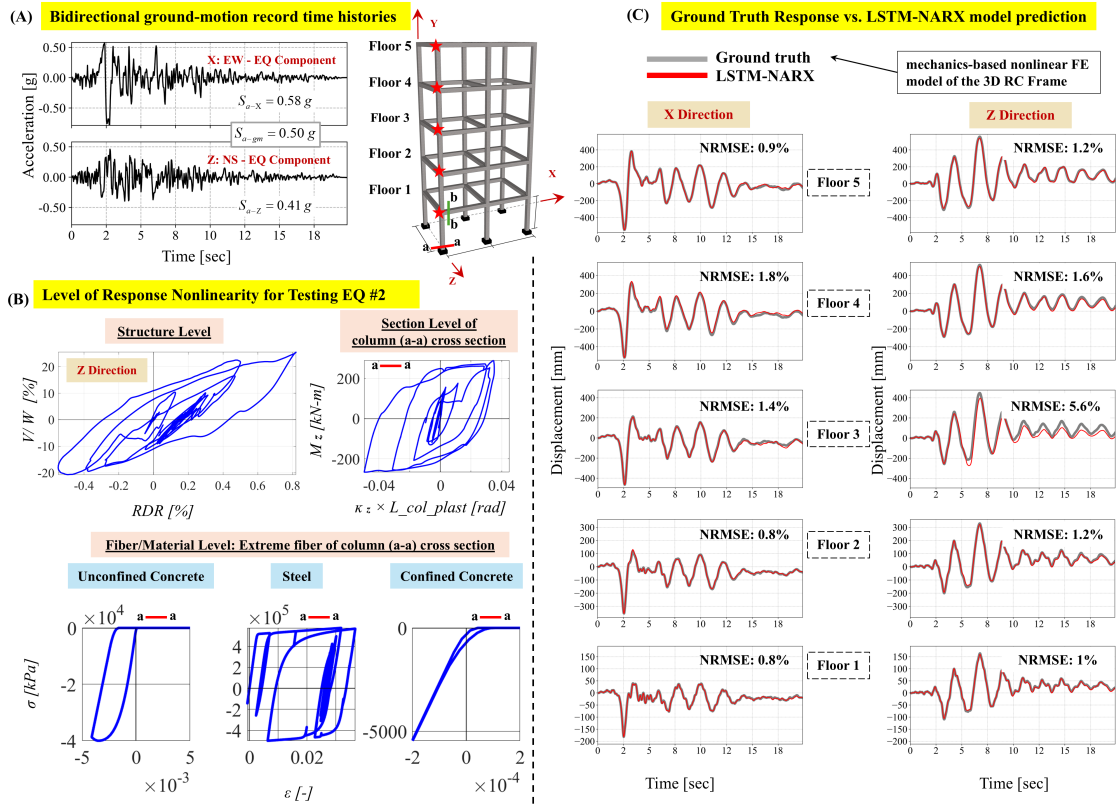


Fig. 8 LSTM-NARX predictions for testing bidirectional earthquake record #2.

Results Discussion

The proposed LSTM-NARX framework provides excellent predictions of the relative displacement response time histories in both the X and Z directions of the 3D reinforced concrete structure. The overall maximum normalized root mean square error (NRMSE) of approximately 5.6% indicates minimal discrepancy between the LSTM-NARX predicted relative displacement time histories and the ground truth values generated using the mechanics-based FE model. This demonstrates strong reliability and predictive capability of the LSTM-NARX model, highlighting the portability and scalability of the framework. The structure exhibits varying levels of nonlinearity when subjected to the testing bidirectional input ground motion records, resulting in moderately nonlinear (Fig. 8) and highly nonlinear (Fig. 7) responses at the structural, section, and material levels. The section-level responses at the base of the column indicate plastic hinge rotations of 0.1 rad and 0.04 rad for the first and second testing examples, respectively. At the fiber/material level at the base of the column, the concrete shows signs of cracking in tension and yielding with some spalling in compression. This condition prompts the steel fibers (reinforcement) to provide post-cracking strength. In general, 3D models better represent real-world structures, enabling a better representation of the structural behavior. During NLTHAs, the nonlinear dynamic response can be substantial, involving significant permanent deformation and pronounced material and geometric nonlinearities. Despite these complexities, the proposed LSTM-NARX model, combined with the CAE ground motion selection process, is able to capture the dynamics of the bidirectional input-output sequences over time via its recurrent connections, thereby preserving the causality inherent in dynamic system behavior modeled across time series.

Long-term time series, such as earthquake ground motions and the corresponding structural responses often display complex variations in period and trend. At the onset of an earthquake, the structure typically remains within the linear elastic range, with its displacement response oscillating around the zero axis. However, once yielding occurs, plastic deformations may result in a permanent shift in the axis of oscillation—commonly referred to as a baseline shift—in the displacement time histories. For instance, the fourth-floor response in the first testing earthquake shows such a baseline shift, with a residual displacement of approximately 100 mm in the X direction. Yet, the LSTM-NARX model, supported by the CAE process, successfully captures this residual drift. The model effectively learns both short- and long-range dependencies within the selected memory length w for output response prediction. The LSTM-NARX architecture enables the model to capture periodic cycles and baseline shifts in long-term trends—80 sec and 18 sec, respectively, for the two testing examples—over extended memory intervals. This improves the model's generalization capability by mitigating overfitting to short-term noise, despite fluctuations in frequency content across different earthquake excitations applied to the surrogate model.

Conclusions

This paper examines the application of the LSTM-NARX (Long Short-Term Memory Nonlinear AutoRegressive model with eXogenous inputs), a type of recurrent neural network (RNN) architecture that combines the strengths of LSTM cells and the NARX approach, to recursively estimate the relative displacement time histories responses of a 3D RC structure in both X and Z direction using external bidirectional input ground motion time histories. The training dataset consists of bidirectional earthquake ground-motion records selected using a convolution autoencoder. These records span a broad range of intensity to generate both linear and nonlinear responses used to train our LSTM-NARX model. After 3,000 epochs, the predicted responses from the LSTM-NARX model, which mimics a traditional time-stepping scheme, closely matched the true responses from unseen bidirectional input motions, with a low relative root-mean-square error. In essence, the LSTM architecture through its recurrent weights mirrors the dynamic response of our application example. The cyclic interplay between its cells and across its layers allows information from previous time steps to shape the current state and output. These recurrent weights, shared across all time steps, allow the LSTM tunable architecture to capture the sequential dynamics accurately over time, resulting in a strong match to the mechanics-based FE model responses.

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Chapter 16

Pre-existing Telecommunication Optical Fiber Cables for Characterizing Bridge Modal Frequency with Temperature Variation

Jatin Aggarwal, Doyun Hwang, Jingxiao Liu, Biondo Biondi, and HaeYoung Noh

Abstract Long-term bridge health monitoring enables early damage detection, helping to prevent sudden failures. Traditionally, this is achieved by installing dedicated sensors such as accelerometers, geophones, strain gauges, and tiltmeters on bridges to collect performance data. However, this approach incurs high installation and maintenance costs and requires sensors to be physically installed on every bridge of interest, making it difficult to scale. To overcome these challenges, we introduce a new long-term bridge health monitoring approach that leverages pre-existing telecommunication optical fiber cables coupled to the bridge for continuous monitoring. These optical fiber cables were originally installed for communication purposes. We turn these cables into virtual strain sensors by sending light pulses into them, which backscatter from naturally occurring impurities based on the principle of DAS. These backscattered light pulses are phase-shifted by the strains in the optical fiber cables induced by bridge vibration. This method significantly reduces costs by using pre-existing infrastructure. However, since these fibers are placed in conduits and measure bridge vibrations indirectly through fiber-bridge interaction, accurately extracting bridge information becomes challenging. To tackle this challenge, we formulate dynamic interaction between bridge, conduit, and fiber systems which enables us to consistently identify bridge frequency under the influence of changing temperatures. Specifically, we show that the DAS response of the fiber can be decoupled into bridge harmonic motion and conduit harmonic motion, resulting in separate peaks in the frequency domain corresponding to bridge modal frequency and conduit modal frequency. Based on this insight, we can track changes in peaks corresponding to the bridge modal frequency due to temperature variations, separately from those corresponding to the conduit modal frequency. To evaluate our approach, we continuously monitored a 28m-steel composite bridge over Sand Hill Road in Palo Alto, CA, over eight months (Dec 2022 to July 2023). Our approach successfully identified the bridge's first three modal frequencies with 0.36%, 0.25%, and 0.13% errors, respectively, as compared to the conventional accelerometers installed on the bridge. Further, we observe a negative correlation between bridge modal frequencies with seasonal temperature variations due to a decrease in elastic modulus of the bridge with an increase in temperature.

Keywords Bridge health monitoring · Distributed acoustic sensing · Modal testing · Natural frequency · System identification

Introduction

Bridges are critical infrastructure for a country's economic development, facilitating connections between people, roadways, businesses, and more. However, continuous deterioration of bridges due to operational use and environmental conditions throughout their service life poses a risk of sudden failure. Such failures can lead to loss of connectivity, increased traffic

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congestion, longer travel times, and business disruptions, along with significant economic burdens from repair costs or the need for new bridge construction. According to the recent ASCE 2021 report card for infrastructure, 7.5% of bridges in the USA (around 46,500) are in poor condition, while 47% (around 291,000) are in fair condition. These bridges require more frequent monitoring compared to the current standard of inspections once every two years [1]. Given these challenges, there is a growing need for a scalable and ubiquitous bridge health monitoring (BHM) technique that can continuously track bridge deterioration across multiple structures in a city. Continuous monitoring enables early damage detection, helping to prevent sudden failures. Traditionally, dedicated sensors such as accelerometers, geophones, strain gauges, tiltmeters, and velocimeters have been installed on bridges to collect performance data [2, 3, 4]. While these sensors provide detailed and continuous data, they are associated with high installation and maintenance costs, as well as traffic interruptions. Furthermore, the physical installation of these sensors on every bridge makes scalability difficult. To improve scalability and cost efficiency, many researchers have explored drive-by vehicle-based sensing methods for bridge health monitoring [5, 6, 7]. While these methods offer the potential to monitor numerous bridges, they do not provide continuous bridge health monitoring data.

To address the need for scalable and continuous bridge health monitoring techniques, we introduce a new long-term bridge health monitoring approach that leverages pre-existing telecommunication optical fiber cables coupled to the bridge for continuous monitoring. Telecommunication optical fibers are typically bundled together and laid underground across modern cities for communication purposes. These bundled optical fiber cables, inserted into conduits and attached to bridges, often include unused cables—referred to as “dark fiber”—that do not carry telecommunication signals. We transform these dark fibers into virtual strain sensors by sending light pulses using the principle of Distributed Acoustic Sensing (DAS). When a coherent laser pulse is injected into one end of the fiber using an instrument called an interrogator, Rayleigh backscattering occurs from the naturally occurring impurities in the optical fiber and are collected back into the interrogator [8]. When a section of the fiber is strained, the optical phase of the backscattered light changes as it returns to the interrogator. This optical phase change is quasi-linearly proportional to the strain in the fiber, allowing the system to function as a virtual strain sensor. Since the speed of light within the optical fiber remains constant, the location of the backscattering along the fiber can be determined based on the time it takes for the light to return to the interrogator. By collecting backscattered light from multiple locations along the fiber, we convert them into an array of virtual strain sensors. A single interrogator can monitor up to 100 kilometers of fiber, with a virtual strain sensor positioned every meter—effectively providing up to 100,000 sensors along the fiber length. This method is highly cost-efficient as it leverages existing optical fiber infrastructure to monitor strain across multiple bridges [9, 10]. However, since the fiber cables housed in the conduits are not directly coupled to the bridges due to the existing telecommunication infrastructure, it becomes challenging to accurately infer bridge-specific information.

To address this challenge, we first formulated the dynamic interaction of the bridge-conduit-fiber system to gain insights into identifying bridge frequencies and characterizing the influence of temperature variations on the estimated bridge parameters. Specifically, we demonstrated that the total strain response of the fiber can be decoupled into two components: bridge harmonic motion and conduit harmonic motion. This decoupling results in distinct peaks in the frequency domain that correspond to the bridge’s modal frequency and the conduit’s modal frequency. With this insight, we are able to track changes in the peaks associated with the bridge’s modal frequencies, even in the presence of temperature variations, separately from those corresponding to the conduit.

To validate our approach, we conducted a field test involving the continuous monitoring of the Sand Hill Bridge in Palo Alto, CA, over an eight-month period from December 2022 to July 2023. Our system successfully identified the first three bridge modal frequencies with errors of 0.36%, 0.25%, and 0.13% respectively, compared to the conventional accelerometers installed on the bridge. Furthermore, we demonstrated the seasonal variation in the bridge modal frequencies and observed that all three modal frequencies exhibited a negative correlation with the average daily temperature, which is due to a decrease in the elastic modulus of the bridge with the increase in the temperature.

Pre-existing Telecommunication Optical Fiber Cables as Distributed Acoustic Sensing Systems

Telecommunication optical fiber cables are often bundled and laid underground across modern cities for communication purposes. These bundled cables are also inserted into conduits and coupled with bridges. However, many of these bundled fibers do not carry telecommunication signals and are referred to as “dark fiber” cables.

We repurpose these dark fiber cables into DAS systems by sending light pulses through them using a device called an interrogator, as illustrated in Figure 1. When a coherent laser pulse is injected into one end of the fiber via the interrogator, Rayleigh backscattering occurs due to naturally occurring impurities in the optical fiber [8]. When a section of the fiber experiences strain, the optical phase of the backscattered light from that section changes. This phase change is quasi-linearly

proportional to the strain in the fiber, thus acting as a virtual strain sensor. The location of the backscattered light can be determined based on its time of arrival back into the interrogator, as the speed of light remains constant within the fiber. By collecting backscattered light from various locations, the system effectively creates an array of virtual strain sensors.

This approach has gained popularity for urban monitoring, such as traffic monitoring [11], as well as for geophysical applications like subsurface characterization [12, 13, 14]. Modern interrogators can sense up to 100 kilometers of fiber, creating a virtual strain sensor every meter. These systems can continuously collect strain data over the years without interfering with regular telecommunication signals and without requiring the on-site maintenance or installation typical of conventional sensors like accelerometers, geophones, or strain gauges on bridges. Furthermore, these telecommunication optical fibers are inserted into conduits that are coupled to bridges. This coupling enables the fibers to capture bridge vibrations [9].

By leveraging the existing infrastructure of telecommunication optical fibers, this sensing method offers a scalable, low-cost solution for continuous bridge health monitoring, allowing multiple bridges to be monitored simultaneously without sensor installation or maintenance on every bridge of interest. However, because the conduits are not directly attached to the bridges, accurately extracting bridge dynamic information becomes challenging under changing environmental conditions, such as temperature fluctuations. To address this, we formulate the dynamic interaction between the bridge, conduit, and fiber, which provides insights into consistent monitoring of bridge frequencies, even in varying temperature conditions.

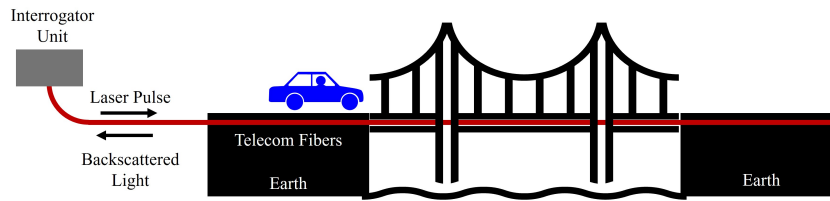


Fig. 1 Representation of pre-existing telecommunication optical fiber cable buried to the ground and coupled to the bridges.

Theoretical Formulation of Dynamic Interaction between Bridge Conduit Fiber System

The fiber conduits are not directly connected to the bridge due to the existing telecommunication infrastructure which makes it hard to infer bridge information consistently. Figure 2 shows examples of pre-existing telecommunication optical fiber cables coupled to the bridge where fiber conduits are coupled to the bridge in several ways. To address this, we formulate the dynamic interaction between the bridge, conduit, and fiber which provides insights into consistent monitoring of bridge frequencies, even in varying temperature conditions. It is shown that the total fiber strain response can be separated into two parts: the harmonic motion of the bridge and the harmonic motion of the conduit. This separation results in distinct peaks in the frequency domain, representing the modal frequencies of the bridge and conduit.

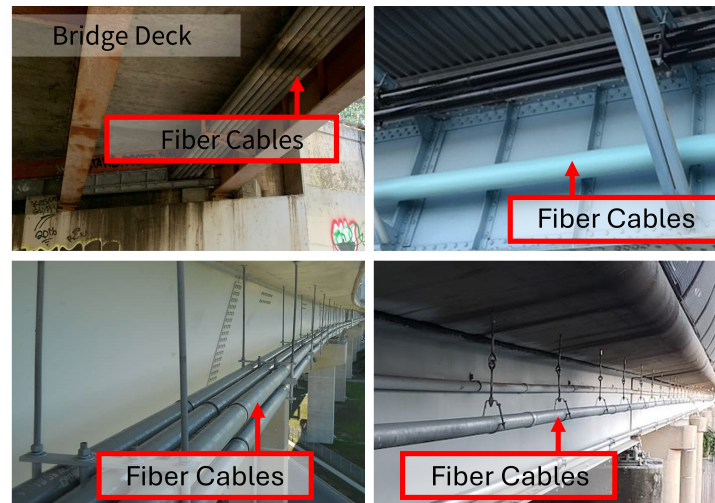


Fig. 2 Examples of pre-existing telecommunication optical fiber cable conduit coupled to the bridges.

Figure 3 shows the pre-installed optical fiber conduit coupled to a simply supported beam of length L with points of attachment at x_A and x_B from one end. To formulate the dynamic interaction between bridge fiber conduit system, We have made some simplified assumptions, as listed below.

- The bridge is assumed to be an Euler beam with a constant cross-section.
- Damping of the beam and conduit is not considered.
- The n^{th} mode shape of the beam is considered to be $\sin(n\pi x/L)$

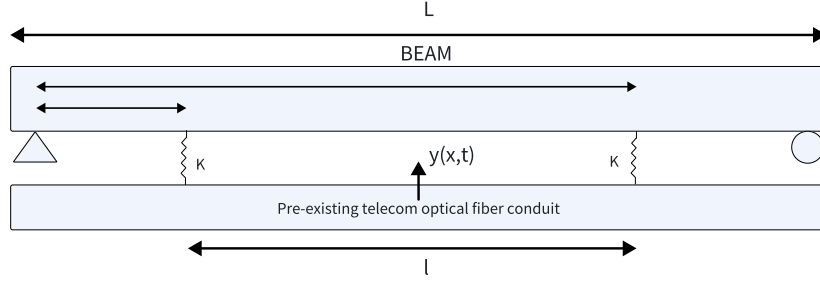


Fig. 3 Pre-existing telecommunication optical fiber cable conduit coupled to the simply supported beam of length L at locations x_A and x_B away from one end of the beam.

By solving the equation of the motion of the beam, the dynamic response of the beam at the location x_A and x_B can be resolved into:

$$u_A(t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{\pi x_A}{L}\right) \sin(w_{bn}t) \quad \text{and} \quad u_B(t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{\pi x_B}{L}\right) \sin(w_{bn}t) \quad (1)$$

where, w_{bn} is the bridge modal frequency for the n^{th} mode and A_n is the amplitude for the n^{th} mode harmonic motion of the beam. Now, the equation of the motion for the conduit can be resolved into:

$$EIy''''(x,t) + \rho A_n \left[\left(\frac{x}{l}\right) \ddot{u}_A(t) + \left(1 - \frac{x}{l}\right) \ddot{u}_B(t) + \ddot{y}(x,t) \right] = f(x,t) \quad (2)$$

where, E , I , ρ , and A are elastic modulus, moment of inertia, density, and area of the fiber conduit. $y(x,t)$ is the vertical displacement at location x at time t , and $f(x,t)$ is the external forcing function applied to the conduit.

$$f(x,t) = K[u_A - y(0,t)]\delta(x) + K[u_B - y(l,t)]\delta(x-l) \quad (3)$$

where K is the connection stiffness between the beam and conduit fiber. $\delta(x)$ is the Dirac delta function. Assuming Y_i is the mode shape for the conduit and G_i is the harmonic function for the conduit for the i_{th} fundamental mode, we can solve the Eq. 2 to obtain:

$$\ddot{G}_i + \omega_{ci} G_i = \frac{1}{l} (\ddot{u}_A - \ddot{u}_B) + \frac{a_1}{a_2} \ddot{u}_B \quad (4)$$

where, $\int_0^l Y_i^2 dx = a_1$ and $\int_0^l x Y_i^2 dx = a_2$. Solving for n_{th} mode of the bridge acceleration using the Duhamel's integral, G is solved to obtain with $A w_{bn}^2 \left(\frac{1}{l} (\sin \pi x_A / L - \sin \pi x_B / L) + \frac{a_1}{a_2} \sin \pi x_B / L \right) = C$

$$G_i = \frac{1}{\omega_{ci}} \int_0^t C \sin(\omega_{bn}\theta) \sin(\omega_{ci}(t-\theta)) d\theta \quad (5)$$

$$G_i = \frac{C}{\omega_{ci}^2 - \omega_{bn}^2} \left(\sin(\omega_{bn}t) - \frac{\omega_{bn}}{\omega_{ci}} \sin(\omega_{ci}t) \right) \quad (6)$$

From the derivation, we draw the following conclusions.

- The DAS response is decoupled into the bridge harmonic motion and conduit harmonic motion.
- The frequency and amplitude of the bridge vibrations in DAS responses, which are separated from conduit vibration, vary with changes in bridge conditions and environmental factors such as temperature.

This formulation provides us insight into the decoupling of the bridge and conduit vibration, which leads to the bridge and conduit having distinct peaks in the frequency domain. Therefore, any shifts in the bridge frequency peak due to temperature are due to changes in the bridge properties.

Furthermore, bridge parameters such as fundamental frequency are estimated from the optical fiber cable strain responses through the Stochastic Subspace Identification Algorithm (SSI-data) [6]. This SSI-data method allows for the identification of system parameters from strain data (output-only) without requiring input data that works for operational system identification. It assumes the input load to be stochastic which is valid for bridges as traffic load. This is because traffic load is highly variable and unpredictable which makes it stochastic in nature. In addition, this method is based on reliable numerical algorithms that involve QR decomposition and SVD to process large data sets and don't need complex optimization techniques.

Field Evaluation: Long-term Evaluation of a Steel Composite Bridge

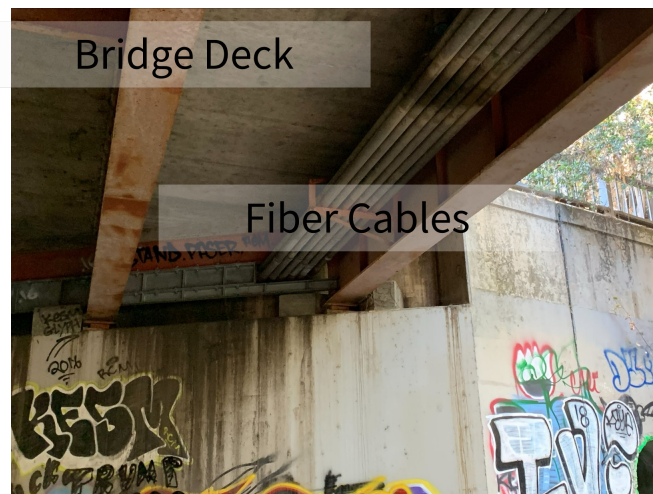
This section presents the field evaluation of our system, conducted through an experiment on a 28-meter steel composite bridge in Palo Alto, CA. We begin by detailing the field setup, followed by a presentation of the results and a discussion of the findings.

Field Setup

The field experiment was conducted on a 28-meter steel composite bridge with a 30° skew angle located on Sand Hill Road in Palo Alto, CA. This bridge supports both roadway traffic and a pedestrian walkway over San Francisquito Creek, as shown in 4a. Telecommunication fiber cables, housed in conduits beneath the bridge deck and coupled to the structure, are depicted in Figure 4b. To monitor the bridge, we utilized an Optasense ODH-3 interrogator installed approximately 4 kilometers away from the bridge. The system operated with a gauge length of 16.3 meters, channel spacing of 8 meters, and a sampling rate of 250 Hz. Additionally, conventional accelerometers (PCB 354C03) were deployed along the bridge for one hour on two separate days to validate the accuracy of our proposed bridge monitoring method. These validation tests were performed on the afternoon of August 11, 2022, and the morning of August 23, 2022. We continuously collected and analyzed data from the telecommunication optical fiber cables for a period of 8 months, from December 2022 to July 2023, to monitor the bridge over time. The SSI-data algorithm was employed to estimate the bridge frequencies, with hourly optical fiber strain data used to derive these estimates.



(a)



(b)

Fig. 4 Experimental setup showing (a) composite steel bridge and (b) bridge deck and optical fiber cables inside the conduit over SanFrancisquito Creek, Palo Alto, CA.

Results and Discussion

This subsection presents the evaluation results and discussion of our system's performance in continuous bridge health monitoring using pre-existing telecommunication optical fiber cables. We first validate the modal parameter estimates derived from the optical fiber cables by comparing them with results obtained from conventional accelerometers. Following this validation, we present the long-term monitoring results, which track the seasonal changes in the bridge's first three modal frequencies. Additionally, we characterize estimated frequencies with varying average daily temperatures.

Table 1 presents a comparison of the modal frequencies estimated from the telecommunication optical fiber cables and conventional accelerometers. For afternoon testing on 11th August, the bridge frequencies were estimated with errors of 0.18%, 0.31%, and 0.14% for the first three modes, respectively. In the morning testing on 23rd August, the errors were 0.54%, 0.19%, and 0.12% for the same modes. These results demonstrate that the telecommunication optical fiber cables accurately capture the bridge frequencies, closely aligning with the measurements from conventional accelerometers.

Table 1 Estimated modal frequency from conventional accelerometer and telecommunication optical fiber cables data collected for two days.

Day	Day 1 (08/11/22)		Day 1 (08/23/22)	
	Acc	Telecom Fiber (Error %)	Acc	Telecom Fiber (Error %)
Modal Parameter	Frequency, Hz			
1 st Mode	4.338	4.33(0.18%)	3.956	3.935(0.54%)
2 nd Mode	6.103	6.122(0.31%)	5.868	5.879(0.19%)
3 rd Mode	7.855	7.866(0.14%)	7.654	7.663(0.12%)

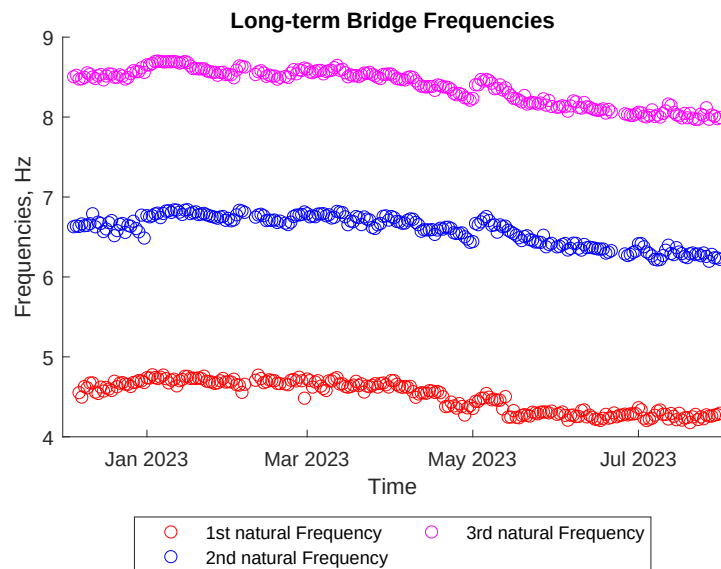


Fig. 5 Scatterplot of estimated bridge modal frequencies for the period of 8 months through telecommunication optical fiber cable strains.

Figure 5 presents a scatter plot showing the variations in the first three fundamental bridge frequencies, estimated daily using telecommunication optical fiber data over an 8-month period (from December 2022 to July 2023). The plot reveals seasonal trends in the frequencies of all three modes, with frequencies increasing during the winter months up until February due to the decrease in temperature, and then decreasing as the temperature rises toward summer. Figure 6 the scatter plot shows the relationship between the estimated bridge frequencies and the average daily temperature, with each point representing a daily estimated frequency. The first three fundamental frequencies display a negative correlation with temperature, with coefficients of determination (r^2) of 0.66, 0.55, and 0.63 for the first, second, and third modes, respectively. The negative correlation arises because as the temperature increases, the elastic modulus of the bridge material tends to decrease, leading to a reduction in the fundamental frequencies.

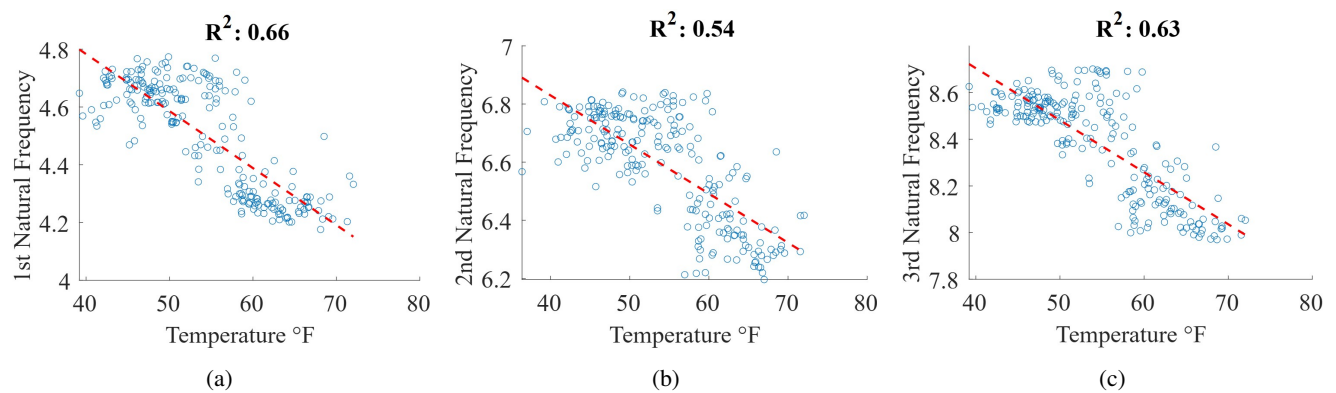


Fig. 6 Scatterplot of the estimated a) first, b) second, and c) third natural frequencies with daily average temperature variation for period of 8 months. The frequencies have a negative correlation with average daily temperature with r^2 as 0.66, 0.55, and 0.63 respectively.

Conclusion

In summary, we introduced a method for continuous bridge monitoring using pre-existing telecommunication optical fiber cables. Due to the pre-existing infrastructure, the conduits carrying the optical fiber cables are not directly attached to the bridges, making it challenging to consistently infer bridge frequencies. To address this, we formulated dynamic interaction of the bridge conduit fiber system which revealed that the total strain in the fiber can be decoupled into components representing the harmonic motion of both the bridge and the conduit. This separation allows us to track changes in the bridge frequencies caused by environmental variations by distinguishing between the frequency peaks from bridge and conduit vibrations. We successfully tested our system in the field by continuously monitoring a 28-meter steel composite bridge in Palo Alto for 8 months. The system captured seasonal changes in the bridge's modal frequencies and found that all frequencies exhibit a negative correlation with temperature, which can be attributed to the decrease in the elastic modulus of the bridge materials as the temperature rises.

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Chapter 17

Body Weight Estimation through Bed Structural Vibrations



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Abstract Accurate body weight estimation is essential for patient care. Weight estimation is needed to inform medical treatments, such as medication dosages, nutrition needs, and the settings of mechanical ventilators. Current weight estimation methods involve using stand-on weight scales, in-bed pressure plates, or visual estimations, which require the patient to leave the bed, are often damage-prone, or are inaccurate. To overcome these challenges, we introduce a novel method for estimating human body weight through the structural vibrations of the bed board, i.e., the plate supporting the bed, excited by a speaker attached to the bed. Our method provides continuous body weight measurement in a non-intrusive way and does not require the patient to leave the bed while measuring the weight. This method is based on the intuition that the patient's weight affects the vibration response of the bed board under excitation. We first analytically formulate the inverse linear relationship between the body weight and the bed board's frequency response based on structural dynamics. Based on this, we extract the frequency spectrum of bed vibrations as a feature to develop a physic-informed machine learning model for body weight estimation. Our method is evaluated on a hospital bed with both stacked weights (simulating human body weight) and a human subject holding different weights of dumbbells. We achieved an average estimation error of 0.68 kg (0.93% error rate) for the human body weight estimation task in a hospital bed. This corresponds to a 36.8% error reduction compared to the baseline method without incorporating the inverse linear relationship.

Keywords Body weight · Structural vibrations · Humans · Hospitals · Body weight · Structure dynamics · Human health

Introduction

Estimating body weight is essential for developing treatment plans in clinical settings. It informs the calculation of medication dosages, the determination of nutritional needs, and the management of fluid levels [1–3]. Inaccurate estimation of body weight can result in overdosing or subtherapeutic dosing, leading to adverse drug reactions or inadequate treatment efficacy [4]. Moreover, it may prompt inappropriate nutritional interventions, potentially resulting in malnutrition and impeding recovery in patients, especially those who are critically ill or undergoing surgical procedures [5]. Additionally, errors in body weight measurement may lead to excessive or insufficient fluid administration, which can cause severe complications such as pulmonary edema, dehydration, and renal failure [3].

Current body weight measurement methods are not suitable for patients with limited mobility in hospital beds due to their critical illness. Limitations include high cost, intrusiveness, or the potential harm to bedridden patients. Standard weight scales, both digital and mechanical, provide straightforward and precise measurements for healthy individuals but are not accessible for patients unable to stand. Specialized devices, such as pressure mats, bed scales, and chair scales, offer alternatives that allow body weight measurement without requiring the patient to stand [6, 7]. However, these methods require the assistance of medical staff to position the patient on the pressure platform, which can pose a risk of injury to

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vulnerable individuals. Additionally, these devices are expensive, with pressure beds, for instance, costing approximately \$3,000 [6, 8].

To overcome these limitations, we develop a novel method for estimating human body weight using speaker-induced bed vibrations. The main idea of this method is that when a person lies on a bed, their body weight affects the vibration response of the bed board. As a result, the vibration response patterns differ depending on the weight applied. By analyzing the vibration responses collected by vibration sensors attached to the bed board, we can estimate the body weight of the person lying on the bed. Compared to other body weight measurement methods, our method is non-intrusive and cost-effective, with minimal risk of interference to patients.

The main research challenge lies in the indirect and complex relationship between bed vibration response and the body weight applied to the bed board. Unlike weight scales and pressure sensors, which measure weight through the invariant relationship between the applied force and internal electronic elements, the relationship between the applied weight and bed vibration is indirect and complex. The main intuition of our method is that the vibration excited by the speakers travels through the bed board whose frequency response is affected by the body weight applied to it and then captured by the vibration sensor attached to the bed. This complex process is influenced by multiple variables, including the bed's material and structural properties, as well as the position of the body and the sensor, complicating the modeling and analysis.

To characterize this indirect and complex relationship, we model and formulate the effect of human body weight on the bed's vibration frequency response using both structural dynamics theories and empirical data. The formulation and analysis indicate an inverse linear relationship between body weight and frequency response, guiding the selection of features and the model for estimating human body weight. Based on this, we develop a physic-informed machine-learning model to realize human body weight estimation using speaker-induced bed vibrations.

To evaluate our method, we conduct two types of experiments using a standard hospital bed: one with stacked steel weights to simulate human body weight and another with a person lying on the bed. The stacked steel weights experiment includes a broader range of weight options, allowing for more varied testing conditions. The human body weight experiment is conducted with a person holding dumbbells of varying weights. Our method achieves a mean absolute error of 0.119 kg (5.3% error rate) for the stacked weights estimation and 0.68 kg (0.93% error rate) for the human body weight estimation.

Our contributions are as follows:

- We develop the first human body weight estimation method based on speaker-induced bed vibrations, enabling a non-intrusive and cost-effective way of measuring body weight, with minimal risk of interference to in-bed patients.
- We model and formulate the effect of human body weight on the bed's frequency response using both structural dynamics theories and empirical data. Guided by this analysis, we develop a physics-informed machine-learning model for estimating human body weight.
- We evaluate our method on a hospital bed using two types of experiments: one with stacked steel weights to simulate human body weight, and another with human subjects. Both experiments yield promising results, showing significant improvements compared to baseline models.

In the remainder of this paper, we first review existing literature on body weight estimation methods and explore approaches that use structural vibrations to estimate loads. Subsequently, we detail the modeling and characterization of the relationship between body weight and bed vibrations. Following this, we present our body weight estimation method. Finally, we discuss the evaluation experiment and analyze the results of body weight estimation.

Related Works

Various methods are developed for measuring human body weight, including stand-on scales, manual observation, and pressure-based bed scales, each with advantages and limitations. The stand-on scale, a traditional and widely used method, utilizes mechanical or electronic components to provide precise weight measurements and additional body composition metrics such as body fat percentage and muscle mass. Despite its affordability and accuracy, this method requires standing, which is quite challenging or even impossible for in-bed patients or patients who may have physical limitations. In clinical environments, professionals often estimate body weight through visual assessment [9, 10]. While convenient, this technique is subjective and can vary significantly based on the observer's experience, often leading to inaccuracies exceeding 10 kg [11]. Pressure-based bed scales embedded in mattresses facilitate passive, continuous monitoring of body weight, offering a non-intrusive option for in-bed patients [12, 13]. However, this method requires medical staff to assist in positioning patients on the pressure bed, which may pose risks to vulnerable patients. In addition, this device can be costly priced at around \$3,000 [8]. In this study, we develop a novel system that measures human body weight through speaker-induced

bed vibrations. Our method is non-intrusive, cost-efficient (with each geophone costing around \$65), with a minimal risk of interference to patients.

Structural vibrations are effective for estimating loads on structures and have been utilized for diverse applications including estimating vehicle weights on roads, detecting structural damage, and measuring animal weights [14–21]. In vehicle weigh-in-motion systems, vibrations induced by a vehicle as it traverses a sensor-equipped road segment are analyzed to determine the vehicle's weight [14–16]. The dynamic response of the pavement to the vehicle's load reflects the vehicle's weight, as different weights induce distinct dynamic interactions between the vehicle and the road surface, thereby generating unique vibration patterns. Structural damages, often simulated by adding weights to a structure, can also be detected through vibrations [17–19]. This is based on the similar impact of weight and damage on the structure's natural frequency, which can be captured through the structural vibrations. Additionally, recent studies have utilized structural vibrations to measure the weight of animals and other objects [20, 21]. These works support that vibrations contain rich information about weight, inspiring further investigation into using bed vibrations to estimate human body weight.

The Relationship between Body Weight and Bed Vibrations

To capture the indirect and complex relationship between human body weight and bed vibrations, we model and formulate the effect of the human body weight on the frequency response of the bed based on structural dynamics theories. The bed board is modeled as a rectangular thin plate with a thickness much smaller than its width and length and the human body is modeled as a static load on the plate (see Figure 1). In this model, we assume that the bed board satisfies the conditions of Kirchhoff–Love plate theory [22]. Both the excitation source and the sensor have areas small enough compared to the bed board to be modeled as points. The bed board is defined as a rectangular area with length a and width b (Without loss of generality, assume a is along the x-axis and b is along the y-axis.). The weight of the human body is simulated by a load m_0 at location (x_0, y_0) , and the vibration sensor is positioned at (x, y) . The excitation source stimulates the plate at the point (x_e, y_e) with the excitation time history function of $f_e(t)$. In addition, for the modeling simplicity, we assume that the bed has simply supported boundary conditions and static initial conditions, and the damping effect is negligible.

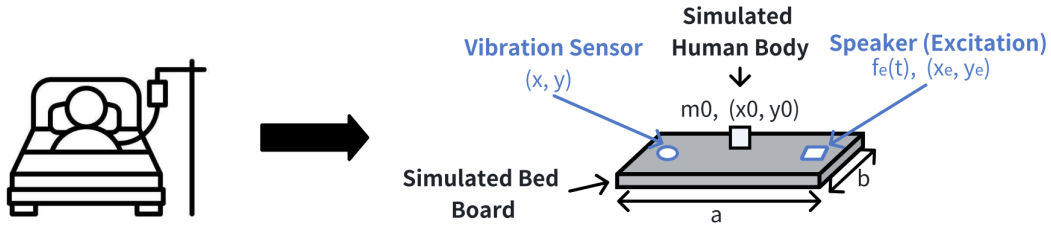


Fig. 1 Simplified Model Representing the Relationship Between Human Body Weight and Bed Vibrations.

We analytically formulate the relationship between the bed board's frequency response and the applied load (human body weight) using structural dynamics theories. The bed board's vibration can be written as Equation 1 [23]:

$$D\nabla^4 w(x, y, t) + \mu \frac{\partial^2 w(x, y, t)}{\partial t^2} = f_e(t)\delta(x - x_e)\delta(y - y_e) + \delta(x - x_0)\delta(y - y_0)\left(\frac{m_0}{S}g - \frac{m_0}{S}\frac{\partial^2 w(x, y, t)}{\partial t^2}\right). \quad (1)$$

In this equation, $w(x, y, t)$ denotes the vertical displacement of the bed board, representing its vibration at time t and location (x, y) (the location of the sensor). The parameters D, ρ, ν correspond to the bed board's flexural rigidity, mass per unit area, and Poisson's ratio, respectively. S denotes the contact area between the load and the bed board. We solve this equation using Galerkin's method [24, 25], which assumes $w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn}(t)\phi_{mn}(x, y)$. With the simply-supported boundary condition and a static initial condition, the basis functions $\phi_{mn}(x, y)$ can be written as $\phi_{mn}(x, y) = \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$. The solution to Equation 1 is detailed in Equations 2, 3, and 4 as follows:

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} q_{mn}(t), \quad (2)$$

where the $q_{mn}(t)$ satisfy:

$$\ddot{q}_{mn}(t) + \omega_{mn}^2 q_{mn}(t) = \frac{f_e(t) \sin \frac{m\pi x_e}{a} \sin \frac{n\pi y_e}{b}}{\frac{ab}{4}\mu + m_0 \sin^2 \frac{m\pi x_0}{a} \sin^2 \frac{n\pi y_0}{b}} \propto \frac{1}{1 + \frac{4m_0}{\mu(ab)} \sin^2 \frac{m\pi x_0}{a} \sin^2 \frac{n\pi y_0}{b}} \sim \frac{1}{m_0} \text{ if } m_0 \gg \mu(ab), \quad (3)$$

where ω_{mn}^2 is defined in:

$$\omega_{mn}^2 = \frac{\frac{ab}{4} D[(\frac{m\pi}{a})^4 + (\frac{n\pi}{b})^4 + 2(\frac{m\pi}{a})^2(\frac{n\pi}{b})^2]}{\frac{ab}{4} \mu + m_0 \sin^2 \frac{m\pi x_0}{a} \sin^2 \frac{n\pi y_0}{b}}. \quad (4)$$

m and n denote the mode numbers of the vibration modes. The corresponding natural frequency for mode m, n is specified in Equation 4. As indicated in Equation 3, the amplitude of vibration for each mode is proportional to $\frac{1}{m_0}$ when $m_0 \gg \mu(ab)$, where $\mu(ab)$ represents the weight of the bed board.

The analysis shows that the frequency spectrum of structural vibrations provides crucial information for estimating human body weight. Moreover, the frequency spectrum is inversely proportional to m_0 when the human body weight significantly exceeds the weight of the bed board. This observation suggests utilizing the frequency spectrum coefficients as features for weight estimation analysis, and $\frac{1}{m_0}$ as the target for linear estimation, which is subsequently converted back to estimate m_0 .

This theoretical relationship is further validated through laboratory experiments on a hospital bed. In the experiment, various weights are placed on the hospital bed, and the bed's structural vibration signals, excited by the speakers, are captured by vibration sensors which are attached to the bed board. First, it is observed that vibration patterns are distinguishable with varying loads as shown in Figure 2 (a) (each line representing one of the repeated experiments). Second, increasing the load by increments of 0.5 kg results in progressively smaller variations in the FFT frequency spectrum as the load increases. This trend is attributed to the frequency spectrum magnitude changing inversely with the mass, as detailed in Equation 3. In Figure 2 (b), the 260 Hz frequency component is used to demonstrate how the changes in the FFT component decrease with increasing weight, following the $\frac{1}{m_0}$ relationship. To show the linearity of this relationship, we plot the reciprocal of the weight against the FFT component and fit a linear curve. The resulting correlation coefficient value of 0.9915 indicates an inverse linear relationship between the weight applied to the bed and the frequency component of the bed vibrations.

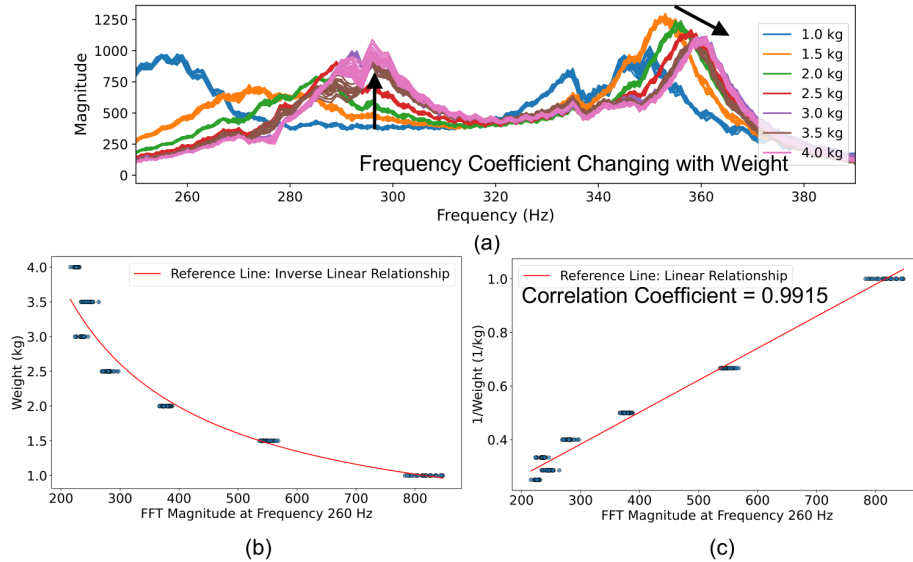


Fig. 2 Characterization of Bed Vibration Frequency Response with Varying Weights on Hospital Bed. (a) FFT spectrum with various weights placed on the bed, indicating various vibration patterns; (b) Scatter plot depicting the relationship between the FFT magnitude at 260 Hz and the corresponding weights; (c) Regression analysis showing the relationship between the FFT magnitude at 260 Hz and the reciprocal of the weight, fitted with a linear model (correlation coefficient: 0.9915)

Body Weight Estimation Using Bed Structural Vibrations

Our method consists of three main steps: bed vibration excitation and sensing, physics-informed bed vibration processing, and body weight estimation modeling (see Figure 3). The main insight of our method is that the bed's structural vibration response varies with different body weights applied on the bed board. By analyzing these vibration responses, captured by sensors attached to the bed board, we can estimate the body weight on the bed structure.

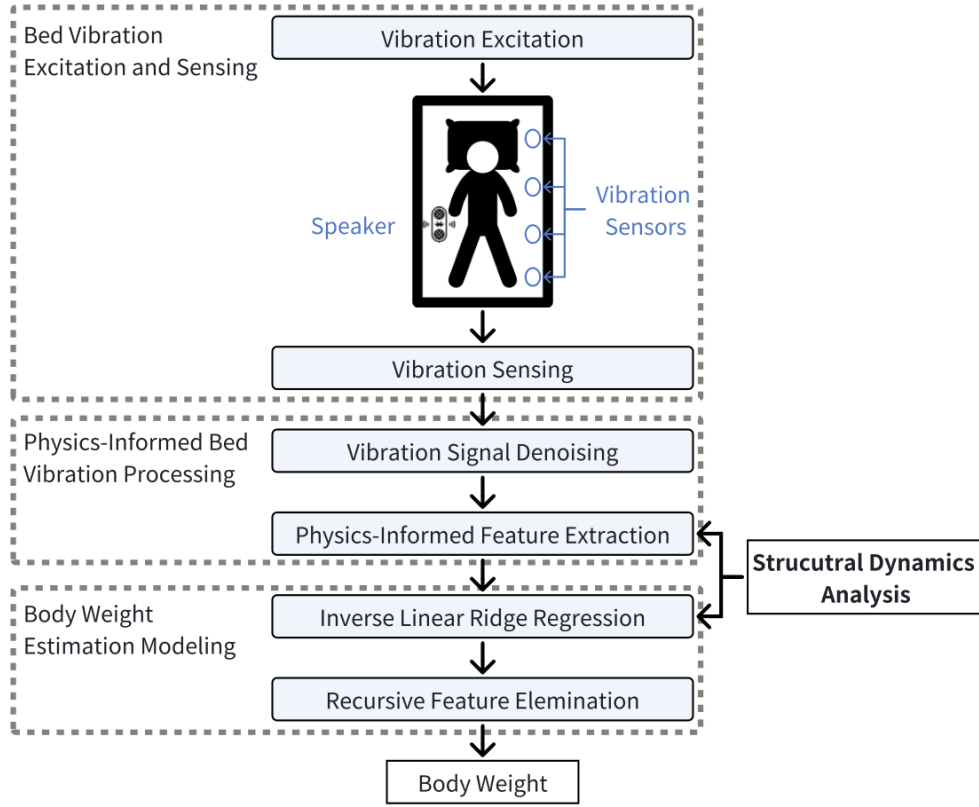


Fig. 3 Bed Vibration-based Body Weight Estimation Method Overview

Bed vibration excitation and sensing

We attach the speaker and vibration sensors to the bed board to excite and collect the bed structural vibrations. The bed vibrations are induced by speakers emitting chirp signals, which traverse the bed board and are then captured by the vibration sensors on the other side of the bed. The vibration response of the bed board is influenced by the human body weight as discussed in the previous section. Thus the signals received by the vibration sensors contain the information of the human body weight applied to the bed structure. The chirp signal is chosen for excitation as it includes a broad frequency range and thus can excite diverse vibration modes. The excitation source and sensors are positioned on opposite sides of the bed to capture structural vibrations to estimate human body weight. A vertical vibration sensor is used because the vertical axis predominantly characterizes bed vibrations, which are significantly greater than those in the horizontal direction [26]. In addition, we avoid the placement of the sensor on the bed board's supporting points to ensure effective measurement of the vibrations [27].

Physics-informed bed vibration processing

The physics-informed bed vibration processing mainly includes two steps: first, we mitigate the impact of noise on the vibration signal through denoising; second, we extract features from the vibration signals, guided by the formulation results derived from structural dynamics theories (as discussed in the previous section).

To reduce noise in vibration signals, we first record vibrations without excitation to establish an environmental noise profile. A threshold in the frequency spectrum is set based on this noise profile, and a gate is employed to attenuate frequencies predominantly associated with noise. For each timeframe of the captured vibrations in the later experiments (the vibrations excited by the speaker), the algorithm masks frequencies below this threshold [28, 29]. This helps in retaining frequencies for the actual vibration signals while reducing those that are likely to be noise.

Guided by the formulation results based on structural dynamics theories, the absolute values of Fourier transform coefficients are extracted as features for inferring human body weight. As discussed in the previous section, these coefficients reflect the load (body weight) applied to the bed structure. A frequency band covering the chirp signal's frequency range is selected, with an expanded scope to cover additional vibrations resulting from the non-ideal performance of the speaker.

The frequency coefficients from all sensors are concatenated as features for body weight estimation modeling to incorporate the weight information from all four plates that comprise the bed board.

Body weight estimation modeling

We develop a physics-informed machine learning model for body weight estimation guided by the analytical formulation results based on structural dynamics theories. Inspired by the inverse linear relationship between frequency components and the body weight formulated in Equation 3, the label for learning is selected as the reciprocal of body weight ($\frac{1}{m_0}$) during training. In addition, we employ a linear regression model to learn the relationship between the reciprocal of body weight and the frequency components features extracted. The linear regression model is selected because it effectively captures this linear relationship by fitting a linear line between the observed frequency components and corresponding weights [30]. Due to the limited data available, we utilize linear regression with ridge regularization to penetrate large coefficients in the model, thereby reducing overfitting [31].

To enhance model performance and efficiency with limited data, we employ Recursive Feature Elimination (RFE) to reduce the feature dimensions [32]. Initially, we train a model using all available features and rank them by importance. The least significant feature is removed, and the model is retrained with the remaining features. This iterative process continues until optimal performance is achieved. RFE is effective in identifying the most important features for body weight estimation. Additionally, it helps mitigate overfitting and reduces data requirements due to the decreased number of features.

Evaluation with Laboratory Bed Experiment

We evaluate our method in a laboratory setting by conducting two types of experiments on a standard steel hospital bed: 1) experiment with stacked steel weight to simulate human body weight; and 2) experiment with a human subject. The first one includes a broader range of weights while the second one is closer to the real clinical settings. This section first outlines the experiment setups, followed by a discussion of the results obtained.

Laboratory experiment setup

The experiments are conducted on a standard hospital bed with a bed board consisting of four plates (see Figure 4). The dimensions of the bed are 2.31 m $\times$ 0.91 m, and the widths of the individual plates are 87 cm, 36 cm, 32 cm, and 33 cm. SM-24 geophone sensors (vertical vibration sensors) are attached to the side of the plate to capture bed vibrations. The signals from the geophones are amplified by the hardware module to enhance the signal-to-noise ratio. Vibrations are excited using a one-second chirp signal emitted from a speaker, with a frequency range from 10 Hz to 1000 Hz to cover the bed's primary natural frequencies. In experiments with stacked steel weights, the sensor and the speaker are located on the same plate, with the speaker positioned at the center of the plate (See Figure 4 (a) (b)). The cylindrical steel weights (steel weights placed in a plastic cylindrical container) weigh 0.5 kg each. They are vertically stacked for varying weight requirements, and placed between the sensor and the speaker. In the experiment, the load varied from 1 kg to 4 kg in 0.5 kg increments. For each weight situation, the chirp signal is repeated 30 times.

To approximate real-world clinical scenarios, we conduct experiments with a student volunteer weighing 71.4 kg, with additional variable weights provided by dumbbells. Due to the larger size of the human body, using a single plate is impractical. Therefore, one sensor is placed on each of the four plates (four sensors in total), and the speaker is attached to one of the plates to induce bed vibrations. Features extracted from the vibrations of the four sensors are concatenated to estimate the human body weight. The following five experimental conditions are included: a human lying on the bed; a human lying on the bed while holding one 3-pound dumbbell; a human lying on the bed while holding two 3-pound dumbbells; a human lying on the bed while holding a 5-kilogram dumbbell; and a human lying on the bed while holding a 7.5-kilogram dumbbell. Each condition is tested with five trials, during which the participant lies supine on the bed. After one experiment round, he stood up and then lay down again for another trial. Each trial included 30 chirp signal excitations (750 samples in total). This experiment was conducted in accordance with the approved IRB protocol (No.: IRB-76030).

Evaluation results

Our method achieves a mean absolute error of (5.3% error rate) when estimating the weight of stacked steel and 0.68 kg (0.93% error rate) when estimating human body weight (see Figure 5). During the training of the linear ridge regression model, we utilize the vibration data from the maximum and minimum weight from the data collected as the training set,

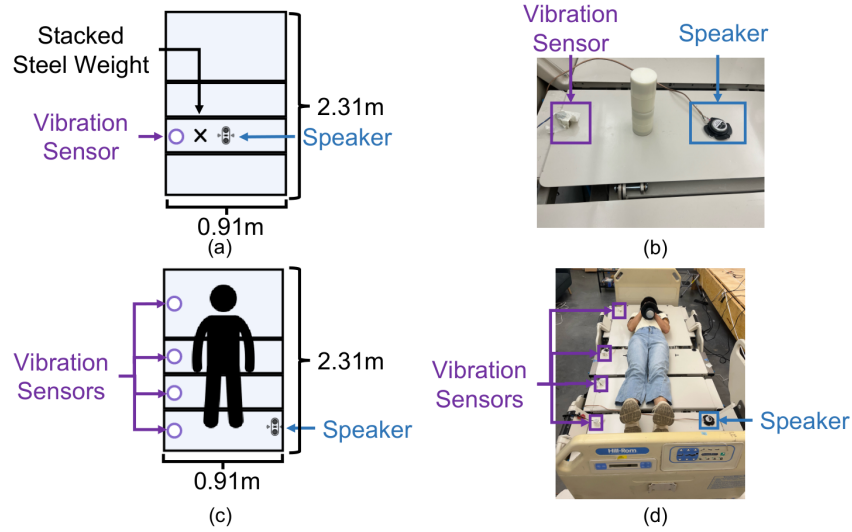


Fig. 4 Laboratory Setup for Estimating Weight through Bed Structural Vibrations: (a) Schematic representation of the experiment setup with stacked steel weights. (b) Laboratory experiment setup with a hospital bed loaded with stacked steel weights (0.5 kg each) placed in a cylindrical container. (c) Schematic representation of the experiment setup for human body weight estimation. (d) Laboratory experiment setup demonstrating the experiment with a human subject on the bed to estimate body weight via bed structural vibrations.

while the weights in between are used as the test set for evaluation. For estimating stacked steel weight, the model is trained using the vibration data corresponding to 1 kg and 4 kg, with the remaining weights forming the test set. In the estimation of human body weight, data for individuals weighing 71.2 kg (without carrying any additional load) and 78.7 kg are used for training, while the other data are used as the basis for testing. The comparison between the estimated and actual weights is shown in Figure 5. Overall, the estimated weights are generally close to the 1:1 line representing the ideal situation of weight estimation. The estimation of human body weight generally exhibits greater errors compared to the estimation of stacked steel weights. This is likely due to the larger contact area of the human body on the bed, which may not be uniformly distributed. The larger contact area may cause deviation from the point load assumption used in structural dynamics theory, potentially causing greater inaccuracies in linear ridge regression methods. Although the estimation of human body weight shows larger errors and greater variation, it remains acceptable for clinical settings.

To prove the effectiveness of regularization and the utilization of the inverse strategy, we compare the weight estimation errors against several common regression models and conduct ablation tests. As illustrated in Figure 6, our method

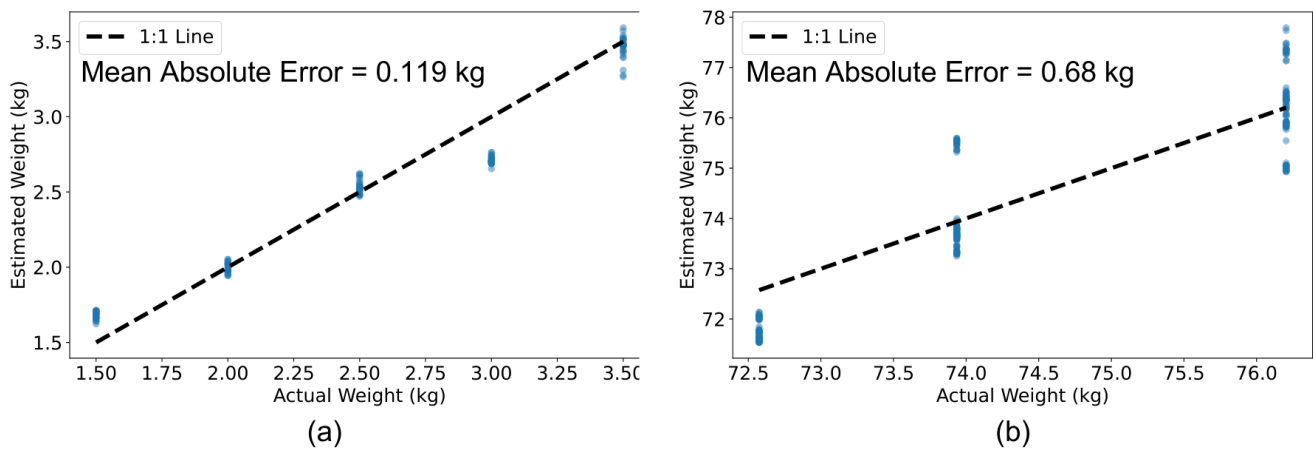


Fig. 5 Evaluation of Weight Estimation Models: (a) Scatter plot comparing the estimated and actual weights of stacked steel weights. (b) Scatter plot comparing the estimated and actual human body weights

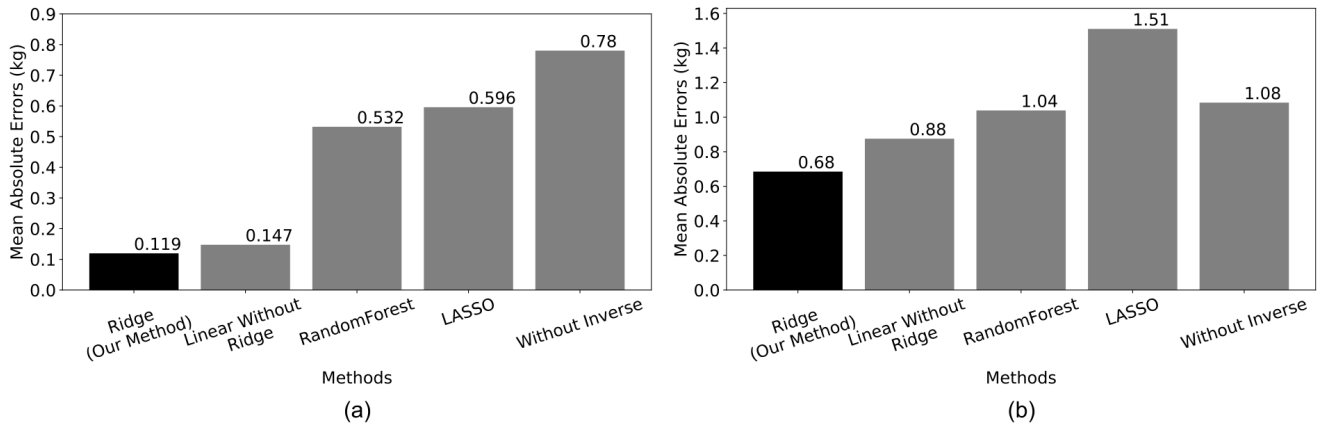


Fig. 6 Comparative Analysis of Regression Models for Weight Estimation. Our method (ridge regression), demonstrates superior performance over linear regression, random forest, and LASSO. Additionally, we evaluate the method that uses direct weight labeling for training ridge regression model (“Without Inverse”s). (a) Mean absolute estimation errors for stacked steel weight estimation. (b) Mean absolute estimation errors for human body weight estimation.

outperforms all other regression models in both the estimation of stacked steel weight and human body weight. Furthermore, in comparison to results obtained by linear regression without ridge regularization, our method achieves significant reductions in estimation error: 19% for the stacked steel weight estimation task and 22% for the human body weight estimation task. Given the linear relationship between the frequency spectrum and the reciprocal of the weight, we employ the inverse strategy which utilizes the reciprocal of the weight as labels during training. Compared to the estimation results which use the weights directly as labels in ridge linear regression, our method shows an error reduction of 84.7% for the stacked steel weight estimation task and 36.8% for the human body weight estimation task.

Conclusions

In summary, we develop a novel method for estimating human body weight using the speaker-excited bed structural vibrations. The main research challenge is the indirect and complex relationship between bed vibrations and body weight. To address this challenge, we model and formulate the inverse linear relationship between human body weight and the bed’s frequency response based on both structural dynamics theories and empirical data. Guided by this analysis, we extract the frequency spectrum as a feature and develop a physics-informed machine-learning model for body weight estimation. We evaluate our method through laboratory experiments using a standard hospital bed. Our results demonstrate a mean absolute error of 0.119 kg (5.3% error rate) when estimating stacked weight and 0.68 kg (0.93% error rate) when estimating human body weight. This body weight estimation method provides a non-intrusive and cost-efficient method with minimal risk of harm for in-bed patients in clinical settings, thereby facilitating the development of more effective treatment plans.

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