Effects of Coil Misalignments on the Magnetic Field and Magnetic Force Components between Circular Filaments

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Abstract

Wireless transfer of electrical energy between air-cored circular coils can be achieved using contactless inductive power transfer (CIPT) system. Notwithstanding, the aim of designing CIPT systems is not always realized because of coil misalignments between the primary and the secondary coils. Based on this information, the author of this paper analyses the effects of lateral and angular misalignments on the magnetic field and the magnetic force between filamentary circular coils which are arbitrarily positioned in space. This investigation is achieved based on advanced and relevant models formulated in the literature. Detailed results obtained using SCILAB application software is given.

Keywords: Circular Filaments, Coil Misalignments, Magnetic Fields, Magnetic Forces.

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1 Introduction

Contactless inductive power transfer (CIPT) systems are a modern technology used for transferring electrical energy over a relatively large air-gap via high frequency magnetic fields. The wireless transfer of electrical energy to the on-board battery storage system of electric vehicles (EVs) is obtained via CIPT transformer which consists of air-cored coils. Air-cored CIPT systems are preferred to an iron-cored type due to the design objectives of controllability. In addition, high power transfer capability may be achieved since the compensation networks of the primary and the secondary coils are not considered independently [1]. However, the purpose of CIPT systems is not always attainable due to lateral and angular misalignment between the primary and the secondary coils.

A number of authors [2–8] have dealt with the computation of magnetic force between coaxial circular coils using analytical methods. However, the current focus is on the computation of magnetic force between circular coils with lateral and angular misalignments [9–16]. With the use of finite element and boundary element methods, magnetic fields and forces can be accurately and rapidly computed [17, 18]. Notwithstanding, the authors in [19] argue that the computation of these important physical quantities can be solved using semi-analytical methods since they considerably reduce the computational time and the enormous mathematical procedures. Also, it is argued that the formulated models obtained using the Lorentz law represent the simplification of the models obtained by the Biot-Savart law and the mutual inductance approach [20]. Finally, it is concluded that the formulated model is easy to understand, numerically suitable and easily applicable for engineers and physicists.

In this paper, the authors investigate the effects of arbitrary lateral and angular misalignment on the magnetic field components which exist when current flows through the primary coil and the magnetic force components exerted on the current carrying conductor. The computations of these important physical quantities between circular filaments arbitrarily positioned in space are achieved based on the advanced and relevant models given in [19]. To achieve this task, this paper is organized as follows. Section 2 presents the advanced and relevant models formulated in the literature. In section 3, results obtained using SCILAB are given. Section 4 discusses the results and section 5 concludes the paper.

2 Formulated Advanced and Relevant Models

This section presents the 3-D space positions of circular filaments with coil misalignments, geometric configurations and common notations for circular filaments and the advanced and relevant models for computing the magnetic field and magnetic force components between circular filaments.

2.1 3-D Space Positions for Circular Filaments with Coil Misalignments [19]

Shown in Figure 1 are the circular filaments with coil misalignments. The secondary side (i.e., the smaller circle) which is placed in an inclined plane $x'Cy'$ is laterally and angularly misaligned whereas the centre of the primary coil is placed at the plane xOy with the axis of z along the axis of the larger circle. The general equation for the smaller circle is given as

$$\lambda = ax + by + cz + D = 0 \quad (1)$$

The centre of the secondary coil is defined in the plane λ , $C (x_C, y_C, z_C)$ and the coordinates of the point D_S which are given as

$$D_1 \left[x_C - \frac{abR_S}{Ll}, y_C + (a^2 + c^2) \frac{R_S}{Ll}, z_C - \frac{bcR_S}{Ll} \right]$$

or

$$D_2 \left[x_C + \frac{abR_S}{Ll}, y_C - (a^2 + c^2) \frac{R_S}{Ll}, z_C + \frac{bcR_S}{Ll} \right]$$

where

$$L = (a^2 + b^2 + c^2)^{0.5}, l = (a^2 + c^2)^{0.5} [10].$$

The following are the 3-D space positions required for the computation of the magnetic field and the magnetic force between circular filaments with lateral and angular misalignments [19]:

- The primary coil of radius R_P is placed in the plane xOy ($z = 0$) with the centre at O (0, 0, 0). An arbitrary point $B_P(x_P, y_P, z_P)$ of this coil has parametric coordinates which are given as

$$\begin{aligned} x_P &= R_P \cos t \\ y_P &= R_P \sin t \\ z_P &= 0 \end{aligned} \quad t \in (0, 2\pi) \quad (2)$$

- The differential element of the primary coil is given by

$$d\vec{l}_P = R_P \left(-\vec{i} \sin(t) + \vec{j} \cos(t) \right) dt \quad t \in (0; 2\pi) \quad (3)$$

- The unit vector N (i.e., the unit vector of the axis z') at the point C (centre of the secondary coil) laying in the plane λ , is defined by

$$\vec{N} = \{n_x, n_y, n_z\} = \left\{ \frac{a}{|\vec{n}|}, \frac{b}{|\vec{n}|}, \frac{c}{|\vec{n}|} \right\} \quad (4)$$

$$|\vec{n}| = (a^2 + b^2 + c^2)^{0.5} = L$$

- The unit vector between two points C and D_S placed in the plane λ is

$$\vec{u} = \{u_x, u_y, u_z\} = \left\{ -\frac{ab}{lL}, \frac{l}{L}, -\frac{bc}{lL} \right\} \quad (5)$$

$$L = (a^2 + b^2 + c^2)^{0.5}, \quad l = (a^2 + c^2)^{0.5}$$

- The unit vector v is defined as the cross product of the unit vectors N and u as follows

$$v = \vec{N} \times \vec{u} = \{v_x, v_y, v_z\} = \left\{ -\frac{c}{l}, 0, \frac{a}{l} \right\} \quad (6)$$

$$l = (a^2 + c^2)^{0.5}$$

- An arbitrary point $E_S (x_S, y_S, z_S)$ of the secondary coil has parametric coordinates which is a well-known parametric equation of circle in 3-D space and is given as

$$\begin{aligned} x_S &= x_C + R_S u_x \cos \phi + R_S v_x \sin \phi \\ y_S &= y_C + R_S u_y \cos \phi + R_S v_y \sin \phi \quad \phi \in (0, 2\pi) \\ z_S &= z_C + R_S u_z \cos \phi + R_S v_z \sin \phi \end{aligned} \quad (7)$$

- The differential element of the secondary coil is also given by

$$d\vec{l}_S = R_S \left\{ l_{Sx} \vec{i} + l_{Sy} \vec{j} + l_{Sz} \vec{k} \right\} d\phi \quad \phi \in (0, 2\pi) \quad (8)$$

where

$$\begin{aligned} l_{Sx} &= -u_x \sin \phi + v_x \cos \phi \\ l_{Sy} &= -u_y \sin \phi + v_y \cos \phi \\ l_{Sz} &= -u_z \sin \phi + v_z \cos \phi \end{aligned}$$

2.2 Geometric Configurations and Common Notations [10]

The geometric configurations for circular filaments with air-core are given in Figures 2 – 4.

Figure 1 Circular filaments with lateral and angular misalignment [10]

Figure 2 Circular filaments with lateral misalignment only [10]

Figure 3 Circular filaments with lateral and angular misalignment [10]

Figure 4 Circular filaments with arbitrary lateral and angular misalignment [10]

Regarding Figure 2, $\theta = 0$ and $\phi = 0$ at axes $y-z$ and $y'-z'$ coplanar. d is the horizontal distance between their centres, h is the vertical distance between their centres, ρ is the lateral misalignment (i.e., distance between their centres), ϕ is the variable rotation angle at any point of the secondary coil. Also, in Figure 3, $\phi = 0$ since axes $y-z$ and $y''-z''$ are coplanar. θ is the angle of inclination between circular coils. Finally, regarding Figure 4, there are no coplanar axes. Therefore, to achieve the aim of this paper, Figure 4 is considered amongst other geometric configurations.

The following common notations are introduced in [10] to make easier link with [21]:

- The centre of the secondary coil must be taken at point

$$C (x_C = 0, y_C = d, z_C = h)$$

where

$$\begin{aligned} d &= x_2 \sin \theta \\ h &= x_1 - x_2 \cos \theta \\ \rho &= \sqrt{d^2 + h^2} \\ \cos \phi &= h/\rho \end{aligned} \tag{9}$$

x_1 is the vertical distance when axis z'' intersects with axis z from the origin O and x_2 is the distance between the intersection and the centre of the secondary coil

- The equivalence between Grover's latitude and longitude angles θ and ϕ and the a , b and c parameters defining the secondary coil plane is that of a spherical Cartesian system of coordinates [10, 21] which is

$$\begin{aligned} a &= \sin \phi \sin \theta \\ b &= -\cos \phi \sin \theta \\ c &= \cos \theta \end{aligned} \tag{10}$$

To investigate the effects of lateral and angular misalignment between circular filaments that are arbitrarily positioned in space, the authors of this paper re-stated equations (5–8) in terms of the common notations given in equations (9–10).

2.3 Models for Computing Magnetic Field and Magnetic Force Components

The advanced and relevant mathematical models for computing the magnetic field and the magnetic force components between circular filaments with lateral and angular misalignment are given in [19] as follows:

- The final form of the magnetic field in an arbitrary point $D_S(x_S, y_S, z_S)$ produced by the primary coil of the radius R_P carrying current I_P is

$$\begin{aligned} B_x(x_S, y_S, z_S) &= -\frac{\mu_0 I_P z_S x_S k}{8\pi\sqrt{R_P}(x_S^2 + y_S^2)^{5/4}} L_0 \\ B_y(x_S, y_S, z_S) &= -\frac{\mu_0 I_P z_S y_S k}{8\pi\sqrt{R_P}(x_S^2 + y_S^2)^{5/4}} L_0 \\ B_z(x_S, y_S, z_S) &= \frac{\mu_0 I_P k}{8\pi\sqrt{R_P}(x_S^2 + y_S^2)^{3/4}} S_0 \end{aligned} \quad (11)$$

- The final form of the magnetic force components between circular filaments is also given as

$$\begin{aligned} F_x &= \frac{\mu_0 I_P I_S R_S}{8\pi\sqrt{R_P}} \int_0^{2\pi} I_x d\phi \\ F_y &= \frac{\mu_0 I_P I_S R_S}{8\pi\sqrt{R_P}} \int_0^{2\pi} I_y d\phi \\ F_z &= \frac{\mu_0 I_P I_S R_S}{8\pi\sqrt{R_P}} \int_0^{2\pi} I_z d\phi \end{aligned} \quad (12)$$

where

$$\begin{aligned} I_x &= \frac{k}{(x_S^2 + y_S^2)^{5/4}} \left[z_S y_S l_{S_z} L_0 + \sqrt{x_S^2 + y_S^2} S_0 l_{S_y} \right] \\ I_y &= -\frac{k}{(x_S^2 + y_S^2)^{5/4}} \left[z_S y_S l_{S_z} L_0 + \sqrt{x_S^2 + y_S^2} S_0 l_{S_x} \right] \\ I_z &= \frac{k}{(x_S^2 + y_S^2)^{5/4}} z_S [x_S l_{S_y} - y_S l_{S_x}] L_0 \\ k^2 &= \frac{4R_P \sqrt{x_S^2 + y_S^2}}{(R_P + \sqrt{x_S^2 + y_S^2})^2 + z_S^2} L_0 = 2K(k) - \frac{2 - k^2}{1 - k^2} E(k) \end{aligned}$$

$$S_0 = 2\sqrt{x_S^2 + y_S^2} K(k) - \frac{2\sqrt{x_S^2 + y_S^2} - \left(R_P + \sqrt{x_S^2 + y_S^2}\right) k^2}{1 - k^2} E(k)$$

$K(k)$ and $E(k)$ are the complete integral of the first and second kind [22, 23].

3 Results Obtained

Shown in Figures 5–12 are the results obtained based on Eq. 11 and Eq. 12. These simulations are achieved using SCILAB application software [24] and the data used is shown in Table 1.

4 Discussion of Results

The effects of coil misalignments such as the lateral ρ and angular θ on the magnetic field and the magnetic force components between circular filaments arbitrarily positioned in space are shown in Figures 5–12.

The computation of the results obtained is achieved by computing equations (11) and (12) with respect to the variable rotation angle ϕ at any point of the secondary coil. Shown in Figures 5–12 are the results obtained for the magnetic field B_x , B_y and B_z components which exist when current flows through the primary coil and the magnetic force F_x , F_y and F_z components exerted on the current carrying conductor. Based on the data given in Table 1, it is clearly seen that the values of B_x , B_y , B_z and F_x , F_y , F_z components decrease and increase at certain variable rotation angles at any point of the secondary coil.

Table 1 Data used for Simulation [10]

Variable Parameters			
$x_1(\text{m})$	$x_2(\text{m})$	$\rho(\text{m})$	$\theta(\text{degree})$
0.05	0.0125	0.04	30
0.10	0.0250	0.08	45
0.15	0.0375	0.14	60
0.20	0.0500	0.19	75
Constant Parameters			
R_P	0.16m		
R_S	0.10m		
μ_0	$4\pi \times 10^{-7} \text{ H/m}$

Figure 5 Magnetic field components when $\rho = 0.04m$, $\theta = 30^\circ$

Figure 6 Magnetic force components when $\rho = 0.04m$, $\theta = 30^\circ$

Figure 7 Magnetic field components when $\rho = 0.08m$, $\theta = 45^\circ$

Figure 8 Magnetic force components when $\rho = 0.08m$, $\theta = 45^\circ$

Figure 9 Magnetic field components when $\rho = 0.14m$, $\theta = 60^\circ$

Figure 10 Magnetic force components when $\rho = 0.14m$, $\theta = 60^\circ$

Figure 11 Magnetic field components when $\rho = 0.19m$, $\theta = 75^\circ$

Figure 12 Magnetic force components when $\rho = 0.19m$, $\theta = 75^\circ$

This analysis shows that in order to obtain the full potential of CIPT systems, the challenges encountered due to arbitrary lateral and angular misalignments between filamentary circular coils must be considered in the CIPT transformer model to be designed for the CIPT system.

5 Conclusion

Unlike plug-in connectors contactless inductive power transfer (CIPT) system is a modern technology which enables electrical energy to be transferred over a relatively large air-gap via high frequency magnetic fields. With the CIPT transformer, electrical energy can be transferred from the primary to the secondary air-cored circular coils. Notwithstanding, its full advantage is not always achievable due to arbitrary lateral and angular misalignments between its coils. Based on this information, this paper examined the consequences of these coil misalignments on the magnetic field and the magnetic force components between circular filaments which are arbitrarily positioned in space.

The computations are based on the advanced and relevant models formulated in the literature. The results obtained using SCILAB application software show that as the lateral and angular misalignment increase the magnetic field and magnetic force components between the circular filaments decrease and increase at certain variable rotation angles at any point of the secondary coil.

This study shows that the issues regarding arbitrary lateral and angular misalignment between circular filaments must be tackled by considering these misalignments in the model developed for the CIPT transformer.

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